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Development of a Bio-Mathematical Crew Fatigue Model for Business Aviation Operators.

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Against the backdrop of the current aviation accident trend which poses human factor as their main cause, pilot fatigue is a pivotal issue which may jeopardize mission safety. At current times, the development of a Fatigue Risk Management System (FRMS) for Business Aviation Operators (BAOs) is not required by EASA, albeit already mandatory according to UK and FAA regulations. The aim of this paper is to present a Bio-Mathematical crew fatigue model which can assist BAOs in the creation of a proper FRMS, based on objective science-based biological models. The already challenging conditions linked to the evaluation of fatigue are compounded by the conditions which BAOs are affected by: 24-hour-a-day activities, night flights, irregular and unpredictable flight schedules, extended wakefulness and changes of time zone. Factors such as these challenge human physiology and can lead to performance-impairing fatigue and increased risks to safety. The lack of instrumental examination providing an objective value of fatigue status makes it difficult to get a complete picture of the crew's mental and physical state. This paper describes the main underlying elements employed in the model creation and how the model has been adapted for BAOs specific requirements. The model is backed up by simple however effective algebraic relationships which take into consideration several influences leveraging pre-existing scales: prediction of alertness, cumulative fatigue duty time and sub-standard sleep quality.

Keywords: Fatigue, Business Aviation Operators, Bio-Mathematical, FRMS, LOSA

1. Introduction

The interrelation between human factors (HF), fatigue, Crew Resource Management (CRM), pilots and operators is an issue of central importance to guarantee the highest safety standards in aviation operations. As the vast and important role of technology in every field of engineering continues to gain more traction, improving the software and hardware performance and capabilities, human physiology remains unchanged. In fact, humans are left to handle increasingly higher workloads, different and multifaceted tasks in a disruptive and safety-critical environment, such as the one pilots are so accustomed to. Nowadays, most would assert that HF-related causes make up, at least in part, for a staggering 70-80 per cent of the whole causes of aviation accidents, while only the remaining 20 per cent is related to other reasons (e.g. equipment malfunctions, mechanical problems, etc.) Shappell and Wiegmann (1996).

Moreover, if the incidence of aircraft accidents caused simply by technical malfunction has reduced significantly over the last 40 years, those caused in part by human mistakes have declined at a considerably slower rate. More advanced studies Shappell and Wiegmann (2003) analyzed that ”80 per cent” more in depth, finding that the underlying cause of human factor errors is mainly linked to skill-based errors, according to the HFACS taxonomy Shappell and Wiegmann (2000). The consequence is clear: something still has to be done Wiegmann and Shappell (2001). The same consideration can be made specifically for fatigue, which is generally considered as part of the bigger human factor category. Definitions of fatigue vary depending on the operational field. The ubiquitous definition from ICAO, which highlights the multifaceted nature of this issue, states that fatigue is *“a physiological state of reduced mental or physical performance capability resulting from sleep loss or extended wakefulness, circadian phase, or workload (mental and/or physical activity) that can impair a crew member’s alertness and ability to safely operate an aircraft or perform safety-related duties.”*ICAO (2015) The problem is definitely widespread and resounding news are appearing also on general purpose newspaper, placing the painstakingly achieved reputation of the aviation sector as the safest one at sake [a] Fatigue is a subtle enemy which the aviation sector has to fight since it poses as one of the most significant threats to air safety. This paper introduces the reasons why fatigue should be even more central thanks to several references to surveys and previous study. After a brief overview on the regulatory framework, the proposed model is presented, along with the relative hypotheses and limitations.

[a] https://abc7chicago.com/pilots-fall-asleep-pilot-during-flight-ita-airways-sleeping/11915028/, https://edition.cnn.com/travel/article/pilots-reported-to-fall-asleep-ethiopian-airlines/index.html

2. Why Do We Have To Care About Fatigue?

2.1. *Fatigue in numbers*

There are countless published studies involving statistical data and surveys which highlight the central importance of fatigue for mission safety and conclude that solving this issue is a pivotal part of that complex system of physical and normative barriers which every day guarantee the safety of flights. Fatigue management is essential to guarantee flight safety and neglecting it can lead to hazardous conditions: for instance some estimations place fatigue as the probable cause of 21–23 per cent of major aviation accidents investigations Caldwell (2005); Marcus and Rosekind (2017); Gaines et al. (2020).

Moreover, a non-peer-reviewed survey Singh (2023) carried out by the indian-based non governmental organization ”Safety Matters Foundation”, which sparked outrage in the general public in 2022, shows that, among the more than 500 interviewed pilots, more than 54 per cent of them suffer from severe excessive daytime sleepiness (following the Epworth Sleepiness Scale Johns (1991)). Furthermore, 66 per cent respondents admitted that they have fallen asleep without consent of the other pilot.

2.2. *The problem is more extended than we think*

Contrarily to what one may think, fatigue affects pilot flying long routes, ultra long routes Berg et al. (2020), but also short-haul flights Honn et al. (2016); Flynn-Evans et al. (2018) alike even if for different reasons. If long and extra long flights affect pilots for the extended wakefulness and attention required, short flights are usually linked with more hectic scheduling, fostering and hours of wakefulness. Unfortunately, BAO pilots are affected by both flight types and hence they suffer problems specific to all flight duration. Moreover, one should not think that fatigue is only related to flight operations carried out by pilots or crew. In fact, the issues related to fatigue can be traced back to higher levels in an airline or airport organization. For instance, the authors in Taneja (2007)

and Berg et al. (2020), highlight how little information on fatigue is given to military and commercial airline pilot, thus leading to dangerous and hazardous behaviours. On top of that, not only is fatigue considered a very central issue for what concerns safety and performance, but fatigue is often related to serious illness and negative long-term effects, such as obesity Goel et al. (2013), cardiovascular problems Goffeng et al. (2019), mental health Pellegrino and Marqueze (2019).

3. FRMS, Fatigue Assessment Methods and Fatigue Models

In Annex 6— Operation of Aircraft, Part I—International Commercial Air Transport—Aeroplanes ICAO requires member states to put in place regulations for managing fatigue rooted to scientific principles, based on scientifically-based prescriptive regulations or FRMS. FRMSs lay their roots on the methodical identification of work-related fatigue risks and they are composed of predictive (adaptive work schedules), proactive (monitoring personnel fatigue in real time), or reactive (identifying fatigue to prevent hazardous events) procedures Sprajcer et al. (2022). ICAO defines FRMS as "a data-driven means of continuously monitoring and managing fatigue related safety risks, based upon scientific principles and knowledge as well as operational experience that aims to ensure relevant personnel are performing at adequate levels of alertness". Being safety measures, some approach even combine FRMS and Safety-Management-Systems (SMS). In the current situation, as far as fatigue is concerned, BAOs are considered within the Reg EU 965/2012, which considers flight time limitations and there are no EASA requirements for FRMS. However, in those countries (USA, Canada and UK) where the LOSA audit is legally binding, the subjective fatigue evaluation is not considered reliable enough for a FRMS. As a consequence, this inconsistency is considered as a minor non compliance and it requires BAOs to adopt objective fatigue modelling to be considered as valid input for further procedures. As a result, an analysis of possible methodologies already in use for commercial aviation has been carried out in order to find possible approaches which could be used also in the BAO sector. Scientific research on methods to assess fatigue is very prolific, given the wide range of applications that they can be employed for. As mentioned before, they can be divided in subjective and objective methods. While the former category handles subjective data gained from surveys, the other one focuses on mathematically backed up strategies. There is a wide variety of subjective (self reported) methodology for fatigue estimation; the interested reader should consider reading Gawron (2016) for a detailed review about these methods. The most famous ones are: Karolinska Sleepiness Scale Åkerstedt and Gillberg (1990), Epworth Sleepiness Scale Johns (1991), Crew Status Scale (Samn-Perelli Scale) Samn and Perelli (1982) etc. Albeit essential for gathering data, the employment of self reported methods and subjective fatigue ratings can be an unreliable and biased measure of fatigue Petrilli et al. (2006). Many studies have highlighted issues related to acquiescence, courtesy and social desirability bias, as well as the general understanding of the survey from the interviewed person Gawron (2016). Some papers have tried to proposed possible solutions to standardize these tests Podsakoff et al. (2003). On the other hand, there are less objective models which, however can be grouped under the "Bio-mathematical" umbrella. In fact, they are still based on biological behaviours and data which are linked together with mathematical relations. These models can be even used to estimate the quantity of sleep obtained by pilots during specific flight patterns Roach et al. (2004). Some famous ones are: Fatigue Avoidance Scheduling Tool (FAST), Qinetiq model Belyavin and Spencer (2004), Fatigue Audit InterDyne (FAID) Roach et al. (2004), Fatigue Index Tool (FIT) and the Sleep, Activity, Fatigue, and Task Effectiveness (SAFTE) Model Hursh et al. (2004) or even the SAFTE/FAST. There are even some studies where Machine Learning techniques have been applied to fatigue management Hooda et al. (2022); Chen and Liu (2022).

4. The Proposed Model

The solution proposed in this paper, whose flow chart is reported in Fig. 1, is a bio-mathematical model built as a combination of three pre-existent methods: the Fatigue Alertness Model (Qinetiq-RAF model) (in green), the cumulative fatigue duty time model (ICAO, IATA, IFALPA) (in orange) and the substandard sleep quality model (USAF - Samn-Perelli) (in magenta). Each one of the three branches outputs a Fatigue Index (FI) in the Samn-Perelli fatigue scale: FI_{AL}, FI_{DT}, FI_{SQ} for the three models respectively. The three values are then summed up using a weighted sum. A particular weight, called Sleep Weight Loss Coefficient (SWLC) is used to multiply the FI_{SQ} and to calculate the overall sum, which is then rounded off, thus obtaining the final fatigue index FI_{TOT}. In the next paragraphs, each method will be approached separately.

One of the most used fatigue scale is the Samn-Perelli one, which was established in the eighties. It is a Samn-Perelli Fatigue Checklist Samn and Perelli (1982). The checklist is a seven-point Likert scale and it will be extensively used as a reference scale in this work.

4.1. *Hypotheses and limitations*

Despite being useful to estimate fatigue when little to no data is available, bio-mathematical fatigue models have some limitations that have to be noted in order to be employed consciously.

- First of all, BMMs are not capable of estimating point fatigue, but they output an average fatigue level
- At the current state, this model is not able to consider possible common fatigue countermeasures taken to fight fatigue (e.g. caffeine, energy drinks, stimulants, physical exercise, controlled rest, tactical naps etc)
- This model is not able to take into account psychological, personal and internal factors which may cause increased fatigue.

On top of that, a very critical review on BMM can be found in Dawson et al. (2017), where some interesting insights on BMM and the relative limitations are reported, warning about a systematic implementation of these models and the reference to a superimposed threshold. Basically, among BMMs main issues, the most critical one is the definition of safe/unsafe threshold. However, this problem is avoided in this specific case, as the output of the model is related to the Samn-Perelli fatigue scale. Furthermore, the importance of post-implementation surveillance is emphasized.

4.2. *Fatigue Alertness Model (Qinetiq-RAF model)*

This model is the result of a protracted effort by Qinetiq through the continuous improvement of existing models based on alertness experimental data obtained in several surveys Belyavin and Spencer (2004); CAA (2007). This very model has been already implemented in a commercial stand alone product: the aforementioned SAFE model. As far as the model presented in this paper is concerned, this branch receives a bi-dimensional input:

- Time On Awake (TOA): intended as the time, expressed in hrs in the range 0-24, at which the crew member woke up;
- Time since Last Sleep (TLS): expressed in hrs in the range 0-18, is intended as the time occured between the end of the last sleep and the start of the duty.

Thanks to these two input variables, the model is able to estimate the degree of alertness (and hence fatigue) based on two principles: on the one hand, alertness varies along the day in a sinusoidal like trend; on the other hand, alertness diminishes almost exponentially as the TLS gets higher Belyavin and Spencer (2004). It is assumed that an individual is fully rested at the beginning of a duty. The two input variables are then employed as coordinates (X, Y) to obtain a value from a fatigue matrix. In order to get suitable values inside the matrix, TOA and TLS are re-scaled Eq.1 with reference to the maximum matrix dimensions $Map_Y = 36$ $Map_X = 48$ using DX and DY, which are described in Eq. 2. After that, the result

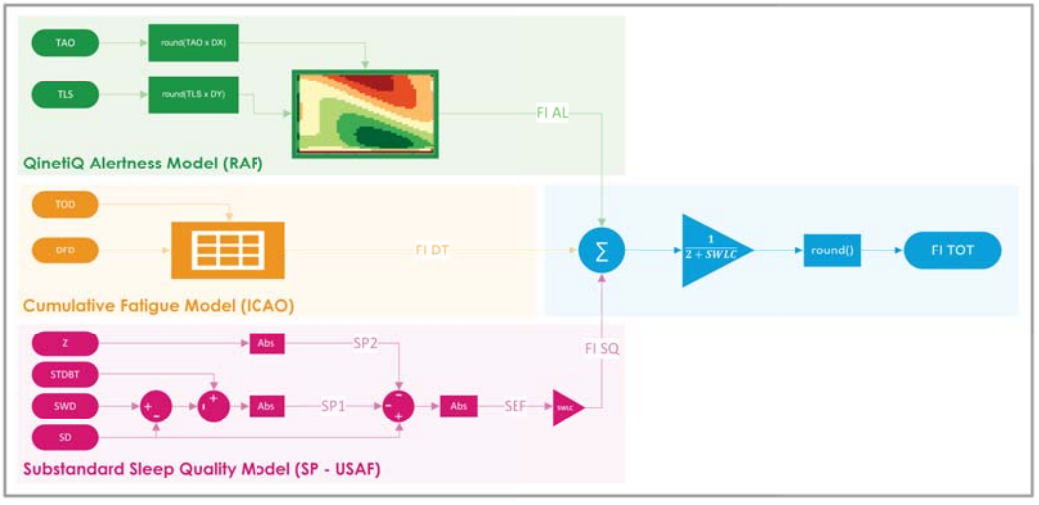


Fig. 1. Overall diagram of the fatigue model, showing the three branches.

is rounded off, in order to get integer values.

$$X = round(TAO \cdot DX)$$
$$Y = round(TLS \cdot DY) \tag{1}$$

$$DX = \frac{Map_X}{TAO_{max} - TAO_{min}}$$
$$DY = \frac{Map_Y}{TLS_{max} - TLS_{min}} \tag{2}$$

Where $TAO_{min} = 0, TAO_{max} = 18, TLS_{max} = 24, TLS_{min} = 0$ are the maximum and minimum accepted values (in hours).

The matrix has been obtained thanks to equations information reported in Belyavin and Spencer (2004). The model output is an estimate of a crewmember's level of attention during a duty period beginning at any time of day. This number is reported in the Samn-Perelli fatigue scale and it is marked by the name FI_{AL}.

4.3. *Cumulative Fatigue Duty Time Model (ICAO, IATA, IFALPA)*

The ICAO model ICAO (2011); IATA (2015) works on a two dimension input as well:

- Duty Start Time (DST)
- Time On Duty (TOD)

As per the previous model, the result is the estimated FI_DT. As shown in Fig 1, in this case TOD and DFD are already used as inputs for the matrix as they are. The model is based on empirically obtained data recorded through checklists compiled by airline crew for short and medium haul flights. It has to be noted that the fatigue is reported at the top of descent part of the flight, prior to the most dangerous and hazardous part of the flight (i.e. the approach and landing). These surveyed data show a strong interaction between the duty duration and the daily cycle of the circadian body clock and they have been transformed in an appropriate fatigue matrix following the Samn-Perelli scale.

4.4. *Substandard Sleep Quality Model (USAF - Samn-Perelli)*

This last part of the model is used to estimate the fatigue related to the sub standard sleep quality; it was firstly introduced in Samn and Perelli (1982) as the second step of a wider more extended algorithm and tries to estimate the fatigue experienced by a crew member due to sleep loss or as a consequence of poor quality sleep. The main idea is to determine sleep-loss penalty according to the time an individual goes to sleep. In fact, it is deemed that the sleep quality is strongly

reduced when and individual goes to sleep at a time different than the one the other people in the safe time-zone go to sleep at. The Standard Bed Time (STDBT) is considered at 22:30. In order to take into consideration this factors, two different sleep penalties have been considered (SP1 and SP2): the reference values for these tables are taken from Samn and Perelli (1982). Referring to Fig. 1, the sleep start time is obtained subtracting the Sleep Duration (SD) from the Start of Working Day (SWD). This value is then compared to the STDBT and the absolute value of the subtraction is then used to obtain SP1. On the other hand, SP2 is obtained considering as input the difference between Home Time Zone and the Local Sleep Time Zone. Finally, SP1 and SP2 are subtracted from SD to determine the effective sleep (SEF) achieved by the individual. This value is then multiplied by the SWLC, thus calculating FI_{SQ}.

5. Conclusion

In this paper the founding stone of a comprehensive bio-mathematical fatigue model, especially designed for BAO has been presented. In particular, the proposed methodology integrates three different pre-existing fatigue models: the QinetiQ Fatigue Alertness Model, the ICAO cumulative fatigue model and the substandard sleep quality model. The three strategies have been melted together with the aim of mixing the information to obtain a more reliable fatigue index which can be used to assess pilot fatigue conditions. Further steps will include the actual implementation of the proposed model in a programming language and the creation of an appropriate stand-alone app or web app, so that users can seamlessly connect to the service to rate the fatigue and, at the same time, provide the flight dispatcher with updated information. After the creation of a versatile platform, an essential step will be devoted to the algorithm validation thanks to real life operational data in order to assess if the model is actually simulating real conditions. Fatigue is still one of the black holes of the aerospace sector and many efforts have to be devoted in order to understand the underlying causes and allow a more precise quantification so that the aviation can always at the forefront of the safest means of transportation.

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