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Smart construction of fibre-reinforced concrete structures: size-scale effects on minimum reinforcement and plastic rotation capacity

Federico Accornero[1*], Alessio Rubino[2], Giuseppe C. Marano[2] and Alberto Carpinteri[1]

Abstract

Advanced structural design approaches should consider the economic and technological benefits offered by the structural applications of fibre-reinforced concrete. In this framework, it is important to highlight how the ductility of fibre-reinforced concrete structures is strongly dependent on the fibre volume fraction together with the structural size. This crucial coupling induces two reverse ductile-to-brittle transitions in the mechanical response of fibre-reinforced and hybrid-reinforced concrete elements: by increasing the characteristic size of the structure, an increase in its load-bearing capacity can be observed together with a decrease in its plastic rotation capacity. These size-scale effects can be taken into account by an effective fracture mechanics approach represented by the Updated Bridged Crack Model (UBCM), which can provide significant improvements in current Standards and regulations on fibre-reinforced concrete structures.

Keywords Fracture Mechanics, Fibre-reinforced concrete, Minimum reinforcement, Plastic rotation capacity, Size-scale effects, Ductile-to-brittle transitions

1 Introduction

Conventional reinforced concrete (RC) played a crucial role in the global economic and social development of the last fifty years. Although this material is mainly associated with civil engineering structures such as dams, roads, bridges, and railways, its use has been also extended to residential and industrial constructions. RC is realized thanks to the perfect match between linear reinforcing bars and concrete matrix, which guarantee strength and durability, so that it has allowed in the last century the construction of large infrastructures, which have enabled the development of commercial and industrial activities. In addition, RC technology has enabled the construction of high-rise and earthquake-resistant buildings, improving people's quality of life and encouraging population growth in urban areas. Invented in 1849 by Joseph-Louis Lambot, RC technology has been refined in the late nineteenth century, when it spread all over the world. Its use was particularly relevant in the post-war period, when Europe needed to rebuild many of its cities destroyed by the conflict. This material has represented a true revolution in building the new architecture of the last century. Such a success went beyond the mere structural framework, also conditioning the history of architecture and the conception of our modern cities, by defining in an innovative and modern way the building volumes, relegating other forms of construction to the past. In summary, RC has been one of the main driving factors for economic and social development in the

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last fifty years, influencing not only civil engineering and construction, but also architecture and urban planning.

The basic principle of RC behavior lies in the remarkable characteristics of compressive strength of concrete, although it is associated with a limited tensile strength, which requires the use of elements capable of withstanding the tensile forces to be overcome.

In the case of RC, the solution now widely established in the design practice consists in reinforcement steel bars, intended as elements that are embedded before casting of the concrete matrix. Specifically, the stress transmission at the interface between the reinforcement bars and the surrounding concrete matrix plays a vital role in the mechanical behavior of RC. As required by current international rules, the bar-matrix perfect bond results in a reduction of the possible crack widths during the structure lifetime and an increase in the bearing capacity of the structural element.

This technology has been used for over fifty years, with minor variations and always relying on the solidarization of two systems, i.e., concrete matrix with reinforcement layers. In addition, it is worth noting that, since the first experiments on RC, the presence of diffuse reinforcements, such as fibres, variously distributed in the matrix was identified as a significant improvement for the global structural response.

Despite its simplicity, fibre-reinforced concrete (FRC) underwent an interesting diffusion only in the last decades, following the development of technologies to produce metal fibres of suitable characteristics, i.e., mechanical capabilities and sizes, as well as a consistent improvement in FRC production. As a matter of fact, mixing of concrete with fibres still remains a significantly more complex production process than the production of traditional reinforced concrete, especially for the need to ensure adequate orientation and distribution of fibres in the concrete matrix.

Fibre-reinforcements play the key role of bridging the dominant crack and the micro-cracks present in the process zone of the cement matrix, in order to prevent their coalescence and subsequent propagation within the bulk of the material (Fig. 1). These bridging mechanisms offered by fibre-reinforcements represent a first benefit of FRC over the more traditional structural solution represented by RC, in which the bridging action carried out by the steel-bar reinforcement invests the single macro-crack.

In addition, recent studies [1–3] have demonstrated that the above-mentioned bridging mechanisms are mainly due to pull-out and friction between the cement matrix and the reinforcing fibres. The result of this interaction consists in a more effective global response of the structural element, also thanks to the improvement in the mechanical properties of the material, including tensile, compression, and shear strengths, together with fracture toughness.

The flexural response of a FRC structural element, as represented in the load vs deflection diagram of Fig. 2, can be divided into three different stages. Before the fracture process begins, the FRC structural behavior exhibits a linear ascending branch (Stage I). After this, the fibre volume fraction, or V_f, is one of the parameters that

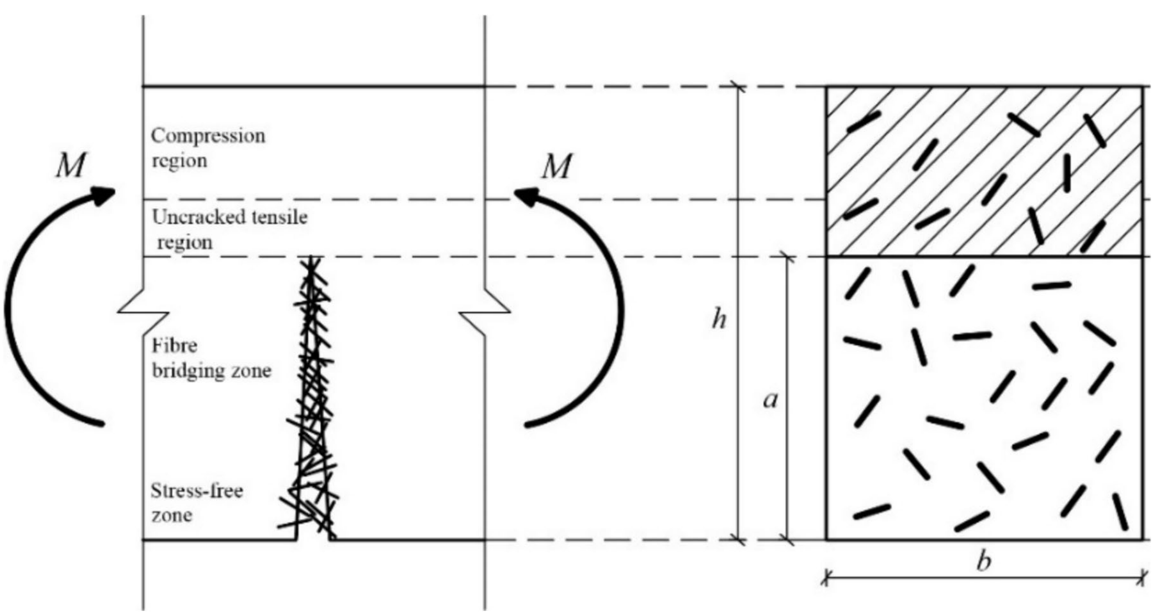

Fig. 1 Interaction between the two crack faces in a fibre-reinforced concrete (FRC) beam: Side view and cross-section

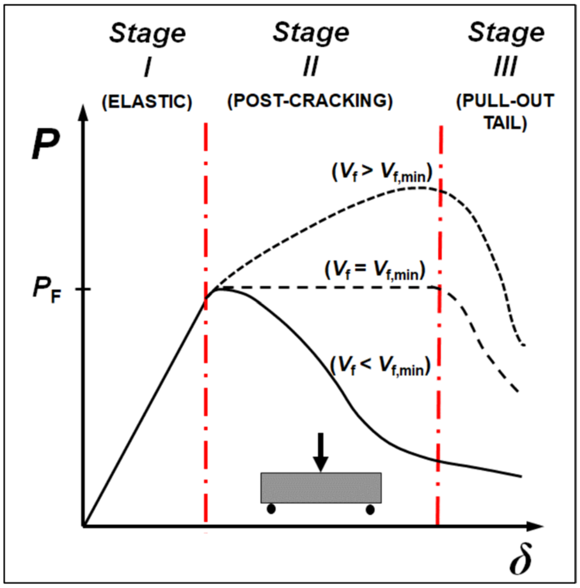

Fig. 2 FRC ductile-to-brittle transitions

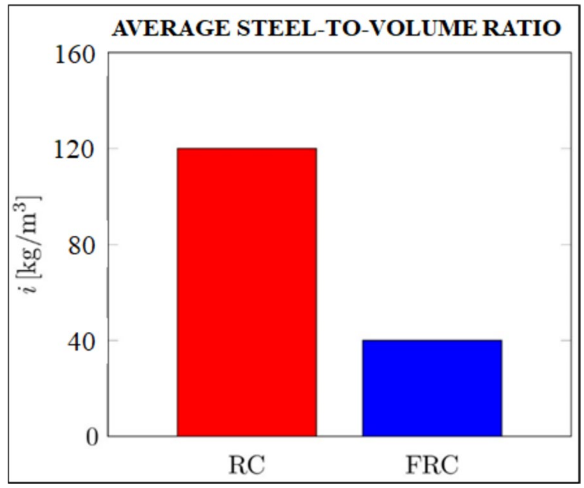

Fig. 3 Average steel-to-volume ratio in RC and FRC structures as required by international regulations [4, 5]

determines the post-cracking regime (Stage II). The so-called minimum fibre volume fraction, $V_{f,min}$, which is necessary to ensure the stability of the fracturing process, can be defined as the optimum condition between the deflection-softening and the deflection-hardening behaviors. This condition occurs when the ultimate capacity of the specimen, P_u, is equal to or greater than the applied load at the onset of crack propagation, P_F. On the contrary, if the fibre volume ratio is not sufficiently high, i.e., $V_f < V_{f,min}$, this intermediate stage may not develop. Ultimately, the fibre pull-out or rupture is represented by the descending tail (Stage III) of the load vs. deflection curve, which characterizes the last phase in the flexural response.

It is interesting to remark that, when $V_f < V_{f,min}$, the FRC composite is identified as Strain-hardening Fibre-reinforced Cementitious Composite (SHCC), or High-performance Fibre reinforced Cementitious Composite (HPFRCC), which is characterized by a hardening post-cracking response with the formation of multiple diffused cracks.

In this framework, it is worth noting that the bending behaviour of the FRC structural elements is governed by: (i) the fibre volume fraction, V_f; (ii) the characteristics of the reinforcing fibres (tensile strength, aspect ratio, and geometric profile) and of the brittle matrix (strength and toughness); (iii) the specimen size-scale.

Despite the obvious advantages, both in terms of material economy and construction technology, the use of FRC is still substantially confined to a minor function, thus attributing to the fibres mixed with the cement matrix a simple controlling role of the Serviceability Limit State. On the other hand, a careful and advanced structural design should take into account the potential of fibre-reinforced concrete also in terms of load-bearing capacity, without resorting entirely to the use of traditional reinforcing bars. An effective comparison between the average steel-to-volume ratio of RC and in FRC for load-bearing capacity is proposed in Fig. 3, where the economic advantage offered by the FRC becomes evident.

Then, a further advantage provided by FRC results in reducing one of the traditional on-site construction phases: in fact, the placing of reinforcements is partially incorporated by the concrete casting phase, differently from what usually happens for RC (Fig. 4).

In this regard, it is important to remark that the mechanical properties of FRC are strictly dependent on concrete mixing method and casting technology, as a consequence of the fibre distribution and orientation in the concrete matrix. In addition, incorporation of fibre-reinforcements in the matrix usually decreases the workability of concrete in a considerable way. Actually, adding large amounts of dispersed fibres to the concrete matrix, without sacrificing its workability, is a major technological problem. High fiber volume ratios require careful consideration of how to modify the concrete mix design, based on the type and dimension of the fibre. Superplasticizers and fine aggregates make this possible. For example, when high fibre volume fractions are added to the concrete matrix, the mix design can be altered by adding more fine aggregates and additives, which results in self-compacting concrete, thus requiring no vibration.

However, because of the fibre-reinforcement bridging action across the crack surfaces, FRC is typically

Fig. 4 Traditional construction site: The placing of reinforcements

characterized by a high residual tensile strength in the post-cracking regime, as well as an enhanced capacity to dissipate energy. When FRC is used to replace stirrups entirely or in part, it can improve the shear behavior of structural elements and relieve reinforcement congestion at critical sections, including beam-column connections in seismic applications. For structural applications, particularly for heavy or light prefabrication, the complete replacement of stirrups with fibres in all the regions needing the minimum amount of shear reinforcement—which is required to increase ductility and provide warning of impending collapse—is highly promising. Additionally, the uniform dispersion of short fibres throughout the concrete matrix provides isotropic characteristics that are uncommon in traditional bar-reinforced concrete materials [6–10].

Finally, it is worth recalling that current structural applications of FRC require, in general, the use of a certain percentage of conventional reinforcements in the form of bars or grids. Fibre-reinforcements can change the bond conditions of conventional steel and grid reinforcements in hybrid-reinforced concrete (HRC) structural systems.

2 The Updated Bridged Crack Model (UBCM): reinforcement and pull-out brittleness numbers

Taking into account the above-mentioned technological context, a thorough knowledge of the mechanical behaviour of FRC can lead to significant improvements in the structural design through an optimization of the material components. Therefore, the Updated Bridged Crack Model (UBCM) represents a Fracture Mechanics model able to effectively describe the structural response of FRC by taking into account both the brittle behaviour of the concrete matrix and the bridging action of the fibres in the fracturing process of the composite [11–17]. In particular, the UBCM can effectively reproduce the complex discontinuity phenomena characterizing the flexural response of FRC structures in the post-cracking regime: snap-back and snap-through instabilities, hyper-strength, progressive slippage of the fibres from the cement matrix, together with the plastic rotation capacity of the structural element.

The UBCM algorithm consists in 3 sets of equilibrium and compatibility equations, the solution of the structural problem being given by a system of $(2n+1)$ equations in $(2n+1)$ unknowns. In particular, we define n elastic equations:

$$\{w\} = \{\lambda_M\}M - [\lambda]\{F\} \tag{1}$$

where M is the bending moment resulting from the applied load, F_i are the bridging forces exerted by the n fibre-reinforcements (Fig. 1), λ_M and λ are the rotational compliances corresponding to the bending moment and to the bridging forces, respectively [11, 15]. Then, we determine n constitutive equations:

$$F_i = F(w_i) \tag{2}$$

representing the bridging action of the fibre distribution, depending on the n crack opening displacements, w_i [15]. Finally, we establish one crack propagation equation:

$$K_I = K_{IM} - \sum_{i=1}^{n} K_{Ii} = K_{IC} \tag{3}$$

where K_{IM} is the stress-intensity factor related to the bending moment M, K_{Ii} is the stress-intensity factor related to the bridging action of the i-th fibre [11], and K_{IC} is the fracture toughness of the brittle matrix. Thus, the driving parameter describing the structural problem is the crack length.

On the other hand, the $(2n+1)$ unknowns involved in the structural problem are: n bridging forces, F_i, n crack opening displacements, w_i, and the bending moment at the onset of crack propagation, M_F.

In addition, the UBCM is able to predict the ductile-to-brittle transition occurring in the structural response of a FRC element, which can be described by the Reinforcement Brittleness Number, N_P [11, 12]. This dimensionless number is function of the fracture toughness of the concrete matrix, K_{IC}, of the slippage strength of the fibres, σ_s, of the fibre volume ratio, V_f, and of the characteristic size of the structural element, h:

$$N_P = V_f \frac{\sigma_s}{K_{IC}} h^{1/2} \tag{4}$$

The Reinforcement Brittleness Number, N_P, turns out to govern the stability of the post-cracking behaviour of the FRC structure, together with its load-bearing capacity, by making explicit a size-scale effect on the minimum fibre volume fraction, V_f, which is proportional to the power-law $h^{-1/2}$.

On the other hand, the plastic rotating capacity of the FRC structural element can be expressed by the Pull-out Brittleness Number, N_w [15], which is function of the modulus of elasticity of the concrete matrix, E, its fracture toughness, K_{IC}, the fibre embedment length, w_c, and the characteristic size of the structural element, h:

$$N_w = \frac{E w_c}{K_{IC} h^{1/2}} \tag{5}$$

3 Size-scale effects on FRC minimum reinforcement and plastic rotation capacity

In Fig. 5a, the flexural responses of different FRC beams are described by means of the UBCM, and by varying the Reinforcement Brittleness Number, N_P. It is worth noting how, by increasing N_P, a transition from a structural behaviour characterized by hyper-strength (red, orange,

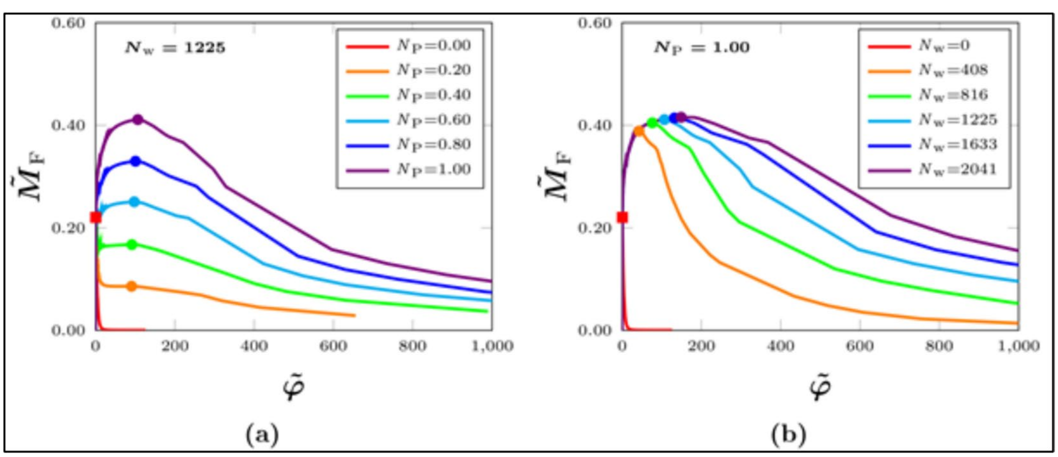

Fig. 5 Size-scale effects in FRC beams. The dimensionless moment of first fracturing is represented by the red square. **a**: Different flexural responses by varying N_P. The minimum reiforcement condition occurs when $N_P = 0.6$ (light-blue curve); **b** Different flexural responses by varying N_w. The plastic rotation capacity increases by increasing N_w

and green curves), to the condition of minimum reinforcement (light-blue curve) or strain-hardening (blue and purple curves) takes place. In all cases, the ultimate behaviour of the beams is described by a final tail due to the progressive slippage of the fibres from the concrete matrix. In general, it can be observed how the load-bearing capacity of the FRC beams is governed by the Reinforcement Brittleness Number, N_P.

On the other hand, in Fig. 5b the flexural responses of different FRC beams are represented by varying the Pull-out Brittleness Number, N_w. It can be seen how the FRC plastic rotation capacity increases by increasing N_w. Also in the cases of high N_w values, plastic plateaus are gradually exhausting by a final tail (softening branch) due to the slippage of the fibers from the concrete matrix.

Furthermore, it is worth noting that, by increasing the structural size, h, two reverse ductile-to-brittle transitions take place: the first one involves a more stable post-cracking process (N_P ↑); the second one entails a reduction in the plastic rotation capacity (N_w ↓) of the FRC beam.

In general, we can acknowledge how an optimization of the FRC components should arise from the reverse size-scale effects involving N_P and N_w.

The most recent developments in the field of structural mechanics have led to realise that the traditional definition of strength—which is defined as the force per unit area that causes failure—needs to be revised. This is especially true when dealing with particularly large or small structures. As a result, size-scale effects have become increasingly important in explaining structural ductility. To determine the ductility or brittleness of the structure via the size, the strength of the material must be compared to other properties, such as the toughness in the event of fracturing processes. Two intrinsic material characteristics and a geometrical characteristic govern the scaling, which is the competition between two distinct collapses described by generalized forces with different physical dimensions. These factors serve as the basic foundation to foresee the structural response [18]. In both the abovementioned dimensionless Brittleness Numbers, N_P and N_w, the size-scale dependence becomes manifest by considering the dimensional mismatch between two generalized forces: the stress, $[\sigma] = [F][L]^{-2}$, and the stress-intensity factor, $[K] = [F][L]^{-3/2}$.

The ductile-to-brittle transition that occurs when the specimen size increases has been observed, even at the laboratory scale. Raising the size-scale causes a transition towards brittle behavior, also if the material and the geometry remain unchanged. This behavior is accompanied by a sharp decrease in the loading capacity of the structure (snap-back or cusp-catastrophe instability) and a rapid crack propagation, which is detected for all materials, including metal, polymers, ceramics, and cement. On the other hand, a ductile behavior with slow crack propagation is observed in specimens with relatively small characteristic size.

4 Numerical vs experimental comparison

Through the numerical vs. experimental comparison proposed in Fig. 6, it can be observed how the UBCM (thick curves) is able to reproduce the experimental behaviour (thin curves [19]) of the FRC beams by varying the characteristic structural sizes (h = 50 mm; 75 mm; 100 mm), thus making the size-scale effect on the minimum reinforcement condition evident. Within the flexural response, this condition is obtained when the ultimate load is equal to the first cracking load, which occurs at the onset of the fracturing process [20].

Following Eq. (4), a scale-dependent evaluation of the minimum fibre volume fraction can be easily obtained as:

$$V_{f,min} = N_{PC} \frac{K_{IC}}{\sigma_s} h^{-1/2} \tag{6}$$

where N_{PC} is the critical value of the Reinforcement Brittleness Number, which can range from 0.5.to 1.3 depending on the initial depth of the beam notch [1, 15, 20]. It is worth noting that the FRC minimum reinforcement condition is not constant, but it varies with the structural size, more precisely according to the power-law $h^{-1/2}$.

It is interesting to remark that an attempt to take into account size-scale effects is mentioned in the latest issue of RILEM Recommendations for the design of FRC structures [21], where the residual flexural stresses, which are evaluated in correspondence to a specific value of crack mouth opening displacement, are adjusted by a coefficient that decreases as the specimen size increases. In reality, such an approach can be referred to the simple scaling law of the strength [22], and it can be quantitatively evaluated in the framework of fracture mechanics by using the Multi-fractal Scaling Law for initially uncracked specimens [23].

According to Eq. (6), in Fig. 7 we can see how the fibre volume fraction able to guarantee the minimum reinforcement condition decreases as the beam size increases, contrary to what is provided by the current Standards and regulations [4, 24].

In this context, a significant lack of information can be detected in both RILEM Recommendations [21] and FIB Model Code [5] that, referring to hybrid-reinforced structures, only acknowledge the presence of fibres providing a reduction in the minimum percentage of bar-reinforcement to limit the crack width at the Serviceability Limit State. On the other hand, no information is given on the fibre volume ratio necessary to

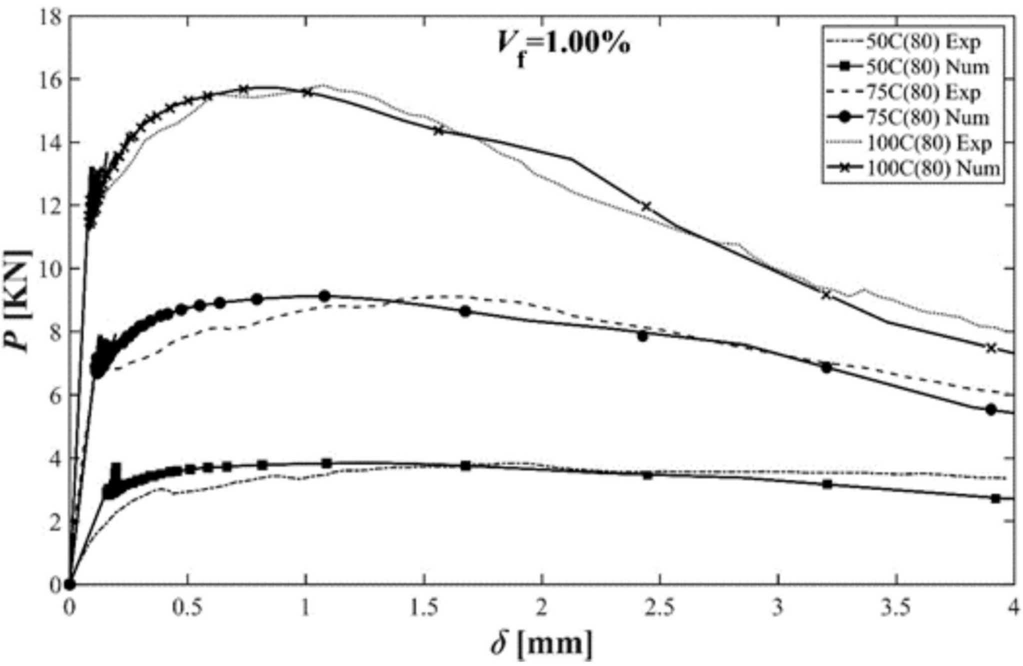

Fig. 6 Flexural behaviour of FRC beams with different sizes ($h = 50$ mm; 75 mm; 100 mm): Numerical (thick) vs. experimental (thin) curves

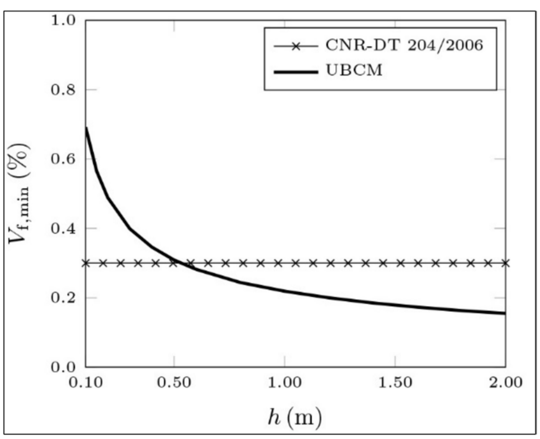

Fig. 7 Size-scale effect on the minimum fibre volume fraction of FRC beams: Comparison between the Italian Standards CNR-DT 204/2006 and the UBCM

obtain a stable post-cracking behaviour at the Ultimate Limit State. In both cases, a quantitative determination of $V_{f,min}$ is not provided.

Thus, a need emerges for national and international codes of practice to undergo a thorough improvement also in view of the crucial aspects highlighted by the most innovative scientific researches in the field of structural engineering.

5 Cohesive (single-phase) crack option

Different models can be found in the current scientific literature describing the post-cracking response of fibre-reinforced brittle-matrix composites, very few of them focusing on equilibrium, constitutive, and kinematic conditions at the cross-sectional level [25–30].

Considering that the effect of fibre-reinforcements is most prominent after cracking, the majority of modeling efforts have been aimed at capturing these phenomena. From the modelling perspective, the following main numerical approaches have emerged [24]: Explicit Fibre-reinforcement Approaches, where the fibres are considered individually in a finite element mesh. The fibre-reinforcement mechanisms can be simulated directly, through fibre-pull out constitutive laws supported by results from experimental tests. In this case, the crack mouth opening displacement can be used to determine the stress components in the fibres crossing the crack by considering the pull out constitutive law. Alternatively, fibers can be considered perfectly bonded to the surrounding concrete matrix, being the strain in a generic fibre obtained from the global displacement field. In addition, a statistical strategy for estimating the fibre distribution and orientation is generally used in the framework of these approaches; Implicit Fibre-reinforcement Approaches, which are more oriented to cohesive crack modelling, aim to identify the cohesive law of the fibre-reinforced brittle-matrix composite, i.e., stress vs crack opening relationship, *σ-w*, in order to model the damaging process of the composite, which is treated as a single-phase material [24]. Ultimately, an inverse analysis applied to experimental data yields a numerical definition of this single-phase constitutive rule, resulting in a process known as "design-by-testing" [31–45].

The fundamental shortcoming with the aforementioned single-phase methods is represented by the fact that the fibrous composite constitutive law is only applicable when a specific number of reinforcing fibers are present. Consequently, these cohesive methods pose a big challenge in the event of a variation in the amount of the reinforcing phase. In such a scenario, comprehensive experimental campaigns involving specimens with varying fiber volume fractions, V_f, are necessary to simulate the FRC composite post-cracking behavior.

More significantly, most of the abovementioned approaches are not able to predict any size-scale effect on the post-cracking behaviour of the composite structure, resulting in a dangerous passage from laboratory-scale to full-scale structural elements.

Such a lack of effectiveness can be overcome by the scale-dependent UBCM approach, in which the brittle matrix and the reinforcing fibres represent its primary and secondary phases, both contributing to the global toughness of the FRC structure.

6 Conclusions

In conclusion, effective and advanced structural design approaches should consider the economic and technological advantages offered by the structural applications of FRC. In this framework, it is important to highlight how the ductility of FRC structures is strongly dependent on the fibre volume fraction together with the structural size, which induces two reverse ductile-to-brittle transitions in the mechanical response of the composite structure: by increasing its size, an increase in the load-bearing capacity of the FRC element takes place, together with a decrease in its plastic rotation capacity. These size-scale effects on the structural brittleness can be taken into account by means of an effective fracture mechanics approach: this is represented by the Updated Bridged Crack Model (UBCM), which can provide significant improvements in current international Standards and regulations also with respect to the structural safety.

Authors' contributions
Conceptualization, Formal Analysis, Writing – Original Draft, and Funding Acquisition: Federico Accornero; Software and Data Curation: Alessio Rubino; Project Administration: Giuseppe Marano; Supervision and Writing – Review and Editing: Alberto Carpinteri.

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Availability of data and materials
The data presented in this study are available on request from the corresponding author.

Declarations

Competing interests
Alberto Carpinteri are editorial board members for Smart Construction and Sustainable Cities and were not involved in the editorial review, or the decision to publish, this article. All authors declare that there are no other competing interests.

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