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DYNAMIC ANALYSIS OF ARTIFACTS: EXPERIMENTAL TESTS FOR THE VALIDATION OF NUMERICAL MODELS

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Abstract

The protection of art goods is an important issue of the seismic engineering. Artifacts are often made by fragile and ancient material, and they can easily present irregular shapes and high slenderness. In these years many contributions have been devoted to their analysis, based on numerical models having different complexity and computational effort. The Finite Element Models represent the most common and versatile approach for the representation of artifacts. Nevertheless, their reliability depends on the numerical assumptions made for the analysis, which requires a wide number of information regarding the dynamic behavior of the artifacts, such as the friction between the analyzed object and its support, the effective damping, the inelastic involvement of material, etc. In this work an experimental campaign has been started aimed at determining the main factors which affect the dynamic representation of artifacts through FEM analysis and simplified models. A large number of experimental tests, both static and dynamic, have been performed, by adopting both real and reduced scale objects.

Keywords: Validation of numerical models, shaking table test, experimental campaign

1 INTRODUCTION

The art goods play a crucial role in defining and understanding the identity of communities, and therefore they should be carefully protected against possible dangers and hazards. The seismic events occurred in the last decades (L'Aquila 2009, Emilia 2012, Centro Italia 2016), however, have induced serious injuries to many monuments and artifacts [1], evidencing their seismic vulnerability and pointing out the importance of increasing the current prevention policy [2,3].

The assessment of the seismic safety level of an art good is achieved by comparing its expected seismic response to the corresponding limit value. The most advanced approach for determining the seismic response of artifact is the Finite Element Method. The adoption of FEM, however, requires many assumptions regarding the mechanical properties of the materials and the boundary conditions to assume for the object and for its restraint. Previous studies developed by some of the Authors [4,5] pointed out the importance played by some quantities, such as the strength and the friction coefficient of the materials, in the achieved collapse mechanism and the expected dynamic response of sculptures. The reliable calibration of the numerical model to use for seismic analysis requires the availability of experimental results. In these years several researchers provided precious contributions to the calibration of numerical models for seismic analyses of artifacts [6-9] . However, the available experimental data regarding art goods are still few, and the calibration of numerical models is an open issue.

In this paper, the first results of an experimental campaign focused on the calibration of FE models to use for representing the seismic behavior of art goods are presented. The campaign, still in progress, consists of two main phases.

The first phase, presented in the Sections 2 and 3, refers to the mechanical properties of some of the materials most used for ancient and current artifacts. The materials have been investigated with reference to their strength (Section 2) and to their attitude to slide over a standing surface (Section 3). The strength of the material has been checked through uniaxial compressive strength (UCS), combined to ultrasound lectures, according to the current Code [10-12] provisions, whilst the friction coefficient has been checked both through static and dynamic tests. Right now, only two materials, i.e. the Carrara marble and the sandstone (*pietra serena*), have been checked. Both the materials are taken from quarries located in the Florentine area, and they can be representative of the sculptures made in the Tuscan Renaissance. Further materials, such as ceramic and glass, should be included in the experimental campaign. At the current time, the friction coefficient has been checked with reference to the marble only, and to standing surfaces made of different materials, such as masonry and mortar, glass, Plexiglas, timber and steel.

The second phase of the survey, whose beginning is planned for September 2019, is focused on real artifacts. The tests will be performed through shaking table, checking the dynamic response of the artifacts to sinusoid acceleration histories and real ground motions. In Section 4 the experimental program is described. The experimental results obtained for the dynamic response of artifacts will be compared to those provided by the FE models. Two different FE models will be adopted in the simulation, and compared to the experimental results, in order to evaluate the role of the single assumptions as a function of the peculiarity of each model.

2 EXPERIMENTAL EVALUATION OF THE MATERIALS STRENGTH

The compressive strength is usually assumed as reference quantity for the assumptions of all the main mechanical properties, such as the Young modulus. The test should be made on all the main materials used for art goods, such as stones, ceramics, etc.. Right now, however, two stones only have been tested, i.e. the Carrara marble and the sandstone ("pietra serena"). The strength of the material has been found through a uniaxial compressive strength (UCS), combined to ultrasound lectures, whilst the friction coefficient has been found through an inclined plane devise.

2.1 Marble

Most part of the marble sculptures made in the Florence area in the XVI Century, were made with Carrara marble. Therefore, the compressive test has been made on six samples taken by a Carrara quarry. The samples have a cylindrical shape, with a diameter of 5.4 cm. The strength was found through a Uniaxial Compressive Strength (UCS) test made through a hydraulic press INSTRON MODEL 5592 with a maximum stress equal to 600 kN, at a constant velocity of 1±0.5 MPa/s, according to UNI EN 1926:2007 and ASTM standards (ASTM 1985). The UCS test was even enhanced by ultrasound lectures. The ultrasound velocity was measured by DSP – UTD 1004 model N034 Boviar, by applying two transducers on the opposite sides of each sample. Figure 1 shows the images of the samples at the beginning of the crushing, whilst in Figure 2 shows the strength values provided by the tests, together with the main data on the samples.

Figure 1. Samples under the UCS test.

	density		Ultrasound lectures		Strength	70							
	KN/m^3	μs	$ \mu s $	$ \mu s $	MPa	60							
M ₁	26.66	19.5	19.4	19.4	55.64	50							
M ₂	26.71	19.6	19.4	19.5	57.99	40							
M ₃	26.50	15.4	15.2	15.3	31.81	STRENGT 30							
M ₄	26.78	18.2	18	17.9	48.29	20							
M ₅	26.84	16.2	16.3	16.2	53.19	10							
M ₆	26.84	16.9	16.8	16.8	58.05	$\mathbf{0}$							
mean	26.72		17.6		50.8		M ₁	M ₂	M ₃	M4	M ₅	M ₆	
C.o.V.	0.5%		9.2%		19.7%		SAMPLES						

Figure 2. Test on the marble: data and results

2.2 Sandstone

The sandstone has been used in Tuscany since the Etruscan age. In the Renaissance it has been widely used for palaces and art goods. Indeed, it is the most valuable (strong and durable) type of the "macigno" stone, that is one of the most common stone of the Florentine area. The color is gray at the extraction, and it becomes ochre with the time, due to the oxidation process. Due to its large diffusions, there have been, along the centuries, several quarries, spread in the country. Each of them has provided material with different colors and mechanical properties. In this survey, four types of sandstone, made by six samples each, have been tested. In Figure 3 the location of the quarries of the checked stone has been shown. The samples have a cubic shape, with a side of 5 cm. Some further tests have been performed to check density, absorption and porosity of the material; in this case, smaller cubic samples, with sides equal to 2 cm, have been adopted.

Figure 3. Location of the quarries supplying the tested samples

Figure 4. Strength of the sandstone samples

An extensive presentation of the results can be found in [13], while Figure 4 and Table 1 show, respectively, the values obtained for the compressive strength of each sample and the main data provided by the test. As can be noted, the strength of the sandstone is much more variable than the marble one; the *Coefficient of Variation* (*CoV*) of the sandstone, indeed, varies between 34% and 52% in the considered samples. Moreover, the strength of the material results to be very sensitive to the extraction quarry: the sandstone coming from the "Bigi" quarry has a strength three times lower that the one of the samples coming from the other

quarries. Even the other mechanical properties, such as porosity and imbibition coefficient, result to be sensitive to the extraction quarry.

Table 1. Test on the sandstone: data and results

3 EXPERIMENTAL EVALUATION OF THE FRICTION COEFFICIENT

The friction coefficient plays a fundamental role in the numerical simulation of the dynamic response of artifacts, since the amount of friction determines the collapse mechanism experienced by the artifact under seismic excitation. At the current time, the friction coefficient has been determined only for the marble samples, i.e. cubes having 10 cm sides. Several materials, instead, have been considered for the standing plane. The experimental campaign, still going on, consists both of static and dynamic tests.

3.1 Static test

The angle between the sample and the standing plane corresponding to the sliding activation has been checked through the device represented in Figure 5. The device has a horizontal fixed plane, and an inclined, adjustable plane where the standing plane is fixed, whose angle

can be read through a goniometer. The test has been performed by checking the sliding angle which activate the sliding of the samples, which are marble cubes of 10 cm sides. Each of the six sides of the samples has a different finishing (shown in Figure 5), i.e. respectively: diamond sawcut, polished, fine chiselled, rough chiselled, gradined and bush hammered.

The test has been performed by fixing the standing plane to the device, placing a cube of marble over it, and gradually increasing the slope until achieving the sliding of the specimen.

Figure 5. Device for determining the friction coefficient and finishing of the sample sides.

The test is aimed to provide data regarding both big sculptures which stand over pedestals or floors, and smaller art goods, exhibited over showcases' shelves. As a consequence, the different materials have been considered for the standing plane. Three planes consisting of masonry covered by mortar, have been considered to represent the interface of sculptures with their pedestals, and four further planes, made respectively of glass, Plexiglas, timber and steel have been considered to represent the interface of small artifacts with shelves.

The masonry planes differ from each other for the covering mortar, that is, respectively, cement, lime and mixed one. In Figure 6 the values provided for the *Friction Coefficient* (*FC*) by the performed tests have been shown. Each result shown in Figure 6 represent the *mean* values of six results, referring to as much samples.

Figure 6. Friction coefficient between the marble samples and the considered standing surfaces Figure 7 shows the values of the *Coefficient of Variation* (*CoV*) found for each 6-samples lecture. As can be noted, the diamond sawcut finishing present the highest value of *CoV*. Such finishing, however, results to be important only for smell objects exhibited over shelves.

Figure 7. Coefficient of Variation found for the Friction Coefficient.

3.2 Dynamic test

A dynamic test through the shaking table has been planned in order to check the effects of the friction coefficient on the dynamic response of the checked systems (samples over standing planes). Previous experimental campaigns [14] pointed out interesting relationships between the friction coefficient and the type of dynamic response (rest, sliding and rocking), as much as the fundamental frequency of the system, and the amplification in the acceleration.

The test will be performed on the same samples and standing planes considered in the static test, by assuming the equipment described in Section 4.1.

4 EXPERIMENTAL BEHAVIOUR OF ARTIFACTS

4.1 Dynamic response of artifact

The dynamic test on handcraft objects will be performed through the bidirectional shaking table at the Disaster Resilience Simulation Laboratory at the Politecnico di Torino, Italy. The structure of the shaking table consists of steel profiles, whereas the upper platform, where specimens can be fixed, is made of aluminium. Two parallel tracks are located side by side and connected through transversal rectangular sections. Tracks' profiles are 3 meters long and the section's size is 40x100x4 mm. Upon the steel profiles there are aluminium guides allowing the motion, along the longitudinal direction, of sliders that support two 600x500x10 mm aluminium platforms. Each track has its own platform, which is moved by a linear electric actuator anchored under it. On the small platforms, other two tracks and platforms are fixed. Type and section of the steel profiles are the same of the bottom ones, while the length is shorter (600 mm). For the transversal motion other two linear electric actuators are anchored under the aluminium platforms. If necessary, a bigger platform $(1500x1500x10 \text{ mm})$ can be installed (Figure 8). The linear electric actuators adopted are manufactured by the company LinMot and each is made of a stator, a slider and a motor. The longitudinal ones have a slider's length of 800 mm and a maximal stroke of 510 mm, whereas the transversal ones have a slider's length of 500 mm and a maximal stroke of 330 mm. The power supply, the two transformers and the four drivers to control the motors are provided by LinMot as well (Figure 9). The drivers are fundamental for the tuning of the motors (i.e. the initial configuration of all the control parameters) to have a response coherent with the input data. This operation is done through the software LinMot-Talk that is also used to switch on the actuators and to bring them in the home position. The software used for the activation and control of the shaking table is LabView. The seismic input is sent to the shaking table through a myRIO device manufactured by National Instruments. This device is physically connected to the motors' drivers and also to an accelerometer, which is located on the platform and allows catching the actual response of the system. Simply, it is possible to connect a USB pen drive containing the seismic signal in terms of displacements to the myRIO device. The LabView code is used to set the input and output sampling rates, to generate a sinusoidal seismic signal or to load a real one, to scale it, to start and stop the motion and finally to compare the data obtained from the accelerometer with the theoretical ones.

Figure 8. Shaking table at Politecnico di Torino.

Figure 9. Control panel and workstation and electronic devices inside the control panel.

Two different loading conditions will be considered in the test. The first one consists of mono-dimensional periodic acceleration histories. The choice of the frequency content to assume will be made on the basis of a preliminary experimental modal analysis [7], which will be perform on the assumed case-studies, by considering different standing surfaces. Different seismic intensities will be considered ranging between 0.10g and 0.35g, i.e. the range of interest of most part of the art goods exhibited in the Italian Museums.

The second loading case consists of real ground motions. The ground motions will be selected in order to be spectrum-compatible to the elastic spectrum provided by the Italian Code for different Italian location. The first considered location is Florence, where is placed the Museum of Bargello, related to the research project (RESIMUS) which promoted the experimental campaign. The dynamic tests will be performed with the main horizontal component of the ground motion, with both the horizontal components and with all the 3 effective components, including the vertical one, in order to check the effects of these assumptions in the numerical analysis. Different ground motions will be considered as seismic input, in order to simulate the effect of the foundation soil on the seismic response of the artifact, according to the current soil classification provided by the Technical Code (NTC 2018).

4.2 Validation of numerical models

It is well known that the reliability of the numerical analysis depends on the assumptions made in the analysis.

Some numerical analyses were performed within the research project "Resimus" [4,5,15- 17] on the statue "Cerere" by Bartolomeo Ammannati. Two advanced Finite Element models have been adopted for representing the artifact. The two models differ from each other both for the material behavior and for the restraint condition: the first model [18] assumes a nonlinear elastic behavior of materials and a perfect continuity between the statue and the pedestal. The second model [19], instead, assumes a linear elastic behavior for the materials, and introduces a proper contact surface between the statue and the pedestal. The results provided by performing a time-history analysis through the two models evidenced the crucial role plaid by the assumed friction coefficient when the detachment between statue and pedestal is taken into account. In the above mentioned analysis, the considered Friction Coefficient has been assumed on the basis of a static test made on the same materials of the artifact.

The planned experimental test through shaking table requires a preliminary geometrical survey of the artifact, in order to obtain the geometrical model to use for the analysis. The geometrical model provided by a photogrammetric or laser-scanner survey is usually very detailed and limited to the object external surface. Therefore, it needs to be simplified and changed in a 3-d model, in order to maintain a satisfactory precision, reducing the computational effort required by the structural analysis. The dynamic test should be performed on artifacts made of different materials, shape and dimensions, in order to check the effectiveness of the numerical simulation at the varying of the conditions.

A third simplified model will be validated through the experimental tests. It consists of a 3 d representation of the dynamic behavior of the statues under the earthquake. Two rigid perpendicular beams have been considered to model the contact surface, and another beam element is used to identify the position of the center of gravity above the contact surface (Figure 10). By defining different lengths of rigid beams, the model is able to take into account the eccentricity as a variable. To consider the effect of friction, a Friction-Pendulum Isolator element has been used to model the contact surface. This element is able to model combination of different conditions varying from at-rest to slide, or from uplift to slam-down for the cases of friction and rocking, respectively. The pendulum radius of the slipping surface was set to zero to consider the flat surface friction. The element models the coupled biaxial friction at contact surface considering the post-slip stiffness. The friction forces are proportional to both external normal force and friction coefficient. The axial force (*P*) is modeled with a compression-only gap element that does not carry the tension force in the case of uplift and it is given by:

$$
P = \begin{cases} K_z d_z & \text{if } d_z < 0 \\ 0 & \text{otherwise} \end{cases}
$$
 (1)

where the K_z is the vertical stiffness in negative axial direction (-*Z*) and d_z is the vertical displacement of the rigid body base at the contact surface. K_z is set to some large value in order to consider the rigidity of the contact surface. The nonlinear behavior is considered for each shear (friction) degree of freedom in *x* and *y* directions. The friction force-deformation relationship is given by:

$$
f_x = -P\mu_x z_x
$$

\n
$$
f_y = -P\mu_y z_y
$$
\n(2)

where f_x and f_y are the friction forces in *x* and *y* directions, μ_x and μ_y are velocity-dependent friction coefficients, and z_x and z_y are internal hysteretic variables. In order to accurately model the problem, the fast and slow friction coefficients are considered as a function of velocity. The initial values of z_x and z_y are zero and they evolve according to following differential equation:
 $\left(\frac{K_x}{\sigma}\dot{d}_x\right)$ differential equation:

tion:
\n
$$
\begin{cases}\n\dot{z}_x \\
\dot{z}_y\n\end{cases} = \begin{pmatrix}\n1 - a_x z_x^2 & -a_y z_x z_y \\
-a_x z_x z_y & 1 - a_y z_y^2\n\end{pmatrix} \begin{pmatrix}\n\frac{K_x}{P \mu_x} \dot{d}_x \\
\frac{K_y}{P \mu_y} \dot{d}_y\n\end{pmatrix}
$$
 for $\sqrt{z_x^2 + z_y^2} \le 1$ (3)

where K_x and K_y are the elastic shear stiffness constants in the absence of sliding, and a_x and *a_y* are binaries parameters deepening on velocity in x and y direction:
 $a = \begin{cases} 1 & \text{if } d_x z_x > 0 \\ 0 & \text{if } d_y z_y > 0 \end{cases}$

$$
a_x = \begin{cases} 1 & \text{if } d_x z_x > 0 \\ 0 & \text{otherwise} \end{cases}, a_y = \begin{cases} 1 & \text{if } d_y z_y > 0 \\ 0 & \text{otherwise} \end{cases}
$$
(4)

Figure 10. Mathematical modeling to evaluate the sliding and rocking behavior using the Friction-Pendulum Isolator element.

5 CONCLUSIVE REMARKS

In this paper, an experimental campaign focused on the calibration of FE models to use for representing the seismic behavior of art goods is presented.

The experimental campaign, still in progress, consists of both static and dynamic tests. In the paper, only the results provided by the static tests have been shown. Such results refer to the mechanical properties of some of the materials most used for ancient and current artifacts (marble and sandstone) and to their attitude to slide over a standing surface, respectively consisting of masonry and mortar, timber, glass, steel and Plexiglas. The strength of the material has been checked through uniaxial compressive strength (UCS), combined to ultrasound lectures, according to the current Code provisions, whilst the friction coefficient has been checked both through static and dynamic tests.

Several dynamic tests will be performed through a shaking table, using both the same samples of the static tests and handcraft objects, compatible for material, shape and scale to common art goods. The results obtained on the handcraft objects will be used for the validation of the presented 3-d numerical model.

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