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# Numerical model of an Yb:Er:Tm:Ho co-doped germanate glass: concentrations optimization and transfer function

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## ABSTRACT

In this paper, we numerically simulate the population of levels of an Yb:Er:Tm:Ho co-doped germanate glass pumped at 980 nm, that could be able to generate broad emission in a wavelength range from 1500 nm to 2100 nm. The aim of this work is to study the possibility of reaching a homogeneous inversion of Er, Tm and Ho, in order to further develop ultra-broadband active devices. We study the influence of a variation in the concentration of the dopants in such a complex system, which exhibits many energy transfer phenomena between different rare earth ions. Furthermore, we computed the transfer function of the system to evaluate the pump noise influence.

**Keywords:** Erbium, Thulium, Holmium, Germanate Glass, Rate Equations, Numerical Model, Transfer Function, Rare-earth

## 1. INTRODUCTION

The optimal doping of a glass doped with multiple rare earths is a complex task, due to the many energy transfer phenomena to be considered which generate interplay between the rare earth elements. Yet multiple doping offers the possibility of ultrabroad gain bandwidth [1-3]. In this paper we consider a germanate glass doped with Yb, Er, Tm and Ho. This glass, if pumped around 980 nm, could be able to generate emission and gain in the wavelength range 1500 nm – 2100 nm from Er, Tm and Ho [3-5]. In such a complex glass, obtaining the emission spectrum homogeneity is quite challenging. Here we propose to study and optimize emission homogeneity using a numerical model based on rare-earth system rate equations, in order to predict the populations of energy levels in relation of rare earths concentrations. Furthermore, the model can compute the transfer function of the system, in order to assess its amplitude stability and to study the effect of the pump noise for future active device design.

## 2. METHODS

### Rate equation model and ion concentration influence

We considered eleven energy levels in total: three energy levels for Tm ($^3H_6$, $^3F_4$, $^3H_5$), three for Er ($^4I_{15/2}$, $^4I_{13/2}$, $^4I_{11/2}$), two for Yb ($^2F_{7/2}$ and $^2F_{5/2}$), and three for Ho ($^5I_8$, $^5I_7$, $^5I_6$). The numerical model is therefore composed of eleven rate equations (1a-1k), one for each energy level, with the number of the *i*-level refers to the order above (#1 is Tm $^3H_6$…#11 is Ho $^5I_6$).

$$\frac{\partial N_1}{\partial t} = +A_{2,1}N_2 + A_{3,1}N_3 - K_{YbTm}N_1N_8 + K_{TmHo}N_2N_9 - K_{ErTm13}N_1N_6 - K_{ErTm12}N_1N_5 \tag{1a}$$

$$\frac{\partial N_2}{\partial t} = +A_{3,2}N_3 - A_{2,1}N_2 - K_{TmHo}N_2N_9 + K_{ErTm12}N_1N_5 \tag{1b}$$

$$\frac{\partial N_3}{\partial t} = -A_{3,2}N_3 - A_{3,1}N_3 + K_{YbTm}N_1N_8 + K_{ErTm13}N_1N_6 \tag{1c}$$

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$$\frac{\partial N_4}{\partial t} = -W_{4,6}N_4 + W_{6,4}N_6 + A_{6,4}N_6 + A_{5,4}N_5 - K_{YbEr}N_4N_8 + K_{ErHo}N_5N_9 + K_{ErTm13}N_1N_6 + K_{ErTm12}N_1N_5 \tag{1d}$$

$$\frac{\partial N_5}{\partial t} = +A_{6,5}N_6 - A_{5,4}N_5 - K_{ErHo}N_5N_9 - K_{ErTm12}N_1N_5 \tag{1e}$$

$$\frac{\partial N_6}{\partial t} = +W_{4,6}N_4 - W_{6,4}N_6 - A_{6,5}N_6 - A_{6,4}N_6 + K_{YbEr}N_4N_8 + K_{ErTm13}N_1N_6 \tag{1f}$$

$$\frac{\partial N_7}{\partial t} = -W_{7,8}N_7 + W_{8,7}N_8 + A_{8,7}N_8 + K_{YbEr}N_4N_8 + K_{YbHo}N_8N_9 + K_{YbTm}N_1N_8 \tag{1g}$$

$$\frac{\partial N_8}{\partial t} = +W_{7,8}N_7 - W_{8,7}N_8 - A_{8,7}N_8 - K_{YbEr}N_4N_8 - K_{YbHo}N_8N_9 - K_{YbTm}N_1N_8 \tag{1h}$$

$$\frac{\partial N_9}{\partial t} = +A_{10,9}N_{10} + A_{11,9}N_{11} - K_{YbHo}N_8N_9 - K_{ErHo}N_5N_9 - K_{TmHo}N_2N_9 \tag{1i}$$

$$\frac{\partial N_{10}}{\partial t} = -A_{10,9}N_{10} + A_{11,10}N_{11} + K_{ErHo}N_5N_9 + K_{TmHo}N_2N_9 \tag{1j}$$

$$\frac{\partial N_{11}}{\partial t} = -A_{11,10}N_{11} - A_{11,9}N_{11} + K_{YbHo}N_8N_9 \tag{1k}$$

Where $N_i$ is the populations of each of the *i*-state levels, K is the energy transfer coefficient between two different rare earth, $A_{i,j}$ is the radiative decay rate of the $i \rightarrow j$ transition and it is given by $A_{i,j} = \beta_{i,j} / \tau_i$ (with $\beta_{i,j}$ is the branching ratio of that transition and $\tau_i$ is the time constant of the state level i), $W_{i,j}$ is the $i \rightarrow j$ transition rate related to the pump and it is given by $W_{i,j} = [\sigma_{i,j}(\lambda_p) / (h*c_0/ \lambda_p)] * P_p * (\Gamma_p/A_d)$ (with $\sigma_{i,j}(\lambda_p)$ is the cross section at the pump wavelength $\lambda_p$, h is the Planck constant, $c_0$ is the speed of light in vacuum, $P_p$ is the pump power, $\Gamma_p$ is the overlap factor between doped region of area $A_d$ and pump profile and here we used 1).

More details about the energy levels considered, the rate equations, and the parameters can be seen in the paper of Falconi et al. [4]. We used a pump power of 10 mW, with a cross section of 78.5 $\mu m^2$. The numerical model is able to compute the populations of the different energy levels in both stationary and dynamic conditions.

In order to study the effect of a variation in the ion concentrations, we decided to start from a set of concentrations [5] and then change them. The starting concentrations were $5.1 \times 10^{25}$ ions/m$^3$ of Yb, $1.7 \times 10^{25}$ ions/m$^3$ of Er, $4.25 \times 10^{24}$ ions/m$^3$ of Tm and $4.25 \times 10^{24}$ ions/m$^3$ of Ho. We chose to keep the Yb fixed, while modifying Er, Tm and Ho in a range of 1 order of magnitude up and down. Tm and Ho were kept equal, so the change interested Er or Tm and Ho concentrations.

The parameters extracted from the model and used to study the effect of the variation were: (1) the absolute population value of the emission energy levels (i.e., $^4I_{13/2}$, $^3F_4$, $^5I_7$), (2) the inversion efficiency (i.e., the percentage of the population in the emission level compared to the total population of the rare earth), and (3) the photon emission rates (i.e., the ratio between the population in the emission level and the time constant of that energy level).

### Transfer function

We implemented a fourth order Runge-Kutta (RK4) algorithm in order to solve the non-linear differential equation system under dynamic conditions. Starting from the stationary condition, we added a sinusoidal noise to the static pump; the additive noise had a frequency ranging from 10 Hz to 100 kHz, and its amplitude was 1% of the pump power (i.e., 0.1 mW). The transfer function has been defined as the variation of the population over the variation of the pump contribute. To assess the correctness of the transfer function obtained through this method, we calculated the analytical expression of it for the Yb alone and the Er alone to be used as ground truth; the analytical expression has been obtained using Laplace transform.

## 3. RESULTS AND DISCUSSION

### Ion concentration influence

Here we report an example of the photon emission rates of Er (Fig. 1a), Tm (Fig. 1b) and Ho (Fig. 1c), the highest difference between these three photon emission rates (Fig. 1d), and the highest difference between the populations of the emission energy levels (Fig. 1e). In order to obtain the spectral homogeneity, the optimal set of concentrations should guarantee a good inversion efficiency combined with a low difference in the populations and photon emission rates of the three rare earths.

As can be seen (Fig. 1d, 1e), as the concentrations of Er, Tm and Ho increase, the area with a low difference, and thus high homogeneity, lies on the diagonal (i.e., where the ratio of Er, Tm, and Ho is about the same as the starting state, 4:1:1).

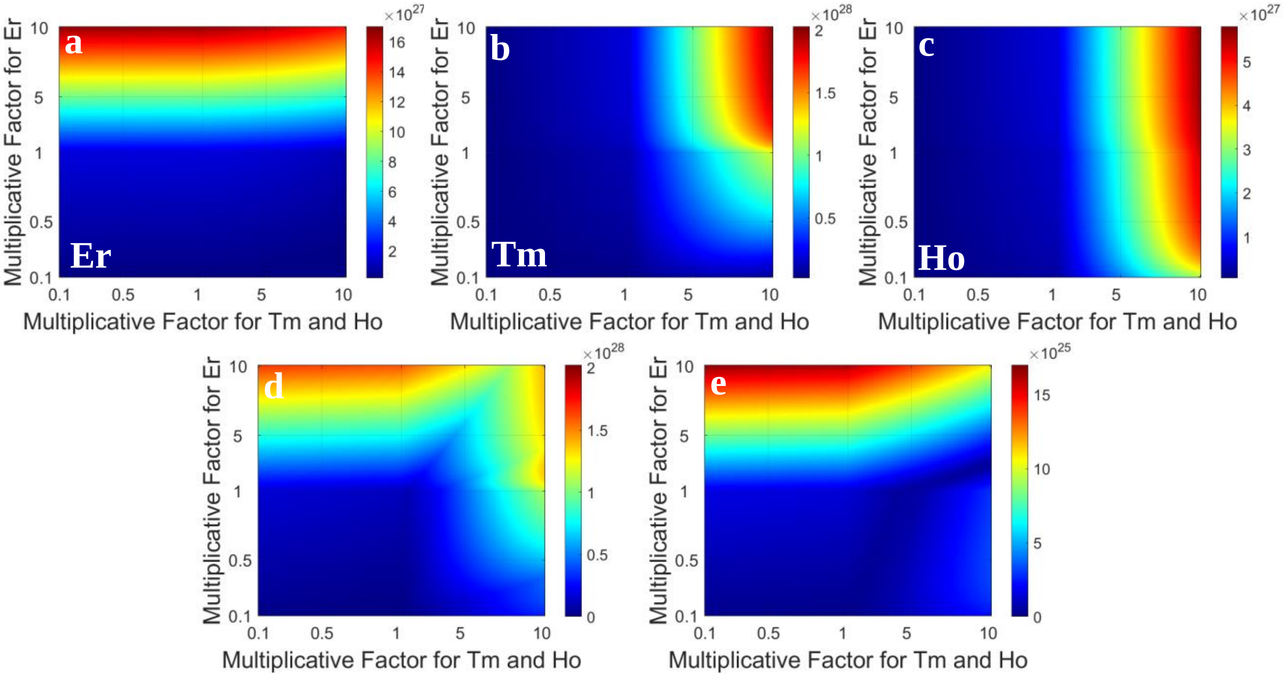

Figure 1. Photon emission rates of (a) Er, (b) Tm and (c) Ho, (d) highest difference between photon emission rates and (e) highest difference between the populations, obtained with a pump power of 10 mW. In all cases horizontal axis is the multiplicative factor of Tm and Ho concentrations and the vertical axis is the multiplicative factor of Er concentration. Both horizontal and vertical axes have a step size of 0.01 in the range 0.1-1 and a step size of 0.1 in the range 1-10

## Transfer function

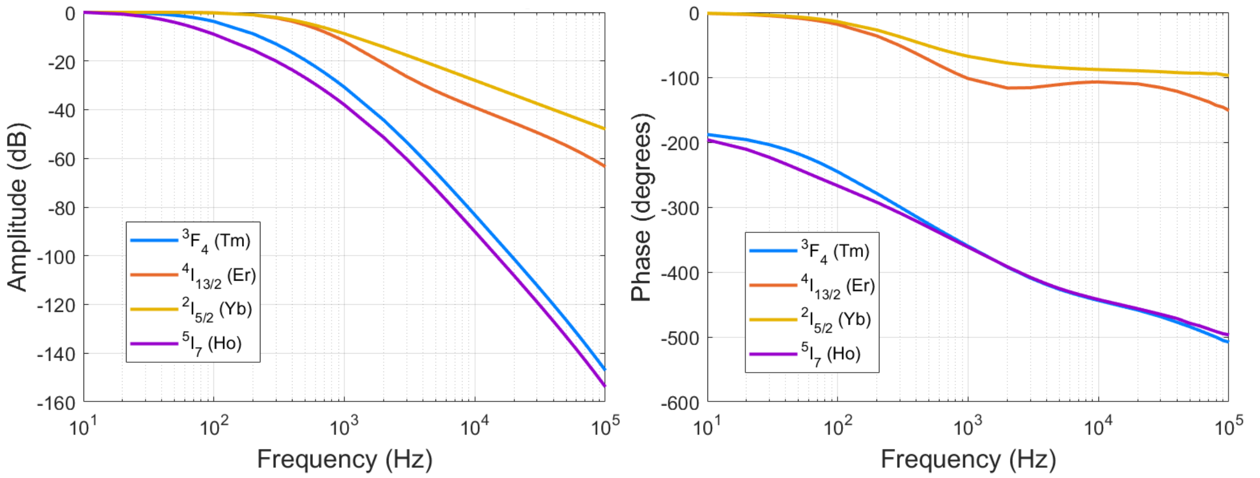

Figure 2. Amplitude and phase of the transfer function of the modelled glass, obtained with a pump power of 10 mW. The transfer function has been computed for 10 frequency values for each decade, from 10 Hz to 100 kHz.

With the transfer function it is possible to analyze a priori the stability of the system against perturbations [6].

From the comparison between the transfer functions obtained from the numerical model and the analytical expression, we can assess the correctness of the numerical model, since amplitude and phase of transfer function are completely overlapped. The only one noticeable effect that we want to mention is that the correctness of the phase depends on the time step used for the RK4 algorithm (e.g., with a time step of 0.1 μs, the phase can be perfectly overlapped to the analytical one up to 10 kHz).

Below we report an example of the computed transfer function (Fig. 2) of the Yb:Er:Tm:Ho glass model related to pump intensity noise, to study the effect of the pump noise on the stability of exited state level populations. As can be seen, the transfer functions have a low-pass filter shape, and none of the frequencies is amplified. From the comparison of the amplitude of Yb and Er as single alone dopant transfer functions against the full system with four rare earths, we noticed Yb and Er are less sensitive to pump fluctuations. Lastly, the poles of the transfer function depend on the energy transfer coefficients and time constants, so we aim the use of the transfer function as an alternative tool to fit experimental data and extract complex spectroscopic parameters, such as time constants or energy transfer coefficients.

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