Influence of the effective strain ratio on equivalent linear ground response simulations

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ABSTRACT

The design of buildings and infrastructures in seismic areas has to account for site effects, as they dramatically affect the expected ground shaking. The equivalent linear approach is commonly adopted for the numerical simulation of seismic site response. This scheme models the nonlinear soil behaviour as a linear viscoelastic medium, characterized by strain-compatible secant shear modulus and damping ratio extracted as a function of an equivalent uniform strain, usually termed effective shear strain. The effective strain is computed as the product between the maximum shear strain and the effective strain ratio, which is a scaling factor conventionally equal to 0.65 or linearly increasing with the magnitude. However, the proposed values are the result of recommendations without a rigorous demonstration and their reliability has been questioned. This study investigates the influence of this parameter on a collection of 1-D ground models, which are subjected to a set of acceleration time histories recorded from earthquakes with different intensities. Models are generated from a database of real soil profiles through a stochastic procedure and they are representative of a broad variety of soil deposits of engineering interest. The study addresses the sensitivity of the predicted ground motion amplification to variations in the effective strain ratio, considering the role of soil deformability, ground motion characteristics, and the investigated amplification parameter. This study contributes towards a more robust prediction of ground motion amplification of soil deposits, enhancing the reliability of design in seismic areas.

Keywords: site effects, numerical analysis, soil nonlinearity, epistemic uncertainty

1 INTRODUCTION

The estimation of the stratigraphic ground motion amplification typically relies on the Equivalent Linear (EQL) approach. This scheme models the nonlinear soil behaviour as a linear viscoelastic medium, characterized by secant shear modulus and damping ratio that are compatible with the strain level in the soil (Idriss & Seed 1968; Seed & Idriss 1969; Schnabel *et al.* 1972). Indeed, a fully NonLinear (NL) approach reproduces the actual changes in soil stiffness occurring during the earthquake, but its complexity limits the use in ordinary applications (e.g., Kwok *et al.* 2008; Stewart & Kwok 2008; Hashash *et al.* 2010; Kaklamanos *et al.* 2015; Régnier *et al.* 2018).

In the EQL scheme, the estimation of the strain-compatible parameters is based on an effective shear strain, which is representative of the actual shear strain history. This value is taken as a fraction of the maximum shear strain, using an effective strain ratio, R, smaller than the unit. The adopted R should account for the characteristics of the strain time history, to avoid excessive deamplification (for instance, when the peak shear strain is much larger than the remainder of the shear strains) or to an underestimation of soil nonlinearity (e.g., when the shear strain amplitude is nearly uniform; Kramer 1996). Conventionally, R is assumed as equal to 0.65 (Idriss 1991; Sun & Idriss 1992; Kottke & Rathje 2009). Alternatively, it can be computed as a function of the earthquake magnitude M, typically through the relationship $R = (M-1)/10$ (Sun & Idriss 1992). On the other hand, several studies based on empirical observations claimed that the recommended value for this parameter may span over a broad range, even from 0.2 to 1.0 (Yoshida *et al.* 2002). Nevertheless, no method is currently available for an a priori estimation of R, to our knowledge. For this reason, some Authors proposed an alternative approach, termed as frequency-dependent EQL. This scheme explicitly models the frequency-dependence of soil parameters and derives the strain-compatible values at each frequency, without involving R (Kausel & Assimaki 2002; Yoshida *et al.* 2002). However, its use is still limited to research applications and the improvement in the estimation accuracy is not significant, compared with the EQL approach (Zalachoris & Rathje 2015).

This paper presents a preliminary investigation on the influence of *R* on the predicted ground motion amplification, considering a collection of 1-D ground models, which are subjected to a set of acceleration time histories with different intensities. Soil models were generated from a database of real profiles through a stochastic procedure and they are representative of a broad variety of soil deposits of engineering interest. To assess the sensitivity of the predicted ground motion amplification to changes in *R*, multiple suits of EQL simulations were run, considering *R* equal to 0.45, 0.65, and 0.85. For each *R*, the estimated amplification was compared with results of NL analyses, that were taken as reference. The influence of *R* was investigated accounting for the soil deformability, ground motion characteristics, and the considered amplification parameter.

The paper starts with a brief description of the procedure of construction of the database of Ground Response Analyses (GRAs), with a focus on the main features of soil models and input motions. This section ends with a list of the considered ground motion amplification parameters. The results section reports an overview of the influence of the investigated parameters, taking the NL simulations as reference. Finally, the results are discussed to highlight the impact of commonly adopted *R* values on the amplification factors predicted by EQL analyses.

2 METHODOLOGY

The stochastic database of GRAs is a collection of the results of 1,421,000 numerical simulations performed over a set of 10,150 1-D ground models, the latter being representative of different stratigraphic conditions. The simulations were performed through the EQL and the NL approach, using the DEEPSOIL version (v.) 7.0 software (Hashash *et al.* 2017). A description of the generation of the ground models, the selection of the seismic inputs and the procedure of interpretation of the results is presented in the following sections. Further details are available in Aimar *et al.* (2020); Aimar & Foti (2021); Paolucci *et al.* (2021).

The models were generated from a collection of 252 real soil deposit profiles, through a Monte Carlo procedure which generated a proper suite of S-wave velocity, V_S, models. For this purpose, the geostatistical model proposed by Passeri *et al.* (2020) was used, referring to the Toro model (Toro 1995) for the statistical parameters. This solution allows the definition of realistic models. The resulting distribution is a population of 10,150 V_S profiles. The resulting realistic models are representative of different soil deposits of engineering interest. The soil nonlinear behavior was described using the formulations for the modulus reduction and damping ratio curves proposed by Darendeli (2001) for clays and sands, Rollins *et al.* (1998) for gravels, and Sun & Idriss (1992) for weathered rocks. The cyclic shear stress–strain relationship was introduced in NL simulations through the modified Kondner–Zelasko model (Kondner & Zelasko 1963; Matasovic & Hashash 2012), the parameters of which were calibrated according to the pressure-dependent hyperbolic model with damping reduction factor (MRDF procedure; Phillips & Hashash 2009). NL analyses also require the definition of a viscous damping ratio component, assumed equal to the small-strain hysteretic damping estimated from the damping curves (Kwok *et al.* 2007), and it was incorporated into the NL GRAs with the frequency-independent damping formulation (Phillips & Hashash 2009).

The generated ground models were subjected to a collection of natural acceleration time histories. The motions are clustered into suites of seven motions, each one compatible with the seismic hazard of 5 locations in Italy, characterized by different levels of seismic intensity (shown in Fig. 1). This solution ensures the representativeness of typical features of the seismicity in Italy.

The numerical analyses were finally performed for each input motion and then logarithmically averaged. The result is a distribution of ground motion amplification for each reference site, as a function of the deformability and the thickness of the soil deposit.

The interpretation of the results referred to synthetic parameters describing the ground motion amplification. On the one hand, this study focused on the peak ground acceleration amplification (*PGAA*), i.e. the ratio between the peak ground acceleration computed on the surface PGA_s and the peak ground acceleration for the reference rock outcrop condition PGA_R (i.e., the input one):

$$PGAA = \frac{PGA_S}{PGA_R} \quad (1)$$

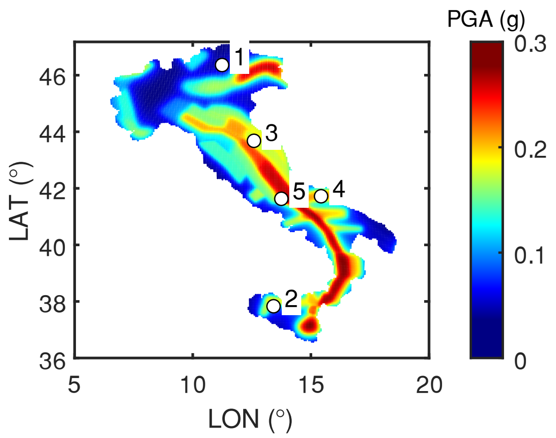

Fig. 1 Position of the reference sites in the Italian seismic hazard map, representing the expected peak ground acceleration for the reference rock outcrop condition, for a return period of 475 years (after Aimar *et al.* 2020).

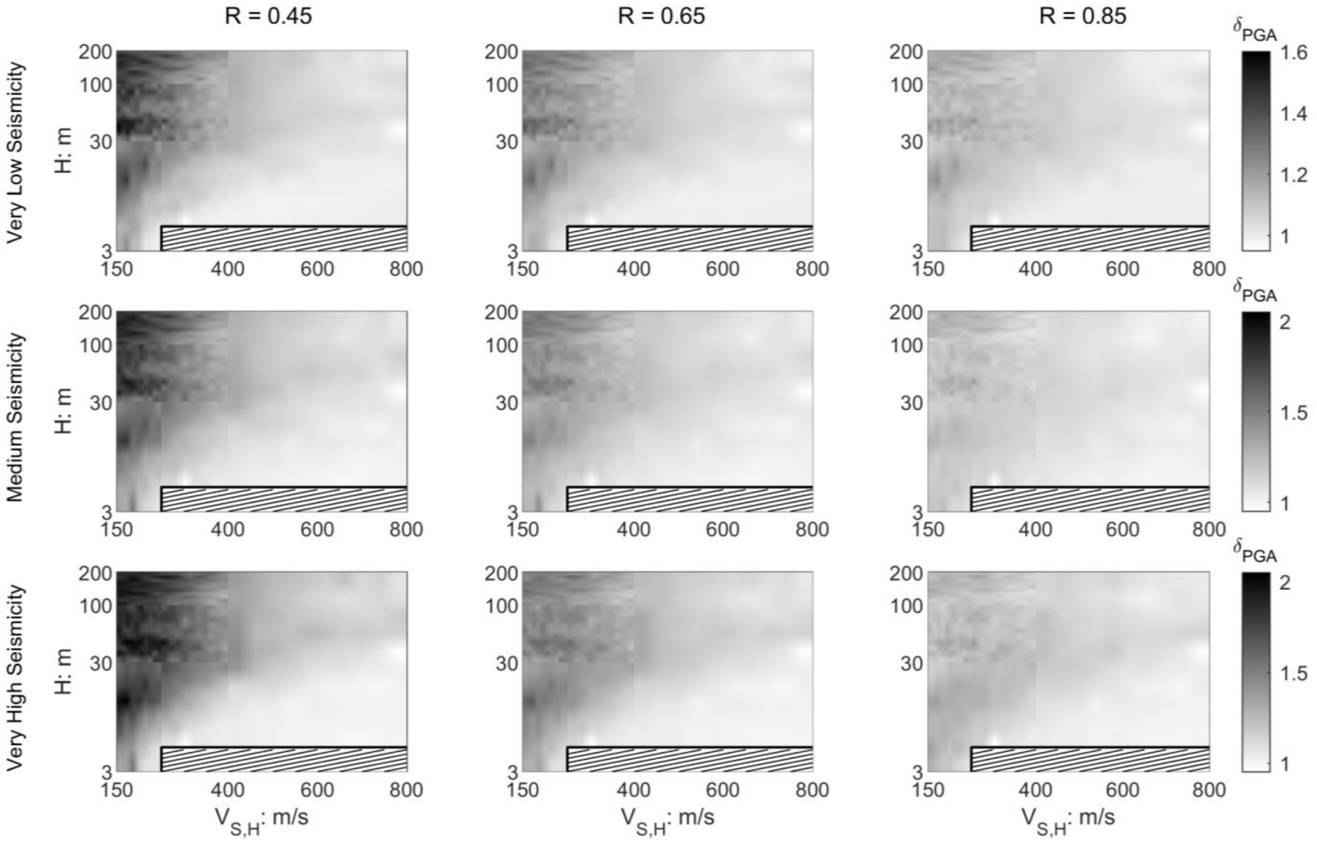

Fig. 2. Distribution of δ_{PGA} as a function of R and seismicity level; the dashed area denotes the region not considered in GRAs.

Indeed, this parameter is relevant for liquefaction assessment (Youd & Idriss 2001) or pseudo-static approaches for estimating earth pressure (Okabe 1924; Mononobe 1929).

Furthermore, this study referred to a short-period spectral amplification factor *SPSA*, defined as ratio between the spectral intensity I_s on the surface and the value for the reference rock outcrop condition I_R:

$$SPSA = \frac{I_S}{I_R} \quad (2)$$

Spectral intensities I_s, I_R are integral quantities of the elastic response spectrum S_e, evaluated for vibration periods ranging from 0.1 s to 0.5 s (Centro per la Microzonazione Sismica e le sue applicazioni 2017):

$$I_j = \int_{0.1\,s}^{0.5\,s} S_e(T)\,dT \qquad j = s, R \quad (3)$$

This parameter is deemed to be relevant for short buildings, for which the EQL approach is typically employed.

In this study, differences between the EQL and NL estimates were quantified through the ratio between the corresponding estimates of the AF *X*, where *X* is PGAA or SPSA. The quantity is denoted as δ_X:

$$\delta_X = \frac{X_{EQL}}{X_{NL}} \quad (4)$$

A value larger than 1 indicates overestimation of the AF from the EQL scheme with respect to the NL approach.

3 RESULTS

Fig.2 shows the distributions of δ_{PGA} as a function of R and seismicity level. The distributions are developed by clustering the results based on the seismic bedrock depth H and the time-averaged shear wave velocity of the soil profile down to the bedrock depth $V_{S,H}$ (European Committee for Standardization 2020).

The larger differences between EQL and NL analyses are observed, as expected, for soft soil profiles characterized by low $V_{S,H}$ values. Such differences increase with increasing level of seismicity as nonlinearity becomes more relevant. For such soils, the assumption of an R value equal to 0.45 leads to severe underestimation of nonlinearity effects and, consequently, to a strong overestimation of the *PGA* amplification factor (i.e., $\delta_{PGA} >> 1$). On the other hand, it is interesting to note that the best estimate of the *PGA* always seems to be associated with an assumed R value of 0.85, for which δ_{PGA} is closer to unity for the whole range of soil profiles and seismicity levels analyzed.

The distributions obtained taking into account the short-period amplification factor are shown in Fig. 3. The main trends previously identified for δ_{PGA} are even more pronounced when considering δ_{SPSA}. For low seismicity levels, the assumed R value is only partially relevant, as the soil response remains within the small-to-moderate strain field.

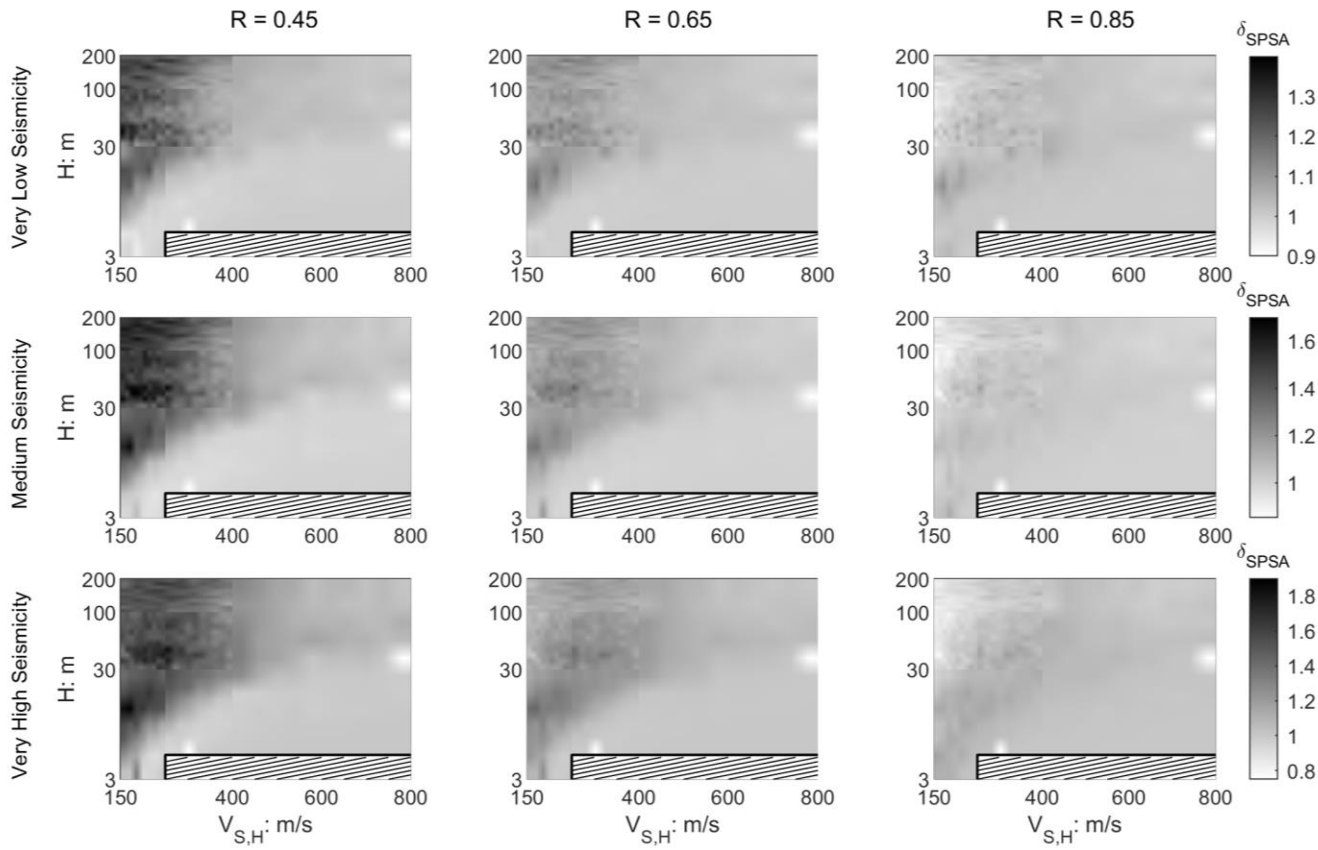

Fig. 3. Distribution of δ_{SPSA} as a function of R and seismicity level; the dashed area denotes the region not considered in GRAs.

However, the differences between EQL and NL analyses increase with increasing seismicity, especially for soft soil profiles. Consequently, the influence of R also becomes relevant. For a very high level of seismicity, a maximum δ_{SPSA} of about 1.9 is observed when assuming $R = 0.45$, which corresponds to a strong overestimation of the EQL analyses compared to the NL analyses over a wide range of vibration periods (namely 0.1s to 0.5s). With increasing R values, δ_{SPSA} tends to decrease, approaching (and sometimes going below) the unity. The best estimate of the short-period spectral amplification factor again seems to be provided by the EQL analyses performed with $R = 0.85$.

In order to systematically interpret the results of the analyses, the data were clustered according to the subsoil classification scheme proposed for the new Eurocode 8 (European Committee for Standardization 2020). Specifically, for each soil profile belonging to a given soil class, the "optimum" R value was identified as the value giving δ_{SPSA} closer to unity (that is, results of the EQL and NL analyses are the same). Then, for each R-value analyzed (namely 0.45, 0.65 and 0.85), the percentage of profiles for which this R value corresponds to the optimal one was calculated. The statistical analysis was then repeated for all the seismicity levels analyzed.

The results of the statistical analysis are presented in the bubble chart of Fig. 4, where the seismicity level is represented according to the 475-years return period peak ground acceleration for the reference rock outcrop condition PGA_R, the latter being an intensity measure characteristic of the seismicity of the site. For a given R value and PGA_R, the size of the scatter points is defined according to the percentage of profiles for which $R = R_{opt}$.

The plot shows that the best estimate of the short-period amplification factor from EQL analyses (with respect to NL analyses) is always obtained by assuming $R = 0.85$, regardless of the soil class and seismicity level considered.

This result is quite surprising, as it would suggest the adoption of an R substantially larger than 0.65, the value commonly adopted for GRAs. Furthermore, it also appears that the R value to be adopted is not significantly influenced by the entity of the seismic action, contrary to the usual recommendations also proposed in the predictive empirical relations for the estimation of R.

However, when considering the R value to be used for EQL analyses in common practice, not only the size of the error should be considered, but also its sign. Looking at the results presented in Fig. 2 for $R = 0.85$, it is indeed worth noting that a significant number of EQL simulations carried out on highly deformable, deep soil profiles have δ_{SPSA} less than unity. For these profiles, the adoption of $R = 0.85$ would result in an underestimation of the short period amplification factor.

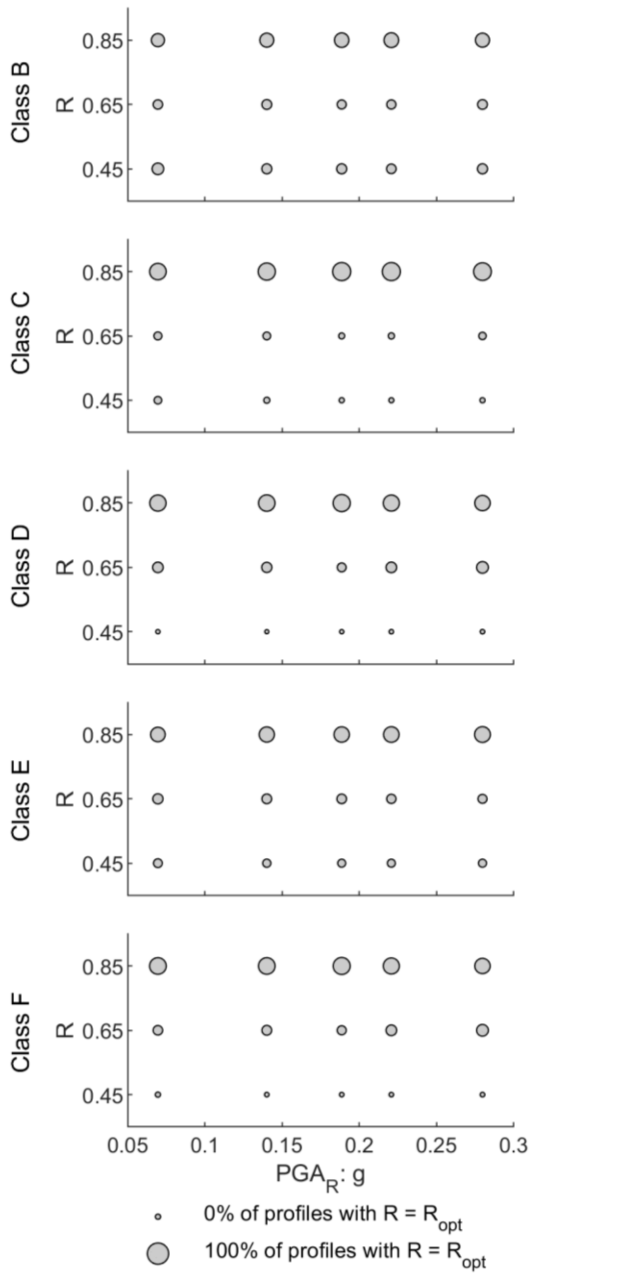

Fig. 4. Bubble chart representing the percentage of profiles for which $R = R_{opt}$ as a function of adopted R value and seismicity level.

Table 1 reports the percentage of EQL analyses that significantly underestimate the short-period amplification factor (i.e., $\delta_{SPSA} < 0.95$) in relation to the subsoil class, the seismicity level and the adopted R value. For low seismicity levels, only a few EQL analyses provide a relevant underestimation of the seismic action, regardless of the adopted R. However, as the entity of the seismic action increases, also the number of profiles increases. Specifically, a moderate percentage of profiles with $\delta_{SPSA} < 0.95$ is observed when R = 0.45-0.65. Conversely, in several cases EQL analyses underestimate the seismic action when R = 0.85. In particular, the percentage increases as moving towards deep and soft soil profiles.

Table 1. Percentage of EQL analyses underestimating short-period soil amplification ($\delta_{SPSA} < 0.95$) by subsoil class.

Subsoil Class	Very Low Seismicity			Medium Seismicity			Very High Seismicity		
	R =			R =			R =		
	0.45	0.65	0.85	0.45	0.65	0.85	0.45	0.65	0.85
B	0.3	0.0	0.3	0.0	0.8	25.7	0.0	0.5	14.3
C	0.6	0.1	2.7	0.2	1.4	42.3	0.1	0.8	32.7
D	0.7	0.3	2.8	0.3	2.8	37.4	0.2	1.5	32.9
E	0.6	0.4	3.7	0.4	5.0	49.7	0.2	3.5	43.4
F	1.0	0.7	7.6	0.7	6.0	59.6	0.3	5.0	52.0

4 CONCLUSIONS

This paper has investigated the influence of the effective shear strain ratio on the amplification factors predicted by EQL ground response analyses. Albeit preliminary, the systematic comparison between EQL and NL analyses has shown a rather interesting overall picture. The "optimum" R value (intended as that which gives the smallest error of estimation in absolute value) is likely to be larger than the conventionally adopted R = 0.65. Furthermore, this value does not seem to be particularly influenced by the entity of the seismic input.

However, looking at the sign of the prediction error, it is quite clear that the assumption of large R values can often lead to an underestimation of the short-period spectral amplification factor. Such an underestimation is more likely to be observed for deep soft soil profiles, especially in regions of high seismicity. Therefore, when defining the R value to be used for EQL analysis, it is necessary to find a balance between the "optimal" value that would produce the smallest estimation error and a practical value that can be effectively used in common practice.

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