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A novel approach for characterizing and enforcing stability of barycentric rational models in the AAA algorithm

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Rational approximation algorithms based on barycentric model structures are some of the methods of choice for learning Reduced-Order Models (ROMs) of large-scale Linear Time-Invariant (LTI) systems in non-intrusive, data-driven settings. In this work, we address the problem of characterizing and enforcing the asymptotic stability of a ROM with transfer function $\hat{H}(s)$ in barycentric form

$$\hat{H}(s) = \frac{N(s)}{D(s)} \in \mathbb{C}, \quad N(s) = \sum_{i=1}^{k} \frac{h_i w_i}{s - q_i}, \quad D(s) = \sum_{i=1}^{k} \frac{w_i}{s - q_i}, \quad D^*(s) = D(s^*), \quad w_i \neq 0 \; \forall i. \tag{1}$$

In the above, q_i, h_i are assumed to be fixed quantities, while the coefficients w_i (the so-called barycentric weights) are model unknowns, to be optimized so that $\hat{H}(s)$ matches samples of the underlying full-order system transfer function, according to a prescribed accuracy criterion, i.e., as in [3]. Our first result is deriving a set of novel algebraic conditions on the barycentric weights w_i's that characterize the stability of $\hat{H}(s)$. This characterization is obtained by proving that, under mild conditions, a ROM with transfer function $\hat{H}(s)$ is asymptotically stable if and only if the denominator $D(s)$ is an *Almost Strictly Positive-Real* (ASPR) transfer function [1], i.e., a transfer function that fulfills the requirement

$$\exists g \in \mathbb{R} : G(s) = \frac{D(s)}{1 + gD(s)} \quad \text{is Strictly Positive Real,} \tag{2}$$

see [2] for further details and proofs. Exploiting this fact, and defining a suitable state-space realization for $D(s)$, requirement (2) is translated into a set of non-convex algebraic constraints involving the unknowns w_i by means of the Positive Real Lemma [1], which provides the proposed characterization.

Our second contribution illustrates how to proficiently exploit the above results for performing reduced-order modeling with guaranteed stable ROMs. We present a constrained version of the AAA (Adaptive Antoulas-Anderson) algorithm [3], in which the unknowns optimization is forced to return solutions that allow the verification of (2), and thus generate stable ROMs. Applying an ad-hoc relaxation strategy, we show that the involved constrained optimization problem can be solved via semidefinite programming, by suitably weighting the linearized error function minimized by the standard AAA iteration. Furthermore, we propose an efficient extension of the resulting algorithm to the Multi-Input-Multi-Output (MIMO) case, and we test experimentally the efficiency and the reliability of the approach, by tackling model order reduction problems arising from different physical domains. Finally, we compare its performance with those of other state-of-the-art methods that use different types of strategies to enforce the stability of rational barycentric models.

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