Numerical Validation of the Multi-Modal Transfer Matrix Method for Hexagonal Unit Cells

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***Abstract*—In this work, we present a modeling procedure for studying periodic structures with a hexagonal lattice. We extend two numerical methods to hexagonal unit cells: the multi-modal transfer matrix method and the method of moments. Both methods are capable of obtaining complex solutions of the eigenproblem. The results of the two methods are found to be in excellent agreement for both the passband and stopband modes.**

I. Introduction

The study of periodic structures with a hexagonal unit cell has gained significant attention in the fields of material science and engineering [1], as well as in the domain of microwave engineering more recently [2]–[4]. A notable property of such structures is high isotropy [1], [3]. Commercial eigenmode solvers are generally used to analyze their dispersion properties. However, these solvers are not able to directly provide information on stopband attenuation, which is crucial for a comprehensive understanding of the periodic structure. Previously, we reported a method to obtain this information through the multi-modal transfer matrix method (MMTMM) [5] and the method of moments (MoM) [6], but the analysis was limited to rectangular unit cells.

In this work, we present and validate two different modeling approaches for periodic structures with a hexagonal unit cell. The first approach is an extension on the MMTMM [5] and the second on the MoM [6]. Both are able to obtain modal solutions with complex-valued wavevectors. Their applicability is demonstrated on a mirror-symmetric holey structure.

II. Modelling

In the application of the MMTMM, we first obtain the coupling between multiple port modes defined at the edge of a convenient supercell [4]. In the present work, CST frequency domain solver is used with hexahedral meshing. The use of the supercell is necessary as it allows all ports to be aligned with the Cartesian axes, a requirement set by the solver. The scattering matrix is then transformed into a transfer matrix to pose the following eigenproblem, as in [5]:

$$[\mathbf{T}]\begin{bmatrix}\mathbf{V}_x\\ \mathbf{V}_y\\ \mathbf{I}_x\\ \mathbf{I}_y\end{bmatrix} = \begin{bmatrix}\exp(-\mathrm{j}\varphi_x)\mathbf{V}_x\\ \exp(-\mathrm{j}\varphi_y)\mathbf{V}_y\\ \exp(-\mathrm{j}\varphi_x)\mathbf{I}_x\\ \exp(-\mathrm{j}\varphi_y)\mathbf{I}_y\end{bmatrix}, \tag{1}$$

where $[\mathbf{T}]$ is the transfer matrix and $\mathbf{V}_{x,y}$ and $\mathbf{I}_{x,y}$ are the effective voltages and currents of the port modes. Information on propagation and attenuation of the electromagnetic field is contained in the phase shifts $\varphi_x = k_x p_x$ and $\varphi_y = k_y p_y$, where k_x and k_y are the complex wavevector components, and p_x and p_y are the x and y periods of the supercell, as depicted in Fig. 1(c).

Equation (1) can be reformulated as a homogeneous system in $\mathbf{V}$ and $\mathbf{I}$. Numerical solutions (φ_x, φ_y) can be computed by searching for complex zeros of the determinant of the matrix of the system. Alternatively, when φ_x or φ_y are constant, the problem can be reformulated as a linear eigenproblem [7], for which there are efficient zero-search algorithms, available in most numerical packages.

The MoM modeling to find the dispersion relation of hexagonal cells closely follows the approach presented in [6], as a solution of the following system:

$$[Z][I] = [0] \tag{2}$$

where $[Z]$ is the impedance matrix, whose elements are a function of the frequency and the wavevector, and $[I]$ contains the coefficients of the (unknown) basis functions of the electric surface current density. The main difference with [6] is in the definition of the lattice vectors as

$$\mathbf{s}_1 = [a, 0, 0] \tag{3}$$

and

$$\mathbf{s}_2 = [a\cos(\pi/3), a\sin(\pi/3), 0]\,, \tag{4}$$

where a is the width of the hexagon. Therefore, the unit cell used with the MoM is a rhombus, as depicted in Fig. 1(b), which is an alternative to the hexagonal primitive cell depicted in Fig. 1(a).

III. Numerical results

The results for passband modes of the MMTMM and MoM simulations can be verified with the commercial solver HFSS Eigenmode Solver (HFSS ES). For validation purposes, a supercell is used with HFSS ES, although a simulation of a primitive cell would also be possible. For MoM, the mesh in Fig. 1(b) is used for simulation. Its size is about one tenth of the wavelength at 25 GHz. The real part of the dispersion

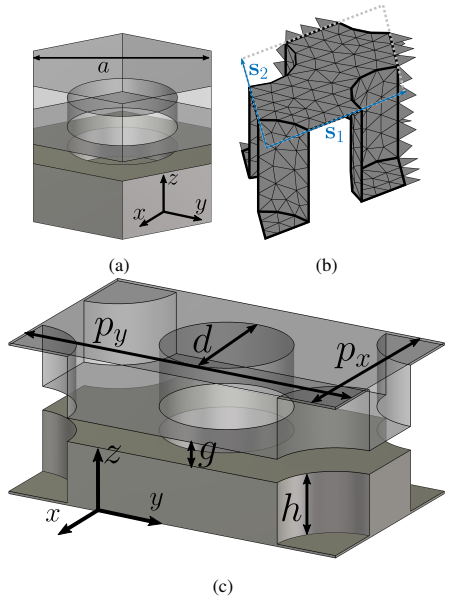

Fig. 1: Geometries under study: (a) hexagonal primitive unit cell, (b) rhombus primitive unit cell (MoM mesh), (c) and the super cell used with MMTMM.

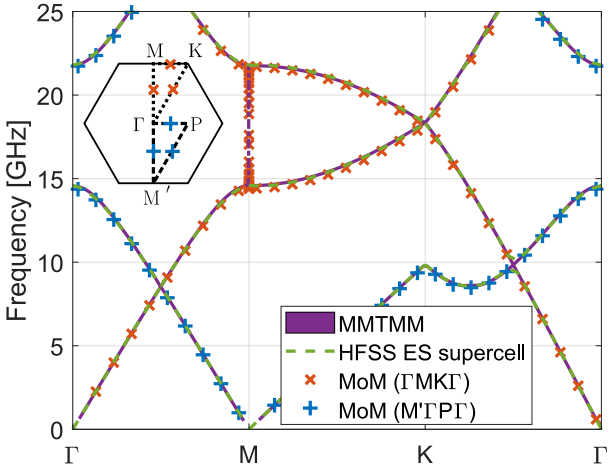

Fig. 2: Dispersion diagram obtained with the three methods. The scanned Brillouin zones are depicted in dashed line in the inset. The dimensions are in millimeters: $a = p_x = 8.7$, $h = 5.22$, $d = 5.394$, $g = 0.05$, $p_y = 2\,p_x \cos(\pi/6)$.

diagram is presented in Fig. 2. For supercell simulations, additional modes appear [4]. For the MoM simulation, obtaining the dispersion diagram of the supercell requires scanning two sections of the Brillouin zone: the irreducible Brillouin zone of the primitive unit cell $\overline{\Gamma\mathrm{MK}\Gamma}$ (represented in orange cross markers), and its image $\overline{\mathrm{M}'\Gamma\mathrm{P}\mathrm{M}'}$ (represented in blue plus markers), as in [4]. The vertical dashed line from about 15-22 GHz at M belongs to an evanescent mode. Its attenuation is presented in Fig. 3, with an excellent agreement between MoM and MMTMM.

IV. Conclusion and Perspectives

In this work, two methods for obtaining both real and imaginary parts of the wavevector in hexagonal structures are presented. Both approaches yield similar results for the

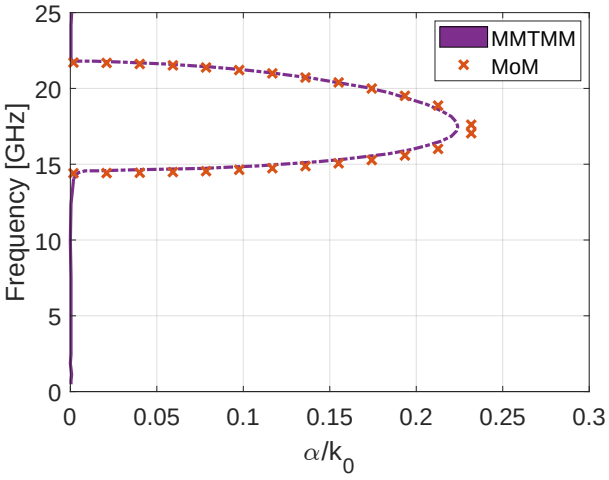

Fig. 3: Attenuation in $\overline{\Gamma\mathrm{M}}$. The dimensions are given in Fig.2.

mirror-symmetric holey unit cell under study, verifying the applicability of the methods. This development allows for a further in-depth exploration of periodic structures with a hexagonal lattice.

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