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Abstract. In this study, a novel optimization method has been applied to a geodesic dome inspired by real-world similar structures in which the environmental and cost impact has been minimized by reducing raw materials at the production stage. To achieve this goal, the Cutting Stock Problem (CSP) has been embedded inside the global optimization procedure of the entire structure. The CSP is one of the most famous combinatorial optimisation problems in the (one-dimensional) bin packing problems (BPP) class. The main objective is to produce d_j copies of each item type j (i.e. elements of the structures with the same cross-sectional Area) by employing the minimum number of bins such that the total weight in any bin does not exceed the capacity. In the civil engineering field, the traditional approach to structural optimization aims to improve the load-bearing capacity and the global performance of the structure itself. This includes, for instance, the maximization of the performance ratio through the minimization of the structure weight. However, this goal doesn't guarantee maximum efficiency in reusing structural elements and minimising waste during the industrial production phase. To overcome these limits, authors propose a stock-constrained structural optimization in which a heuristic search technique is adopted in order to find the best spatial arrangement of elements composing the structure whit the lowest cut-off waste. Finally, considerations have been discussed by comparing the solution obtained by the traditional weight-minimization approach and the stock-constrained one.

Keywords: Optimization · Steel · Genetic Algorithm · Cutting Stock Problem.

1 Introduction

Since the beginning, researchers were focused on optimizing the weight or volume of specific structures by satisfying safety recommendations provided by specific standard regulations. In other words, the traditional approach which is largely adopted by the majority of the Scientific community relies on the minimum

* Supported by ADDOPTML Project, MSCA RISE 2020 Marie Skodowska Curie Research and Innovation Staff Exchange

weight or cost design of structures. However, several authors demonstrated that optimal design derived by this approach leads to unrealistic and no practical technical solutions. Taking into account constructability issues or construction details in the optimization process assumes crucial importance if the feasibility of these optimality approaches has to be proven.

As demonstrated in this study, a considerable portion of construction expenses can be attributed to material waste resulting from the cutting process. Insufficient attention to meticulous cutting design and the minimization of waste during the construction phase can lead to suboptimal cost optimization. Additionally, the production phase may contribute to a higher cost and environmental impact expressed in terms of economic indices and CO2 emissions, respectively.

Construction and demolition activities are known to account for a significant proportion, approximately 23% (see [1]), of the total solid waste stream. This translates to an annual waste volume exceeding 100 million tonnes globally. Numerous surveys conducted worldwide, including those conducted in the United States, corroborate these estimates. Notably, a portion of the waste generated from stock reduction is preventable and stems from improper utilization of materials.

Efficient utilization of supplies could result in a reduction in the number of excess materials acquired, unnecessary craftsmanship, waste generation, as well as the associated costs of transportation and disposal. Therefore, emphasizing the importance of optimizing the cutting process in construction projects becomes crucial for minimizing waste and improving overall efficiency [1].

Indeed, optimizing resource utilization is not only beneficial to industrialists, but it also holds immense significance for the well-being of our planet. The disposal of waste resulting from stock-cutting operations can lead to pollution, while excessive wastage contributes to the depletion of our Earth's invaluable resources. By emphasizing the importance of cutting losses through optimization, we can effectively reduce our environmental impact and safeguard the future of our planet [2].

Cutting losses is possibly the most major source of steel waste. Cutting losses arise when normal steel lengths are shortened to fit the project's required lengths. Shahin et al. [3] pointed out the main causes deriving from cutting losses and provided some suggestions for optimal stock-cutting planning:

- dividing an order into separate, smaller orders typically results in more waste due to fewer cutting alternatives;
- using inefficient cutting patterns in the cutting schedule results in the generation of avoidable waste that could be avoided through better stock-cutting planning;
- using the optimum cutting patterns may result in unavoidable waste which is the minimum waste generated if the optimum cutting patterns are used

In this study, the potentiality of the cutting stock problem (CSP) is demonstrated and a novel optimization framework is developed by adopting a modified Genetic Algorithm (GA). Benefits derived from the proposed method are tested on a structural application case study inspired by a real-world structure.

2 Survey on CSP techniques and applications

In literature, Cutting and packing (C&P) problems appear under various specifications as cutting problems, knapsack problems, container and vehicle loading problems, pallet loading, bin packing, assembly line balancing, capital budgeting and changing coins [4]. All of these problems have essentially the same logical structure. One of the most prominent challenges in this domain is the bin packing problem (BPP), which addresses the optimization of item placement within a container. Practically, the goal is to minimize the number of containers (referred to as bins) required to accommodate a given inventory of goods. Specifically, the BPP can be defined as follows: given a set of n items, each characterized by an integer weight w_j $(j = 1, ..., n)$, and an unlimited supply of identical bins with an integer capacity c, the objective is to effectively allocate the items into as few bins as possible, while ensuring that the total weight packed into each bin does not exceed its specified limit [5].

Moreover, the cutting stock problem can be classified as a one-dimensional and two-dimensional problem [5]-[2].

The one-dimensional cutting stock problem (1D-CSP) involves the extraction of a specified set of order lengths from stock rods of fixed length. The primary objective is to minimize the number of rods (material input) required. On the other hand, the two-dimensional two-stage constrained cutting problem (2D-2CP) focuses on selecting the most valuable group of rectangular objects from a single rectangular plate of infinite length. It is worth noting that in the context of the two-dimensional cutting stock problem, shapes can be either regular or irregular, with the latter scenario referred to as nesting, posing a more challenging problem that demands a more intricate solution [4].

These types of problems fall under the purview of complex combinatorial optimization, constituting a strongly NP-hard problem from a mathematical standpoint. Consequently, numerous approaches have been proposed over the years, including linear programming techniques, heuristics, and metaheuristic methods, to tackle the complexities associated with these problems [6]-[7].

The initial attempts to tackle cutting and packing (C&P) problems can be attributed to Kantorovich in the 1960s [8], although his approach was limited in its applicability to small-scale cases. However, it played a crucial role in shaping problem understanding.

To address C&P problems, various deterministic and heuristic approaches have emerged, utilizing the technique of linear programming (LP) relaxation. These approaches involve solving the problem initially as an LP problem and subsequently converting the solution to an integer form. Relaxation results in removing the integrality constraints of variables, allowing the IP problem to be solved as an LP problem. This transformation converts the NP-hard optimization problem of integer programming into a polynomial-time solvable problem. By solving the related knapsack issue, Gilmore and Gomory [9] provided a novel method to identify the cutting patterns required to enhance the LP solution. They proposed a column generation approach inspired by the procedure developed by Dantzig and Wolfe [10] for decreasing stock and bin packing concerns

(BBP). Because enumerating all possible cutting patterns would take an inordinate amount of time, it reduces valid patterns repeatedly and adds them to the issue based on their contribution to the objective function. The column generation approach made large-scale cutting stock issues solvable in a reasonable amount of time.

In the last decade, soft computing techniques were largely used to solve these kinds of problems. Specifically, Genetic Algorithm [11], Evolutionary approach [13] and simulated annealing [14] still nowadays represent the most promising approach in terms of both computational effort and accuracy of the optimal result obtained.

Few applications have been recognized in the literature in which optimization methods and C&P formulation are adopted simultaneously to solve Civil Engineering problems or, by considering, optimal cutting stocks at the first stage of design and construction. Mainly, researchers focused on 2D structures like frames and trusses with the aim to minimize the waste during the fabrication phase or by reusing stock materials from other structures. Normally, the adopted objective functions were formulated taking into account structural cost parameters and environmental indices. In these works realized by Brutting et al. [15]-[17], comparisons between the 1-to-1 assignment of elements to positions in the truss and a cutting stock approach in which multiple members can be cut from individual elements are pointed out. The authors proposed a procedure to optimize the configuration of stock or kit-of-parts such that its elements can be reused in various structures. This last consideration allowed to spread of the stock of reusing items in many structures and the outcome leads to an ulterior minimization of the waste. The feasibility of the proposed approach was tested by the same authors in [18] for three different real-world application case studies. The proposed method includes form finding and digital fabrication and it applies to the design of trusses, gridshells, and space frames.

Finally, benefits derived from the proposed reusing procedure were illustrated by the same authors in [19] where the problem statement entirely based on a combination of Life Cycle Assessment (LCA) analysis and discrete optimization was introduced.

3 Combined CSP-GA optimization procedure

In this section, an optimization procedure for the minimum structural cost of spatial structures has been developed. The proposed method uses the potentiality of the Cutting Stock Problem for solving combinatorial problems and the effectiveness of the Genetic programming during the exploration and exploitation phase of the algorithm. Then, the performance of the proposed method will be checked in a real case study.

3.1 Problem statement and setting parameters

The statement of the entire optimization process is the following:

$$\min f(x) = W(x) \tag{1}$$

$$\textbf{Where} \quad W(x) = \phi_1 \rho \sum_{g=1}^{g=k} n_g A_g L_g \tag{2}$$

$$\textbf{Subject to} \quad \frac{N_{ED}}{N_{t,RD}} \leq 1 \tag{3}$$

$$\frac{N_{ED}}{N_{c,RD}} \leq 1 \tag{4}$$

$$\frac{N_{ED}}{N_{b,RD}} \leq 1 \tag{5}$$

$$u_{max,x} \leq u_{lim,x} \tag{6}$$

$$u_{max,y} \leq u_{lim,y} \tag{7}$$

The goal of structural optimization is the minimization of the objective function $W(x)$ (see Eq.(2)) correlated to the stock mass e subjected to various structural and serviceability constraints. Specifically, $W(x)$ is the optimization objective function (OF) and represents the total mass evaluated as the sum of the purchased bars' weight for each group g. Specifically, k represents the total number of groups of elements with the same cross-sectional areas. n_g, A_g and L_g are the cardinality, the cross-sectional area and the length of bins belonging to the same group g of elements with the same cross-sectional area A_g, respectively. ρ is the material mass density assumed to be equal for all members composing the structure (in this case the steel density).

Equ.s (3) - (7) represent the structural constraints of the problem.

In detail, strength verifications about tensile stress (without any holes) and compression stress according to Eurocode 3 (EN 1993-12005 and EN 1993-2 2006) are introduced by Equ.s (3) and (4), respectively. Other constraints to satisfy is the maximum deflection along x and y directions (i.e. Equ.s (6) and (7)).

The mentioned-above constraints were considered in the optimization process through a suitable penalty coefficient ϕ_1. In this way, the unfeasible solutions have been penalized with respect to the feasible ones.

The resultant penalty, ϕ_1, applied to the OF is the sum of the single penalty related to each constraint's violation:

$$\phi_1 = \phi_{Nc} + \phi_{Nt} + \phi_{Nb} + \phi_{ux} + \phi_{uy} \tag{8}$$

More in detail, the violation functions are simply equal to the sum of the verification ratios

$$\phi_Q = \sum_{i}^{i=v} \frac{Q^i_{Ed}}{Q_{lim}} \tag{9}$$

Where Q is the constraint parameter (stress or deflections) for an element of the truss i which does not satisfy the constraint condition. The overall penalty related to each solution, ϕ_Q, is the sum of the violation of all the violated items v. Particularly, the effective solicitation or deflection is at the numerator, Q_{Ed}, while the allowable value is at the denominator, Q_{lim}. The displacements of the free nodes in both directions had to be less than $L/300$ according to tumble recommendations largely adopted by practitioners.
The algorithm setting parameters are collected in table 1.

Table 1. Optimization algorithm parameters set by the operator.

Parameter	**Value**
Maximum number of iterations	200
Number of individuals per population	200
Mutations' probability	1%
Stagnation condition	10 iterations

4 Case study: Geodesic dome

In this section, the results obtained by adopting the optimization framework described in the previous section will be shown.

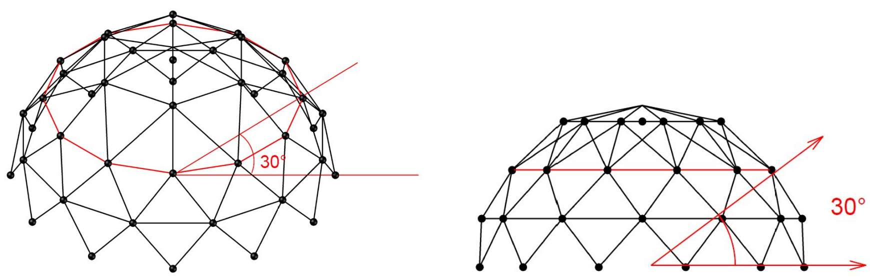
4.1 Description of the case study

The geodesic dome is composed of 102 bars with 7 different lengths and inclinations and connected one each other with 43 joints. The reticulated dome is made of steel with a mass density of $\rho = 2.768 kg/m^3$, elastic modulus $E = 275 GPa$ and constrained by simple supports at each joint of the bottom level. More in detail, each level of the structure is realized by circular hoops, lying in the horizontal plane, and diagonal bars. Specifically, the first and second level is composed of 24 and 12 bars for the diagonals and the circular rings, respectively. Indeed, the third and four level of diagonals is composed of 18 and 6 bars, respectively, connected by a six-bar horizontal hoop. Finally, the kit-of-part is composed of tubular elements (CHS profile) connected with spherical nodes. In Fig. 1, an overview of the structure is shown and the theoretical ring which distinguishes the compressed ring from the tensioned one is reported in red.

The discrete design variables are chosen by selecting the cross-sectional areas from a standard list (EN 10210) with lower and upper bounds assumed to be equal to 182 and 24700 mm^2, respectively.

In the proposed method, the structure is subjected to only self-weight. Hence, the structural weight and, subsequently, the employed number of bins change iteration by iteration once the set of cross-sectional areas is selected by the optimizer.

As demonstrated in previous works of one of the authors (e.g. [20], [21]), adopting a feasible grouping strategy is fundamental for reducing the computational effort and the global robustness of the entire optimization algorithm. Additionally, the feasible combinatorial solutions (i.e. number of bins), obtained by solving the CSP problem, are dramatically reduced. In this work, different cross-sectional areas are selected for each level of diagonals and horizontal rings. In Fig. 2, elements with the same colour are assigned to the same cross-sectional areas. Moreover, the grouping strategy is based on the level of stress in the structure: from the top to the bottom, horizontal rings and diagonals are always tensioned and compressed, respectively, with an increasing level of magnitude. For these reasons, different cross-sections (e.g. colours) were assigned to the diagonals and horizontal members at each level of the structure.



(a) (b)

Fig. 1. Prospective (a) and perspective (b) view of the dome

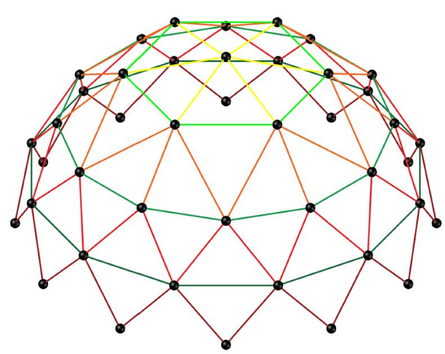


Fig. 2. Grouping strategy for the structural sizing. Each colour is representative of a specific cross-section property.

4.2 Results and discussion

In this section, the results of the optimization process with specific regard to the investigated case study are pointed out. Specifically, two optimization scenarios have been performed:

- Scenario (a): optimization by considering CSP procedure (minimization of purchased steel bars);
- Scenario (b): optimization via traditional approach by minimizing the total weight of the structure without considering the CSP procedure (see [22], [23]).

Since the two approaches adopt different single-objective functions (minimum weight vs a minimum number of bins), the CSP procedure has been performed at the end of (b) such that the total number of bins requested for the assemblage of the optimized structure has been evaluated and a comparison with the optimal number of bins obtained by (a) would be possible. In this way, the benefit derived from incorporating the CSP procedure into structural optimization would be emphasized.
The problem statement described in the previous section has been adopted for the optimal design of the dome and 10 runs have been performed to assess the robustness and accuracy of the method. For simplicity, the authors will show the minimum value of the OF, among all runs, obtained for both scenarios. Taking a look at Fig.3(a)-(b), the advantages in terms of material saving derived from adopting the proposed method are quite evident. The optimal design of the dome obtained with a simple minimization of weight (scenario b) leads to significant material waste demonstrated by the no-optimal cutting pattern of pieces. Due to the adopted grouping strategy, 7 different cross-sectional areas have been selected by the optimizer aiming to maximize the structural performance of each group element. On the contrary, in scenario (a) where the CSP has been solved at each iteration of the optimization process, the total amount of bins requested at the production stage is dramatically reduced and the number of different cross-sections decreased to 4. The effect of coupled CSP with structural optimization becomes more evident through the comparison between the total amount of bins obtained by both the investigated scenarios. On the contrary, the structural mass derived by (b) results to be lower than the one obtained by (a). However, the gain in terms of mass waste evaluated in (a) seems to be more significant than the loss in terms of structural weight detected in (b).

5 Conclusions and future developments

In this work, a novel optimization procedure for the minimum structural waste has been introduced. The feasibility of the procedure has been tested and the convenience to consider constructability issues as cutting patterns during the production phase has been verified. The strategy to adopt the cutting stock procedure as an internal routine embedded in a well-known optimization algorithm,

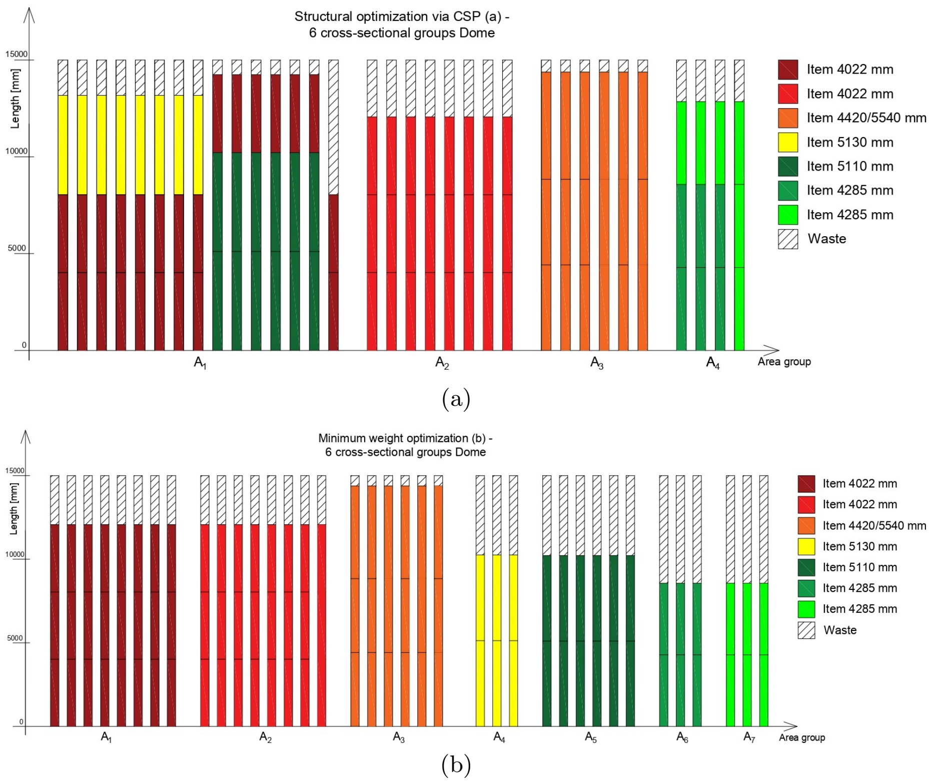


Fig. 3. Optimal cutting pattern obtained by scenario (a) and scenario (b)

such as the Genetic algorithm, allows to achieve a significant reduction of the number of bins with a negligible increase in the structural weight. In future works, several improvements could be realized by considering geometry parameters as the total height of the dome or with new problem formulations entirely based on environmental impact indices.

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