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Supersymmetric smooth distributions of M2-branes as AdS solitons

A. Anabalón,[a,b] **D. Astefanesei,**[c] **A. Gallerati**[d] **and J. Oliva**[a]

[a] *Departamento de Física, Universidad de Concepción, Casilla 160-C, Concepción, Chile*
[b] *Instituto de Física Teórica, Universidade Estadual Paulista, R. Dr. Bento T. Ferraz 271, Bl. II, Sao Paulo 01140-070, Brazil*
[c] *Pontificia Universidad Católica de Valparaíso, Instituto de Física, Av. Brasil 2950, Valparaíso, Chile*
[d] *Politecnico di Torino, Dipartimento di Scienza Applicata e Tecnologia, corso Duca degli Abruzzi 24, 10129 Torino, Italy*

E-mail: anabalo@gmail.com, dumitru.astefanesei@pucv.cl, antonio.gallerati@polito.it, julioolivazapata@gmail.com

Abstract: We show that the singularities of certain distributions of M2-branes can be removed by adding a Wilson loop around a compact direction along the brane. The holographic coordinate naturally ends up before the singularity and the resulting spacetime provides an AdS soliton configuration, albeit possibly supersymmetric. We study the phase space diagram of these M2-branes distributions endowed with Wilson loops and show that different distributions are interconnected, providing a rich structure of phase transitions at zero temperature.

Keywords: Supergravity Models, Extended Supersymmetry, Solitons Monopoles and Instantons, Supersymmetry Breaking

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1 Introduction and discussion

One of the most important open problems in theoretical particle physics is the description of confinement in non-abelian gauge theories. When an holographic description of the quantum field theory exists [1] it was pointed out that this confined phase might have a very simple description in AdS [2]. A Lorentzian section of this everywhere-regular confining metric has negative energy and is perturbatively stable: the configuration is known as the AdS soliton [3].

Perturbative stability is non-trivial, since the AdS soliton entails the existence of fermions with antiperiodic boundary conditions on a spacelike $\mathbb{S}^1$, which in asymptotically flat spaces gives rise to instabilities associated to the production of bubbles of nothing [4]. This, however, is not the case in AdS. Moreover, one can ensure a non-perturbative stability, due the existence of a BPS bound, when the AdS soliton is endowed with a magnetic flux [5, 6]. This opened the possibility to check whether these AdS solitons have more sophisticated supersymmetric configurations. In this regard, some of us recently studied a framework with a dilaton scalar running in the T^3 truncation of the gauged maximal $\mathcal{N} = 8$ supergravity [7]. This kind of construction has also been generalized to 10 dimensions, as new models of holographic confinement [8–11]. The soliton/black hole phase transition has also been discussed in some of these cases [12–14].

In this paper we provide a further generalization of these AdS soliton-like geometries, and advance their understanding as we show that they have a nice interpretation as smoothing singular distributions of M2-branes in eleven dimensions. The latter were found and discussed

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in detail in [15]. Our results encompass some of these brane distributions and allow to construct their phase diagrams. We shall also see how the same distribution is associated to inequivalent spacetimes, and study how the latter can coexist for different values of the boundary conditions on the gauge fields. Indeed, the boundary value of the gauge fields can be interpreted as giving vacuum expectation values (vevs) to currents in a dual field theory.

In the following, we focus on a sector of the $D = 4$, gauged $\mathcal{N} = 8$ supergravity theory, breaking the isometry of the seven-sphere $\mathbb{S}^7$ from SO(8) to SO(4) $\times$ SO(4) for vanishing gauge fields. Hence, it is possible to capture only certain M2-branes distributions, namely those that have the same isometry. The construction is quite similar to what we found in [7]. In this respect, we believe that the connection between brane distributions and regular AdS solitons points to the possibility that various singular brane distributions can be smoothed by giving a vev to a current on the QFT side.

The structure of the paper is as follows. In section 2, we present a consistent truncation of the SO(8)-gauged maximal supergravity framework, its oxidation to eleven dimensions and the associated BPS conditions. In section 3 we reproduce the brane distributions of [15], specialized to the chosen truncation. We also present the change of coordinates that connect the $\mathbb{S}^7$ compactification yielding the STU model with the one that is better adapted to describe the brane distribution. We also introduce a change of coordinates to suitably describe the maximal analytic extension of the M2-brane spacetimes with SO(4) $\times$ SO(4) isometry. It therefore becomes clear that, for every value of the parameters of the distribution, there are two inequivalent and disconnected spacetimes, both of them asymptotically $\text{AdS}_4 \times \mathbb{S}^7$ and singular. In section 4 we construct new spacetime configurations that generalize these distributions and yield a vev for the holographic current. The supersymmetric limit of these new soliton solutions are discussed in section 5, and contrasted to the supersymmetric case in the absence of running scalars. Finally, in section 6 we compute the Euclidean action and energy of the solutions, carefully investigating their phase diagrams.

2 The model

We are interested in studying the dilatonic sector of the STU model of the maximal SO(8)-gauged, $\mathcal{N} = 8$ supergravity with action

$$\mathcal{S} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left(R - \sum_{i=1}^{3} \frac{(\partial \Phi_i)^2}{2} + \frac{2}{L^2} \cosh\left(\Phi_i\right) - \frac{1}{4} \sum_{i=1}^{4} X_i^{-2} \bar{F}_i^2 \right) , \tag{2.1}$$

where $\bar{F}_i$ are two forms, related with gauge fields in the standard way, and we have

$$\bar{F}_i = d\bar{A}_i \,, \qquad X_i = e^{-\frac{1}{2}\vec{a}_i \cdot \vec{\Phi}} \,, \qquad \vec{\Phi} = (\Phi_1, \Phi_2, \Phi_3) \,, \tag{2.2}$$

with

$$\vec{a}_1 = (1,1,1) \,, \qquad \vec{a}_2 = (1,-1,-1) \,, \qquad \vec{a}_3 = (-1,1,-1) \,, \qquad \vec{a}_4 = (-1,-1,1) \,. \tag{2.3}$$

We consider purely magnetic solutions, for which it is consistent to truncate the axions to zero. The Lagrangian (2.1) can be obtained from the compactification of eleven dimensional

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supergravity over the seven sphere with the ansatz [16]

$$ds_{11}^2 = \tilde{\Delta}^{2/3}\, ds_4^2 + 4\, L^2 \tilde{\Delta}^{-1/3} \sum_{i=1}^{4} X_i^{-1} \left(d\mu_i^2 + \mu_i^2 \left(d\varphi_i + \frac{1}{2L} \bar{A}_i \right)^2 \right) , \tag{2.4}$$

$$\begin{aligned} F = & -\frac{1}{L}\, \epsilon_4 \sum_{i=1}^{4} \left(X_i^2 \mu_i^2 - \tilde{\Delta}\, X_i \right) + L\, X_i^{-1}\, {}^{\star_4} dX_i \wedge d\mu_i^2 \\ & - 4\, L^2 \sum_i X_i^{-2} \mu_i\, d\mu_i \wedge \left(d\varphi_i + \frac{1}{2\, L} \bar{A}_i \right) \wedge {}^{\star_4} \bar{F}_i \,, \end{aligned} \tag{2.5}$$

where ϵ_4 is the volume form of the four-dimensional metric ds_4^2, F is the four-form field strength and ^{*_4} is the Hodge dual operator. The four independent rotations on $\mathbb{S}^7$ are parameterized by the φ_i, which are 2π-periodic angular coordinates. We also have

$$\tilde{\Delta} = \sum_{i=1}^{4} X_i\, \mu_i^2 \,, \qquad \sum_{i=1}^{4} \mu_i^2 = 1 \,, \tag{2.6}$$

where the latter μ_i can be parameterised in terms of angles on the 3-sphere as

$$\mu_1 = \sin\vartheta \,, \quad \mu_2 = \cos\vartheta\, \sin\psi \,, \quad \mu_3 = \cos\vartheta\, \cos\psi\, \sin\xi \,, \quad \mu_4 = \cos\vartheta\, \cos\psi\, \cos\xi \,. \tag{2.7}$$

We are interested in considering the higher-dimensional interpretation of some of our solutions using this uplift. In particular, we shall work with a simplified, consistent truncation of the theory, where

$$\Phi_1 = \Phi_3 = 0 \,, \quad \Phi_2 = \sqrt{2}\, \phi \,, \quad \bar{F}^1 = \bar{F}^3 = F^1 \,, \quad \bar{F}^2 = \bar{F}^4 = F^2 \,, \tag{2.8}$$

and we then obtain an action of the form:

$$\mathcal{S} = \frac{1}{\kappa} \int d^4x\, \sqrt{-g} \left(\frac{R}{2} - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4} e^{\sqrt{2}\phi} \left(F^1 \right)^2 - \frac{1}{4} e^{-\sqrt{2}\phi} \left(F^2 \right)^2 + \frac{1}{L^2} \left(2 + \cosh\left(\sqrt{2}\, \phi \right) \right) \right) , \tag{2.9}$$

where

$$F_{\mu\nu}^{\Lambda} = \partial_\mu A_\nu^{\Lambda} - \partial_\nu A_\mu^{\Lambda} \,, \qquad \Lambda = 1, 2 \,. \tag{2.10}$$

The above Lagrangian yields a consistent truncation to the dilaton ϕ only of the $\mathcal{N} = 2$ *minimal coupling* supergravity [17], provided the constraint

$$F^1 \wedge F^1 - e^{-2\sqrt{2}\phi}\, F^2 \wedge F^2 = 0 \,, \tag{2.11}$$

is satisfied. The minimal-coupling model defined by (2.9) is a consistent truncation of the STU model, characterized by a scalar manifold of the form $\mathrm{SL}(2, \mathbb{R})/\mathrm{SO}(2)$.[1]

[1]The special geometry [18, 19] of this smaller theory can be described in terms of a prepotential function $\mathcal{F}(\mathcal{X}^\Lambda) = -\frac{i}{4} \left(\mathcal{X}^0 \right) \left(\mathcal{X}^1 \right)$ and is selected among the class of theories discussed in [20] by choosing $n = 1$, corresponding to $\nu = \infty$. The formulae obtained in the present paper can obtained by choosing the Fayet-Iliopoulos terms of the gauged $D = 4$ supergravity as $\theta_M = \left(\rho^{-1} L^{-2},\, \rho/4,\, 0,\, 0 \right)$, setting $\zeta = 0$ in the dyonic $\nu = \infty$ model considered in [20], having also suitably shifted the dilaton and rescaled the vector fields as described in the same reference.

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2.1 Supersymmetry

The general formulae related to the supersymmetry transformations in $\mathcal{N} = 2$, $D = 4$ supergravity with Fayet-Iliopoulos terms can be found in appendix A of [7], where we also define our spinor conventions.

Minimal-coupling model truncation. In order to see if the solutions preserve part of the supersymmetry, we consider the explicit form of the fermionic variations of the gauged supergravity theory. The latter, once adapted to the minimal-coupling model, read

$$\begin{aligned}\delta\Psi^A_\mu \;=\; & \partial_\mu\epsilon^A + \frac{1}{4}\,\omega_\mu^{ab}\,\gamma_{ab}\,\epsilon^A + \frac{1}{2\,L}\left(A^1_\mu + A^2_\mu\right) i\left(\sigma^2\right)^A{}_B\,\epsilon^B \\ & - \frac{1}{8}\left(F^1_{\nu\rho}\,e^{\frac{1}{\sqrt{2}}\,\phi} + F^2_{\nu\rho}\,e^{-\frac{1}{\sqrt{2}}\,\phi}\right)\gamma^{\nu\rho}\,\gamma_\mu\,\varepsilon^{AB}\,\epsilon_B \\ & + \frac{1}{2}\,\mathcal{W}\,\gamma_\mu\,\delta^{AB}\,\epsilon_B\,,\end{aligned} \tag{2.12}$$

$$\begin{aligned}\delta\lambda^A = & -\gamma^\mu\,\partial_\mu\phi\,\epsilon^A - \frac{1}{2\sqrt{2}}\left(-F^1_{\nu\rho}\,e^{\frac{1}{\sqrt{2}}\,\phi} + F^2_{\nu\rho}\,e^{-\frac{1}{\sqrt{2}}\,\phi}\right)\gamma^{\nu\rho}\,\varepsilon^{AB}\,\epsilon_B - \\ & + \frac{1}{\sqrt{2}\,L}\left(e^{\frac{1}{\sqrt{2}}\,\phi} - e^{-\frac{1}{\sqrt{2}}\,\phi}\right)\delta^{AB}\,\epsilon_B\,,\end{aligned} \tag{2.13}$$

where the superpotential explicitly reads

$$\mathcal{W} = \frac{e^{\frac{1}{\sqrt{2}}\,\phi} + e^{-\frac{1}{\sqrt{2}}\,\phi}}{2\,L}\,. \tag{2.14}$$

We write the chiral spinors in terms of their real and imaginary components, using the spinor conventions specified in appendix A of [7]:

$$\epsilon^A = \mathrm{Re}\,\epsilon^A + i\;\mathrm{Im}\,\epsilon^A\,, \tag{2.15}$$

and introduce the complex spinors

$$\chi_{\mathrm{R}} = \mathrm{Re}\,\epsilon^1 + i\;\mathrm{Re}\,\epsilon^2\,, \qquad \chi_{\mathrm{I}} = \mathrm{Im}\,\epsilon^1 + i\;\mathrm{Im}\,\epsilon^2\,. \tag{2.16}$$

Each of the Killing spinor relations may be expressed as a first-order differential equation. Nevertheless, the two are not independent, and we can solve the spinor equations in just one of them (χ_{R}, for instance). Since in the chosen spinor basis

$$\epsilon_A = \left(\epsilon^A\right)^*\,, \tag{2.17}$$

the Majorana spinors $\epsilon^A_{(\mathrm{M})}$ is expressed as

$$\epsilon^A_{(\mathrm{M})} = \epsilon_A + \epsilon^A = 2\;\mathrm{Re}\,\epsilon^A\,. \tag{2.18}$$

The action of $\gamma^5 = i\,\gamma^0\gamma^1\gamma^2\gamma^3$ on the above Majorana $\epsilon^A_{(\mathrm{M})}$ is

$$\gamma^5\,\epsilon^A_{(\mathrm{M})} = \epsilon_A - \epsilon^A = -2\,i\;\mathrm{Im}\,\epsilon^A\,, \tag{2.19}$$

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so that

$$\mathrm{Im}\,\epsilon^A = i\,\gamma^5\ \mathrm{Re}\,\epsilon^A\,, \tag{2.20}$$

giving

$$\chi_{\rm I} \equiv \mathrm{Im}\,\epsilon^1 + i\ \mathrm{Im}\,\epsilon^2 = i\,\gamma^5\left(\mathrm{Re}\,\epsilon^1 + i\ \mathrm{Re}\,\epsilon^2\right) = i\,\gamma^5\,\chi_{\rm R}\,, \tag{2.21}$$

providing the connection between the complex spinors.

The spinor components can be retrieved from a solution $\chi_{\rm R}$ of the Killing spinor equations as

$$\epsilon^1_{(\rm M)} = 2\ \mathrm{Re}\,\chi_{\rm R}\,, \qquad \epsilon^2_{(\rm M)} = 2\ \mathrm{Im}\,\chi_{\rm R}\,, \tag{2.22}$$

and

$$\begin{aligned}\epsilon^1 &= \frac{(\mathbb{1}-\gamma^5)}{2}\,\epsilon^1_{(\rm M)} = \left(\mathbb{1}-\gamma^5\right)\mathrm{Re}\,\chi_{\rm R}\,,\\ \epsilon^2 &= \frac{(\mathbb{1}-\gamma^5)}{2}\,\epsilon^2_{(\rm M)} = \left(\mathbb{1}-\gamma^5\right)\mathrm{Im}\,\chi_{\rm R}\,.\end{aligned} \tag{2.23}$$

The explicit equations for $\chi_{\rm R}$ have the form

$$\begin{aligned}0 \;=\;& \partial_\mu\chi_{\rm R} + \frac{1}{4}\,\omega_\mu^{ab}\,\gamma_{ab}\,\chi_{\rm R} - \frac{i}{2\,L}\,\left(A^1_\mu + A^2_\mu\right)\chi_{\rm R}\\ &+ \frac{i}{8}\left(F^1_{\nu\rho}\,e^{\frac{1}{\sqrt{2}}\,\phi} + F^2_{\nu\rho}\,e^{-\frac{1}{\sqrt{2}}\,\phi}\right)\gamma^{\nu\rho}\,\gamma_\mu\,\chi_{\rm R} + \frac{1}{2}\,\mathcal{W}\,\gamma_\mu\,\chi_{\rm R}\,,\end{aligned} \tag{2.24}$$

$$\begin{aligned}0 \;=\;& -\,\gamma^\mu\,\partial_\mu\phi\,\chi_{\rm R} + \frac{i}{2\sqrt{2}}\left(-F^1_{\nu\rho}\,e^{\frac{1}{\sqrt{2}}\,\phi} + F^2_{\nu\rho}\,e^{-\frac{1}{\sqrt{2}}\,\phi}\right)\gamma^{\nu\rho}\,\chi_{\rm R}\\ &+ \frac{1}{\sqrt{2}\,L}\,\left(e^{\frac{1}{\sqrt{2}}\,\phi} - e^{-\frac{1}{\sqrt{2}}\,\phi}\right)\chi_{\rm R}\,.\end{aligned} \tag{2.25}$$

To derive analogous conditions on $\chi_{\rm I}$ it is sufficient to multiply the above expressions by $i\,\gamma^5$ from the left.

3 Maximal extension of a M2 brane distribution

Now we will take a singular distribution of M2-branes and show how it can be seen as a four dimensional solution of the theory (2.9). While the solution is known, the maximal extension we provide in this section is new.

Our starting point is the 11-dimensional M2 brane distribution of [15]:

$$\begin{aligned}ds^2_{11} &= H^{-2/3}\left(-dt^2 + dz^2 + d\varphi^2\right) + H^{1/3}\,ds^2_8\,,\\ F^{(4)} &= dt\wedge dz\wedge d\varphi\wedge dH^{-1}\,,\end{aligned} \tag{3.1}$$

where

$$H = \frac{(2L)^6}{\rho^6\,\Delta_0}\,, \qquad \Delta_0 = (H_1\ldots H_8)^{1/2}\sum_{i=1}^{8}\frac{y_i^2}{H_i}\,, \tag{3.2}$$

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with

$$H_i = 1 + \frac{\ell_i^2}{\rho^2} \tag{3.3}$$

and where ds_8^2 is just flat space in disguise,

$$ds_8^2 = \frac{\Delta_0\, d\rho^2}{(H_1 \cdots H_8)^{1/2}} + \rho^2 \sum_{i=1}^{8} H_i\, dy_i^2\,, \tag{3.4}$$

with the constraint

$$\sum_{i=1}^{8} y_i^2 = \text{constant}\,. \tag{3.5}$$

The unconstrained cartesian coordinates are

$$w^i = \rho\, \sqrt{H_i}\, y_i\,, \tag{3.6}$$

and the metric is given by

$$ds_8^2 = \delta_{ij}\, dw^i dw^j\,. \tag{3.7}$$

In order to make contact with the STU model, we specialize to the case when

$$\ell_8 = \ell_1\,, \quad \ell_7 = \ell_2\,, \quad \ell_6 = \ell_3\,, \quad \ell_5 = \ell_4\,, \tag{3.8}$$

and we introduce the new coordinates

$$\begin{aligned} y_1 &= \mu_1\, \cos\varphi_1\,, & y_8 &= \mu_1\, \sin\varphi_1\,, \\ y_2 &= \mu_2\, \cos\varphi_2\,, & y_7 &= \mu_2\, \sin\varphi_2\,, \\ y_3 &= \mu_3\, \cos\varphi_3\,, & y_6 &= \mu_3\, \sin\varphi_3\,, \\ y_4 &= \mu_4\, \cos\varphi_4\,, & y_5 &= \mu_4\, \sin\varphi_4\,. \end{aligned} \tag{3.9}$$

Using this new parametrization, we see that

$$\Delta_0 = (H_1 \cdots H_4)^{3/4} \sum_{i=1}^{4} X_i\, \mu_i^2 = (H_1 \cdots H_4)^{3/4}\, \tilde{\Delta}\,, \qquad X_i = \frac{(H_1 \cdots H_4)^{1/4}}{H_i}\,. \tag{3.10}$$

The eleven-dimensional M2-branes metric now reads

$$\begin{aligned} ds_{11}^2 = \tilde{\Delta}^{2/3}\, \frac{\rho^4 (H_1 \cdots H_4)^{1/2}}{2^4 L^4} \left(-dt^2 + dz^2 + d\varphi^2 + G(\rho)\, d\rho^2\right) \\ + 4\, L^2\, \tilde{\Delta}^{-1/3} \sum_{i=1}^{4} X_i^{-1} \left(d\mu_i^2 + \mu_i^2\, d\varphi_i^2\right)\,, \end{aligned} \tag{3.11}$$

with

$$G(\rho) = \frac{2^6 L^6}{\rho^6\, H_1 \cdots H_4}\,. \tag{3.12}$$

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The distribution is read-off from the harmonic function H, which can be written as

$$H = (2L)^6 \int \frac{\sigma(w')}{|w-w'|^6}\, d^8w' \,, \qquad \sigma = \frac{2}{\pi^4\, \ell_1^2\, \ell_2^2\, \ell_3^2\, \ell_4^2}\, \delta'\left(1 - \rho^2 \sum_{i=1}^{4} \frac{H_i\, \mu_i^2}{\ell_i^2}\right). \tag{3.13}$$

Note that (3.11) indeed coincides with the sphere reduction of the previous section, whenever $\sum_{i=1}^{4} \mu_i^2 = 1$ and the gauge fields vanish. The truncation we are interested in is obtained setting $\ell_3 = \ell_1$, $\ell_4 = \ell_2$. In this latter case, the distribution has support on an ellipsoid in terms of the Cartesian coordinates w_i:

$$\frac{w_1^2 + w_3^2 + w_6^2 + w_8^2}{\ell_1^2} + \frac{w_2^2 + w_4^2 + w_5^2 + w_7^2}{\ell_2^2} = 1 \,, \tag{3.14}$$

which makes the isometry of the distribution manifestly $\mathrm{SO}(4) \times \mathrm{SO}(4)$ invariant. In terms of the intrinsic coordinates of the $\mathbb{S}^7$, this reads:

$$\rho^2 \left(\frac{\mu_1^2 + \mu_3^2}{\ell_1^2} + \frac{\mu_2^2 + \mu_4^2}{\ell_2^2}\right) = 0 \,. \tag{3.15}$$

The term in parenthesis of (3.15) is manifestly positive definite on the $\mathbb{S}^7$. Hence, we learn that the support of the M2 is located at $\rho = 0$. We shall see below that this is not a curvature singularity. Indeed, the curvature singularity is actually located at negative values of ρ^2. We shall cure the latter in the M2 brane distribution and see that this yields a form of an AdS soliton in four dimensions, with a possible supersymmetric limit as in [21].

We find convenient to introduce the following change of coordinates [22, 23]

$$x = \frac{H_2}{H_1} \quad \to \quad \rho^2 = \frac{\ell_2^2 - \ell_1^2\, x}{x - 1} \,, \tag{3.16}$$

where the four dimensional metric (the part multiplying $\tilde{\Delta}^{2/3}$) in (3.11) reads

$$ds_4^2 = \Upsilon(x) \left(-dt^2 + dz^2 + d\varphi^2 + \frac{\eta^2}{x^2}\, dx^2\right), \tag{3.17}$$

with

$$\Upsilon(x) = \frac{L^2\, x}{\eta^2\, (x-1)^2} \,, \qquad \eta^2 = \frac{16\, L^6}{\left(\ell_1^2 - \ell_2^2\right)^2} \tag{3.18}$$

and the four dimensional dilaton reading $\phi = -\dfrac{\ln(x)}{\sqrt{2}}$.

We note that the asymptotic region is located at $x = 1$, namely for $\rho^2 = \pm\infty$. Hence, we find two different disconnected geometries, $0 < x < 1$ and $1 < x < \infty$. In particular, one has

$$\begin{aligned} &\ell_1^2 < \ell_2^2 : \quad x \in (0,1) \;\to\; \rho^2 \in \left(-\infty, -\ell_2^2\right), \quad x \in (1, +\infty) \;\to\; \rho^2 \in \left(-\ell_1^2, +\infty\right), \\ &\ell_1^2 > \ell_2^2 : \quad x \in (0,1) \;\to\; \rho^2 \in \left(-\ell_2^2, +\infty\right), \quad x \in (1, +\infty) \;\to\; \rho^2 \in \left(-\infty, -\ell_1^2\right). \end{aligned} \tag{3.19}$$

Then, in any case, the coordinate x provides a maximal analytic extension of the manifold, unveiling its geometry in a transparent form.

It should also be clear that the singularities in the manifold are not related to the M2 brane distributions. They occur at $\rho^2 = -\ell_1^2$ and $\rho^2 = -\ell_2^2$. In the following, we will study how to remove these singularities. We will also see that the two geometries are actually interconnected from the boundary point of view, as they are both smoothly connected in the phase space description of these distributions in terms of the vevs of a dual current.

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4 Hairy soliton solutions

The model (2.9) admits soliton solutions generalizing the configurations of [23] with the presence of a scalar field. These solutions can be obtained by analytic continuations

$$t \to i\,\varphi\,, \qquad \varphi \to i\,t\,, \qquad Q_\Lambda \to i\,Q_\Lambda\,, \tag{4.1}$$

of the black hole configurations of [20, 22, 24], whose charged planar solutions can be embedded in the STU model [16].[2]

In particular, if we consider the dyonic, charged planar black hole of [20] for $\nu = \infty$, a hairy soliton solution obtained through (4.1) reads[3]

$$\begin{aligned} &e^0 = \sqrt{\Upsilon(x)}\,L\,dt, \quad e^1 = \sqrt{\frac{\Upsilon(x)}{f(x)}}\,\frac{\eta}{x}\,dx, \quad e^2 = \sqrt{\Upsilon(x)\,f(x)}\,d\varphi, \quad e^3 = \sqrt{\Upsilon(x)}\,L\,dz, \\ &\phi = -\frac{\ln(x)}{\sqrt{2}}, \quad A^1 = -P_1\,z\,dt + Q_1(x-x_0)\,d\varphi, \quad A^2 = -P_2\,z\,dt - Q_2\left(\frac{1}{x}-\frac{1}{x_0}\right)d\varphi, \end{aligned} \tag{4.2}$$

with

$$\Upsilon(x) = \frac{L^2\,x}{\eta^2\,(x-1)^2}\,, \qquad f(x) = 1 - \frac{\eta^2(x-1)^3}{L^2\,x}\left(Q_1^2 - \frac{Q_2^2}{x} - \frac{\eta^2}{L^4}\,\frac{P_1^2}{x} + \frac{\eta^2}{L^4}\,P_2^2\right). \tag{4.3}$$

The quantities of the $D = 11$ theory of section 2 can be written for this solution as

$$\begin{aligned} &\Phi_2 = -\log x\,, \qquad \Phi_1 = \Phi_3 = 0\,, \\ &\bar{F}^1 = \bar{F}^3 = Q_1\,dx \wedge d\varphi\,, \qquad \bar{F}^2 = \bar{F}^4 = Q_2\,x^{-2}\,dx \wedge d\varphi\,, \\ &X_i = \left(x^{\frac{1}{2}}\,,\, x^{-\frac{1}{2}}\,,\, x^{\frac{1}{2}}\,,\, x^{-\frac{1}{2}}\right), \\ &\tilde{\Delta} = x^{\frac{1}{2}}\,\sin^2\vartheta + x^{-\frac{1}{2}}\,\cos^2\vartheta\,\left(\sin^2\psi + \cos^2\psi\left(\cos^2\xi + x\,\sin^2\xi\right)\right), \end{aligned} \tag{4.4}$$

which shows that everything is regular for $x \neq 0$ and $x \neq \infty$ in the eleven dimensional geometry. The constraint (2.11) is solved if

$$P_1\,Q_1 - P_2\,Q_2 = 0 \tag{4.5}$$

This indeed coincide with the brane distribution of the previous section when there are no gauge fields.

[2] The STU model [25–27] is a $\mathcal{N} = 2$ supergravity coupled to 3 vector multiplets and characterized, in a suitable symplectic frame, by the prepotential $\mathcal{F}_{\text{STU}}(\mathcal{X}^\Lambda) = -\frac{i}{4}\sqrt{\mathcal{X}^0\,\mathcal{X}^1\,\mathcal{X}^2\,\mathcal{X}^3}$, together with symmetric scalar manifold of the form $\mathscr{M}_{\text{STU}} = (\text{SL}(2,\mathbb{R})/\text{SO}(2))^3$ spanned by the three complex scalars $z^i = \mathcal{X}^i/\mathcal{X}^0$ $(i = 1,2,3)$; this model is in turn a consistent truncation of the maximal $\mathcal{N} = 8$ theory in four dimensions with SO(8) gauge group [28–31].

[3] Note that the coordinates (z,t) of this section are related to the one of the previous section by a factor L.

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Asymptotic expansions. The metric's conformal boundary is at $x = 1$, where the conformal factor $\Upsilon(x)$ features a pole of order two. The metric then describes two distinct spacetimes for $x \in (0,1)$ and $x \in (1,\infty)$, physically identified by different signs of the dilaton field.

Let us restrict to a soliton magnetically-charged configuration, $P_1 = P_2 = 0$. The canonical form of an asymptotically AdS_4 spacetime can be obtained by introducing the change of variable

$$x = 1 \pm \left(\frac{L^2}{\eta\,\rho} - \frac{L^6}{8\,\eta^3\,\rho^3} \right) + \frac{L^4}{2\,\eta^2\,\rho^2} + O(\rho^{-3})\,, \tag{4.6}$$

the sign determined by the choice of branch $x > 1$ or $0 < x < 1$. This gives for the metric quantities

$$\Upsilon(x) = \frac{\rho^2}{L^2} + O(\rho^{-2})\,, \tag{4.7}$$

$$g_{\varphi\varphi} = \Upsilon(x)\, f(x) = \frac{\rho^2}{L^2} - \frac{\mu}{\rho} + O(\rho^{-2})\,, \tag{4.8}$$

$$\mu = \pm \frac{L^2}{\eta} \left(Q_1^2 - Q_2^2 \right)\,, \tag{4.9}$$

the latter μ being the energy parameter of the solution. The dilaton field can be expanded as

$$\phi = L^2\,\frac{\phi_0}{\rho} + L^4\,\frac{\phi_1}{\rho^2} + O(\rho^{-3})\,, \tag{4.10}$$

where

$$\phi_0 = \mp \frac{1}{\sqrt{2}\,\eta}\,, \qquad \phi_1 = 0\,, \tag{4.11}$$

then giving only a non-trivial leading term ϕ_0, which yields a vev in the QFT.

Dual theory. Our focus lies in soliton solutions, identified by the contraction of the φ-circle at point x_0 where

$$f(x_0) = 0\,, \tag{4.12}$$

the soliton configuration existing in the interval between x_0 and the boundary $x = 1$.

Expanding the metric around x_0, together with the above condition (4.12), regularity requires defining a parameter Δ such that

$$\varphi \in [0,\, \Delta]\,, \tag{4.13}$$

where Δ is given by

$$\Delta^{-1} = \left| \frac{1}{4\,\pi\,\eta}\, x\, \frac{df}{dx} \right|_{x=x_0} = \left| \frac{\eta\,(x_0-1)^2}{4\,\pi\,L^2\,x_0^2} \left(Q_1^2\, x_0\,(1+2\,x_0) - Q_2^2\,(2+x_0) \right) \right|\,. \tag{4.14}$$

Magnetically charged configurations feature net magnetic fluxes at infinity,

$$\begin{aligned} \Phi^1_{\rm M} &= \int F^1 = \oint A^1 = Q_1\,\Delta\,(1-x_0) \equiv 2\pi L\,\psi_1\,, \\ \Phi^2_{\rm M} &= \int F^2 = \oint A^2 = Q_2\,\Delta\left(-1 + x_0^{-1}\right) \equiv 2\pi L\,\psi_2\,. \end{aligned} \tag{4.15}$$

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The dilaton field generates a non-zero vev of an operator, in the dual theory, of conformal dimension one, that can be expressed in terms of the fluxes as

$$\langle \mathcal{O} \rangle = \phi_0 = \mp \frac{\pi}{\sqrt{2}\,\Delta}\, x_0^{-1} \left((1+2\,x_0)\,\psi_1^2 - x_0\,(2+x_0)\,\psi_2^2 \right). \tag{4.16}$$

The dual energy momentum tensor is expressed in terms of the energy parameter as [32–35]

$$\langle T_{tt} \rangle = -\frac{\mu}{2\,\kappa\,L^2}\,, \qquad \langle T_{zz} \rangle = \frac{\mu}{2\,\kappa\,L^2}\,, \qquad \langle T_{\varphi\varphi} \rangle = -\frac{\mu}{\kappa\,L^2}\,. \tag{4.17}$$

Finally, from the presence of the gauge fields originates a vev for the currents at the boundary [36]

$$\langle J_1^\nu \rangle = \frac{\delta \mathcal{S}}{\delta A_\nu^1} = -\frac{1}{\kappa}\, N_\mu\, e^{\sqrt{2}\,\phi}\, F^{1\,\mu\nu} \sqrt{|h|} = -\frac{Q_1}{\eta\,\kappa}\,\delta_\varphi^\nu\,, \tag{4.18}$$

$$\langle J_2^\nu \rangle = \frac{\delta \mathcal{S}}{\delta A_\nu^2} = -\frac{1}{\kappa}\, N_\mu\, e^{-\sqrt{2}\,\phi}\, F^{2\,\mu\nu} \sqrt{|h|} = -\frac{Q_2}{\eta\,\kappa}\,\delta_\varphi^\nu\,, \tag{4.19}$$

where N_μ is the outward pointing normal to the boundary metric $h_{\mu\nu} = g_{\mu\nu} - N_\mu N_\nu$.

4.1 Existence of solitons

As seen from a bulk perspective, Q_1, Q_2, and η parametrize the solutions. Nonetheless, from a boundary perspective, it makes more sense to parameterize solutions in terms of the boundary data we hold fixed.

We can take into account fixed charges (or better, fixed currents) holding fixed Q_1/η, Q_2/η, and the period Δ, or fixed fluxes (Wilson loops), holding fixed ψ_1, ψ_2, and the period Δ.

Fixed fluxes. Let us consider soliton configuration in terms of the boundary quantities ψ_1, ψ_2 and Δ. We recast the bulk parameters as

$$\begin{aligned} Q_1 &= \frac{2\pi\,L\,\psi_1}{\Delta\,(1-x_0)}\,, \qquad Q_2 = \frac{2\pi\,L\,\psi_2}{\Delta\left(x_0^{-1}-1\right)}\,, \\ \eta &= \frac{x_0\,\Delta}{\pi\,\left|\psi_1^2\,(1+2\,x_0) - \psi_2^2\,x_0\,(2+x_0)\right|}\,. \end{aligned} \tag{4.20}$$

Substituting into $f(x_0)$, we get

$$f(x_0) = 1 + \frac{4\,x_0\,(1-x_0)\,(\psi_1^2 - x_0\,\psi_2^2)}{\left((1+2\,x_0)\,\psi_1^2 - x_0\,(2+x_0)\,\psi_2^2\right)^2}\,. \tag{4.21}$$

Then, similarly to the Einstein-Maxwell configurations [5, 37], there exists a limited range of parameters characterized by two different solitonic solutions, coalescing at the range extremum, see figure 1.

Fixed charges. Let us now consider the boundary fixed charge framework. We introduce the rescaling

$$q_{1,2} \equiv \frac{\Delta^2}{4\pi^2 L}\,\frac{Q_{1,2}}{\eta}\,. \tag{4.22}$$

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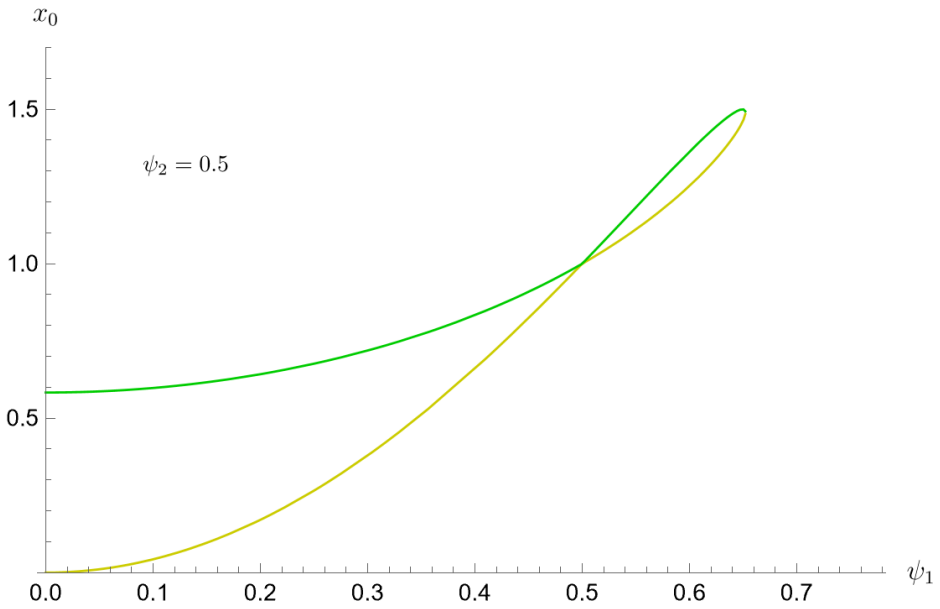


Figure 1. Location of x_0 as a function of ψ_1, for fixed $\psi_2 = 0.5$. We find that, for each value of ψ_1 identifying a configuration, there exist two solutions for x_0, possibly coalescing.

If we rewrite the metric in terms of the rescaled charges and impose regularity around x_0, we obtain for η the expression

$$\eta = \frac{\Delta}{4\pi} \frac{q_2^2\,(2+x_0) - q_1^2\,x_0\,(1+2\,x_0)}{(-1+x_0)\,(q_2^2 - q_1^2\,x_0)}\,. \tag{4.23}$$

The location of soliton configurations now come from the solutions of the equation

$$f(x_0) = 1 - \frac{\left(q_2^2\,(2+x_0) - q_1^2\,x_0\,(1+2\,x_0)\right)^4}{16\,x_0^2\,(-1+x_0)\,\left(q_1^2\,x_0 - q_2^2\right)^3} = 0\,. \tag{4.24}$$

In figure 2 we show how, for every value of the rescaled charges (4.22), from zero to four solutions can be found.[4] The supersymmetric scenario (see section 5) turns out to be simpler, featuring two superymmetric solitons for the same boundary conditions, with the same energy and free energy.

4.2 Relation to earlier solutions

Soliton solutions in a model with a single gauge field were discovered in [5], exploiting a compactification from the $D = 11$ supergravity setup with $\bar{A}_i = \frac{1}{2}A$ $(i = 1, \ldots 4)$ and $\Phi_a = 0$ $(a = 1, 2, 3)$. The hairy solutions we discuss in this paper should then reduce to the scalar-free model [5] for $A^1 = A^2 = \frac{1}{2}A$ and $\phi = 0$.

Let us know construct the explicit connection between the two models. We start with the change of coordinates

$$x = 1 - \frac{\alpha}{y}\,, \tag{4.25}$$

[4]The situation is very different than for pure Einstein-Maxwell-AdS system, where there are only two solitons for each value of the charge [5].

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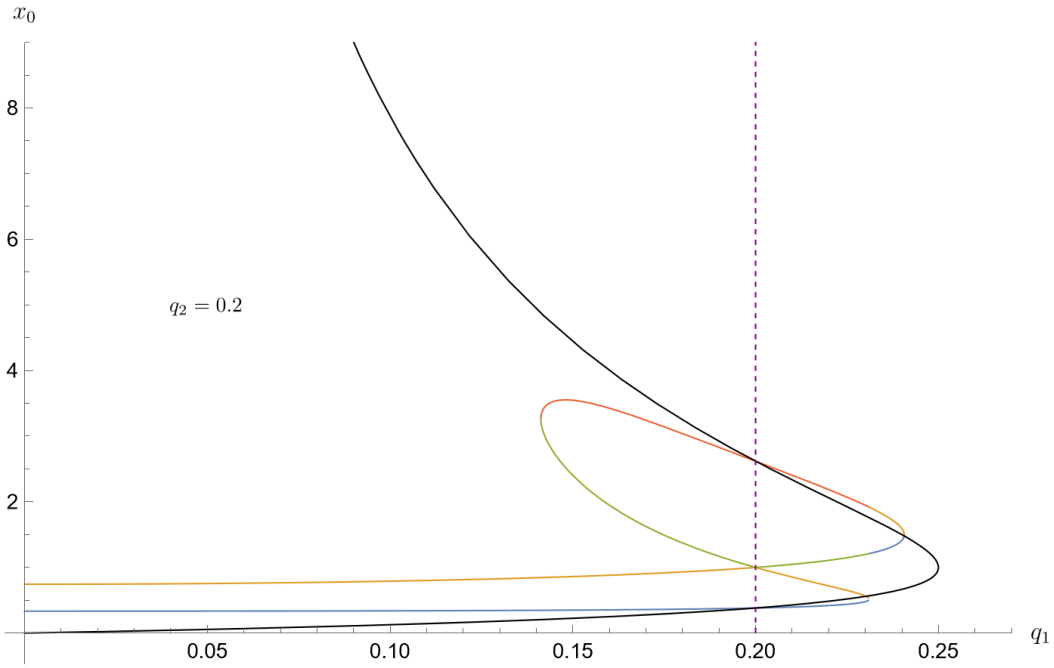


Figure 2. The x_0 solutions for the roots of (4.24) as functions of the rescaled charge q_1. The black line shows the location of the supersymmetric solitons, with the susy condition $q_2 = q_1$ (see section 5). The other coloured lines represent the different roots of (4.24) for fixed $q_2 = 0.2$. The vertical dotted purple line correspond to the value of q_1 satisfiying the susy condition $q_1 = q_2 = 0.2$. As expected, this intersects the coloured solution lines where they intersect the black line. The additional intersection at $x_0 = 1$ corresponds to non-susy solutions with vanishing scalar (see following subsection 5.2).

setting the boundary at $y \to \infty$ and implying for the x_0 point location $x_0 = 1 - \frac{\alpha}{y_0}$. The solution can be then rewritten as

$$e^0 = \sqrt{\Upsilon(y)}\, dt, \quad e^1 = \sqrt{\frac{\Upsilon(y)}{f(y)}}\, \frac{\alpha\,\eta}{y\,(y-\alpha)}\, dy, \quad e^2 = \sqrt{\Upsilon(y)\, f(y)}\, d\varphi, \quad e^3 = \sqrt{\Upsilon(y)}\, dz,$$
$$\phi = -\frac{1}{\sqrt{2}} \ln\left(1 - \frac{\alpha}{y}\right), \quad A^1 = Q_1\, \alpha \left(-\frac{1}{y} + \frac{1}{y_0}\right) d\varphi\,, \quad A^2 = Q_2\, \alpha\, \frac{y - y_0}{(y-\alpha)\,(y_0 - \alpha)}\, d\varphi, \tag{4.26}$$

with

$$\Upsilon(y) = \frac{L^2\, y\,(y-\alpha)}{\alpha^2\, \eta^2}\,, \qquad f(y) = 1 - \frac{\eta^2\, \alpha^3\, (-Q_1^2\,(y-\alpha) + Q_2^2\, y)}{L^2\, y^2\, (y-\alpha)^2}\,. \tag{4.27}$$

To make contact with the model [5], we have to impose $\phi = 0$ while also obtaining the same functional form for non-vanishing gauge fields A^1 and A^2. In order to achieve this result in a well-defined configuration, we consider the simultaneous scalings

$$\alpha \to 0\,, \qquad \eta \to \frac{L^2}{\alpha}\,, \qquad Q_{1,2} \to \eta\, \tilde{Q}_{1,2}\,, \qquad \tilde{Q}_1^2 - \tilde{Q}_2^2 \to \frac{\mu}{\eta\, L^2}\,, \tag{4.28}$$

resulting in a finite well-behaving limit for the solution and a standard form for the conformal factor $\Upsilon \to \frac{y^2}{L^2}$.

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Explicitly, the above limit gives

$$\begin{aligned} &e^0 = \frac{y}{L}\,dt\,, \qquad e^1 = \frac{dy}{\sqrt{f_0(y)}}\,, \qquad e^2 = \sqrt{f_0(y)}\,d\varphi\,, \qquad e^3 = \frac{y}{L}\,dz\,, \\ &\phi = 0\,, \qquad A^1 = A^2 = L^2\,\tilde{Q}_1\left(-\frac{1}{y}+\frac{1}{y_0}\right)d\varphi\,, \end{aligned} \tag{4.29}$$

with

$$f_0(y) = \frac{y^2}{L^2}\,f(y) = \frac{y^2}{L^2} - \frac{\mu}{y} - \frac{L^4\,\tilde{Q}_1^2}{y^2}\,, \tag{4.30}$$

which finally corresponds to the scalar free, single gauge field solution [5] for $Q = L^2\,\tilde{Q}_1$.

5 Supersymmetric solutions

The soliton configuration preserve some part of the supersymmetry if

$$Q_1 = Q_2 \quad \Longrightarrow \quad \mu = 0\,. \tag{5.1}$$

It is possible to achieve the aforementioned condition by solving the Killing spinor equations. We also remark that, as expected, supersymmetry leads to the vanishing of the energy parameter μ, see (4.9), while the reverse is not true (the solution is not supersymmetric for $Q_1 = -Q_2$).

Applying the susy condition (5.1) in the $f(x)$ function gives

$$f(x) = 1 - \frac{Q_1^2\,\eta^2\,(-1+x)^4}{L^2\,x^2}\,, \tag{5.2}$$

and the Killing spinors can be explicitly written as

$$\chi_{\mathrm{R}_{(1)}} = e^{i\,\omega\,\varphi}\begin{pmatrix}\alpha_-(x)\\0\\0\\-i\,\alpha_+(x)\end{pmatrix}, \qquad \chi_{\mathrm{R}_{(2)}} = e^{i\,\omega\,\varphi}\begin{pmatrix}0\\\alpha_+(x)\\-i\,\alpha_-(x)\\0\end{pmatrix}, \tag{5.3}$$

$$\chi_{\mathrm{I}_{(1)}} = e^{i\,\omega\,\varphi}\begin{pmatrix}0\\\alpha_-(x)\\-i\,\alpha_+(x)\\0\end{pmatrix}, \qquad \chi_{\mathrm{I}_{(2)}} = e^{i\,\omega\,\varphi}\begin{pmatrix}-\alpha_+(x)\\0\\0\\i\,\alpha_-(x)\end{pmatrix}, \tag{5.4}$$

where, from eq. (2.21), we have $\chi_{\mathrm{I}_{(k)}} = i\,\gamma^5\chi_{\mathrm{R}_{(k)}}$, and with

$$\alpha_\pm(x) = \sqrt{\frac{\eta}{L}}\,\Upsilon(x)^{\frac{1}{4}}\,\sqrt{1\pm f(x)^{1/2}}\,, \qquad \omega = -\frac{\pi}{\Delta}\,. \tag{5.5}$$

The above relation involving Δ implies the antiperiodicity of the spinors. The chiral form of the spinors can be reproduced as

$$\begin{aligned} \epsilon^1_{(k)} &= \mathrm{Re}\,\chi_{\mathrm{R}_{(k)}} + i\,\mathrm{Re}\,\chi_{\mathrm{I}_{(k)}} = \left(\mathbb{1}-\gamma^5\right)\mathrm{Re}\,\chi_{\mathrm{R}_{(k)}}\,, \\ \epsilon^2_{(k)} &= \mathrm{Im}\,\chi_{\mathrm{R}_{(k)}} + i\,\mathrm{Im}\,\chi_{\mathrm{I}_{(k)}} = \left(\mathbb{1}-\gamma^5\right)\mathrm{Im}\,\chi_{\mathrm{R}_{(k)}}\,, \end{aligned} \tag{5.6}$$

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giving

$$\epsilon^1_{(1)} = \begin{pmatrix} \cos(\omega\varphi)\,\alpha_-(x) \\ i\cos(\omega\varphi)\,\alpha_-(x) \\ i\sin(\omega\varphi)\,\alpha_+(x) \\ \sin(\omega\varphi)\,\alpha_+(x) \end{pmatrix}, \qquad \epsilon^2_{(1)} = \begin{pmatrix} \sin(\omega\varphi)\,\alpha_-(x) \\ i\sin(\omega\varphi)\,\alpha_-(x) \\ -i\cos(\omega\varphi)\,\alpha_+(x) \\ -\cos(\omega\varphi)\,\alpha_+(x) \end{pmatrix}, \tag{5.7}$$

$$\epsilon^1_{(2)} = \begin{pmatrix} -i\cos(\omega\varphi)\,\alpha_+(x) \\ \cos(\omega\varphi)\,\alpha_+(x) \\ \sin(\omega\varphi)\,\alpha_-(x) \\ -i\sin(\omega\varphi)\,\alpha_-(x) \end{pmatrix}, \qquad \epsilon^2_{(2)} = \begin{pmatrix} -i\sin(\omega\varphi)\,\alpha_+(x) \\ \sin(\omega\varphi)\,\alpha_+(x) \\ -\cos(\omega\varphi)\,\alpha_-(x) \\ i\cos(\omega\varphi)\,\alpha_-(x) \end{pmatrix}, \tag{5.8}$$

that, as expected, satisfy the condition

$$\gamma^5\,\epsilon^A_{(k)} = -\epsilon^A_{(k)}\,. \tag{5.9}$$

If we consider the $\mathcal{N}=2$ framework, the existence of four chiral spinors identifies the solution as 1/2 BPS [38, 39]. The solution instead turns out to be 1/8-BPS when referring to the maximal $\mathcal{N}=8$ theory.

5.1 Supersymmetric solutions with fixed fluxes

In order to satisfy the susy condition (5.1), the fluxes must be related as (see eq. (4.15))

$$\psi_2 = -\psi_1/x_0\,, \tag{5.10}$$

that inserted in (4.21) determines the existence of supersymmetric solitons for:

$$x_0 = \frac{\psi_1}{1-\psi_1}\,, \qquad \psi_2 = -1+\psi_1\,, \qquad 0<\psi_1<1\,, \tag{5.11}$$

$$x_0 = -\frac{\psi_1}{1+\psi_1}\,, \qquad \psi_2 = 1+\psi_1\,, \qquad -1<\psi_1<0\,. \tag{5.12}$$

The EM configurations of [5] can be reproduced by taking $\psi_1 = \pm\frac{1}{2}$.

5.2 Supersymmetric solutions with fixed charges

The susy condition in terms of the rescaled charges (4.22) simply reads

$$q_1 = q_2\,. \tag{5.13}$$

The η parameter (4.23) and the metric function (4.24) now simplify in

$$\eta = \frac{\Delta}{2\pi}\,\frac{1+x_0}{-1+x_0}\,, \qquad f(x_0) = 1 - \frac{q_1^2\,(1+x_0)^4}{x_0}\,. \tag{5.14}$$

Then, for every value $|q_1| < \frac{1}{4}$ there exist two charged solitons (one for each branch $x_0 \gtrless 1$) satisfying the condition

$$f(x_0) = 0 \qquad \rightarrow \qquad q_1 = \pm\frac{x_0}{(1+x_0)^2}\,, \tag{5.15}$$

the sign depending on the sign of the charge.

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Relation with pure EM solutions. It is possible to show that this choice of boundary conditions also features non-susy configuratons with vanishing scalar. If we consider the discussion of section 4.2, it is easily found that EM solutions with vanishing scalar of [5] can be achieved from our susy configuration by the scaling $x_0 \to 1$ and

$$q_1^2 - q_2^2 \to 0\,. \tag{5.16}$$

Since we have found susy hairy configuration (5.13) satisfying the above condition but with $x_0 \neq 1$, we realize that in the fixed charges framework we can have both susy hairy and scalar-free EM solutions [5], satisfying the same boundary condition (5.16). The latter EM solutions however are *not* supersymmetric (except when their period Δ is maximum) and correspond to the discussed additional intersection of the vertical dotted purple with the hairy solutions at $x_0 = 1$ in figure 2.

When the period of the scalar-free solutions [5] takes the maximum value $\Delta = \pi\sqrt{\frac{L^3}{Q}}$, the configuration turns out to be supersymmetric, and coincides with our susy hairy soliton solution. In particular, since we have found the relation $Q = L^2\,\tilde{Q}_1$, this corresponds to a period for the hairy solution $\Delta = \pi\sqrt{\frac{L\,\eta}{Q_1}}$, that inserted in (4.22) gives

$$\Delta = \pi\sqrt{\tfrac{L\,\eta}{Q_1}} \quad \Rightarrow \quad q_1 = \frac{1}{4} \quad \Rightarrow \quad x_0^2 = 1 \quad \Rightarrow \quad \eta \to \infty \tag{5.17}$$

having used equation (5.15). This gives again the limit discussed in section 4.2, where hairy configurations reduce to the vanishing scalar solutions [5]. The remarkable implications of this correspondence on the stability of susy configurations will be better illustrated in section 6.3.

6 Phase structure

6.1 Euclidean action

Let us consider the analytic continuation of the soliton metric to a real Euclidean metric $g_{\rm E}$ featuring Euclidean time $\tau \in [0, \beta]$, $\beta = 1/T$ representing the inverse of the temperature T of the configuration. This would give rise to a periodic bosonic solution in β, being the gauge and the dilaton fields invariant under the continuation.

We note that, unless $\beta = \infty$, the thermal partition function is incompatible with supersymmetry, since it needs the fermions to be antiperiodic in τ. As a result, only zero-temperature susy solutions exist, while non-susy bosonic solutions can be defined for all values of β.

The Euclidean action $S_{\rm E}$ can be expressed as [40]:

$$\frac{S_{\rm E}}{V} = I_{\rm bulk} + I_{\rm GH} + I_{\rm BK} + I_{\rm ct} + I_\phi\,, \tag{6.1}$$

where

$$V = \beta\,\Delta\,\Delta_z\,, \qquad \Delta_z = \int dz\,. \tag{6.2}$$

The $I_{\rm bulk}$ term gives the bulk contribution,

$$I_{\rm bulk} = \lim_{\epsilon\to 1^-}\frac{1}{\kappa}\int_{x_0}^{\epsilon} dx\,\sqrt{g_{\rm E}}\left(-\frac{R}{2}+\frac{1}{2}\,(\partial\phi)^2+\frac{1}{4}\,e^{\sqrt{2}\,\phi}\left(F^1\right)^2+\frac{1}{4}e^{-\sqrt{2}\,\phi}\left(F^2\right)^2-\frac{2+\cosh\left(\sqrt{2}\,\phi\right)}{L^2}\right), \tag{6.3}$$

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while $I_{\rm GH}$ is the Gibbons-Hawking term, $I_{\rm BK}$ the Balasubramanian-Krauss counterterm, $I_{\rm ct}$ is a scalar-dependent divergent counterterm and I_ϕ is a finite counterterm. For our solutions, they can be expressed as

$$I_{\rm GH} = -\frac{1}{\kappa}\lim_{\epsilon\to 1^-} K\sqrt{h}\,, \qquad I_{\rm BK} = \frac{2}{\kappa\, L}\lim_{\epsilon\to 1^-}\sqrt{h}\,, \qquad I_{\rm ct} = \frac{1}{2\,\kappa\, L}\lim_{\epsilon\to 1^-}\sqrt{h}\,\phi^2\,, \qquad I_\phi = 0\,, \tag{6.4}$$

where h is the square root of the determinant of the boundary metric $h_{\mu\nu}$ and K is the trace of the extrinsic curvature $K_{\mu\nu} = \mathcal{D}_{(\mu}N_{\nu)}$. The contributions explicitly read:

$$\begin{aligned} I_{\rm bulk} &= \frac{1}{\kappa}\lim_{\rho\to\infty}\left(\frac{\rho^3}{L^4}+\frac{\rho}{8\,\eta^2}-\frac{\mu}{L^2}\right),\\ I_{\rm GH} &= -\frac{1}{\kappa}\lim_{\rho\to\infty}\left(\frac{3\,\rho^3}{L^4}+\frac{3\,\rho}{8\,\eta^2}-\frac{3\,\mu}{2\,L^2}\right),\\ I_{\rm BK} &= \frac{1}{\kappa}\lim_{\rho\to\infty}\left(\frac{2\,\rho^3}{L^4}-\frac{\mu}{L^2}\right),\\ I_{\rm ct} &= \frac{1}{\kappa}\lim_{\rho\to\infty}\frac{\rho}{4\,\eta^2}\,, \end{aligned} \tag{6.5}$$

giving for the Euclidean action

$$\frac{S_{\rm E}}{V} = -\frac{\mu}{2\,L^2\,\kappa}\,, \tag{6.6}$$

where μ given in (4.9). Since there is no entropy associated with these configurations, their free energy corresponds to their energy.

6.2 Fixed fluxes ψ_1 and ψ_2

Let us now consider fixed fluxes boundary conditions. For the hairy soliton configuration, the free energy density is as follows:

$$\begin{aligned} \frac{S_{\rm E}}{V} &= \frac{G_\phi}{\Delta\,\Delta_z} = \frac{M}{\Delta\,\Delta_z} = -\frac{\mu}{2\,\kappa\,L^2} = \mp\frac{1}{2\,\kappa\,\eta}\left(Q_1^2-Q_2^2\right)\\ &= \mp\frac{2\pi^3 L^2}{\kappa\,\Delta^3}\,\frac{\left(-\psi_1^2+x_0^2\,\psi_2^2\right)\left|\left(1+2\,x_0\right)\psi_1^2-x_0\left(2+x_0\right)\psi_2^2\right|}{x_0\left(-1+x_0^2\right)^2}\,, \end{aligned} \tag{6.7}$$

having employed eq. (4.20). Using now the free energy of the AdS soliton of [3]

$$G_0 = -\frac{32}{27}\,\frac{\pi^3 L^2}{\Delta^3\,\kappa}\,\Delta\,\Delta_z\,, \tag{6.8}$$

as a convenient normalization, we show in figures 3, 4 the ratio

$$\frac{G_\phi}{|G_0|} = \pm\frac{27}{16}\,\frac{\left(-\psi_1^2+x_0^2\,\psi_2^2\right)\left|\left(1+2\,x_0\right)\psi_1^2-x_0\left(2+x_0\right)\psi_2^2\right|}{x_0\left(-1+x_0^2\right)^2}\,. \tag{6.9}$$

We note that due to the normalization, G_0, the graphs always tend to a value bigger than -1 in the y axis. This seems to indicate that the AdS soliton is the lowest energy state of General Relativity also within these more general supergravity theories.

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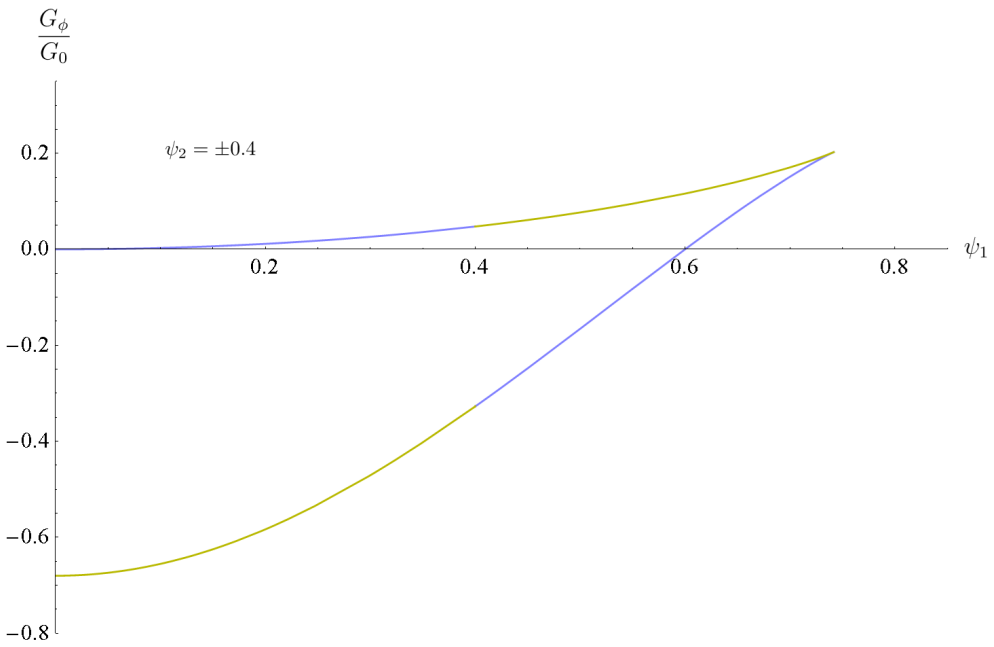


Figure 3. Rescaled free energy $\frac{G_\phi}{|G_0|}$ as a function of ψ_1, for $\psi_2 = \pm 0.4$. The $x \gtrless 1$ branches of the solution are represented by different hues. The shift in color occurs when one of the spacetimes, pertaining to a configuration, transits into another (see section 4).

Figures 3 and 4 show two distinct branches of energies. The upper one ends up at zero energy, when there are no sources. The lower one ends up at a negative energy, and when there are no sources it matches the energy of the AdS Soliton. On the field theory side, the dual fermions are antiperiodic by construction on these solutions. Therefore, in the dual QFT, the higher energy branch corresponds to antiperiodic bosonic fields on the $\mathbb{S}^1$ and the zero energy end point of the higher energy branch is just a representation of the matching of the bosonic and fermionic degrees of freedom, as expected from a supersymmetric dual field theory. The lowest energy solution then has antiperiodic fermions and periodic bosons on the $\mathbb{S}^1$.

It is remarkable that both the higher and lower energy solutions are reproduced in the gravity side by matching the negative vev and the positive vev solutions as can be seen from figure 5, the latter representing the vev $\langle \mathcal{O} \rangle \Delta$ of eq. (4.16) originating from the presence of the dilaton.

Negative values of the vev $\langle \mathcal{O} \rangle \Delta$ in figure 5 correspond to configurations belonging to the $x > 1$ branch, while the positive region is related to the $0 < x < 1$ interval. The susy solution (requiring $\mu = 0$) is found for vanishing value of the free energy, therefore in correspondence with the intersection with the ψ_1 horizontal axis, see section 5.1.

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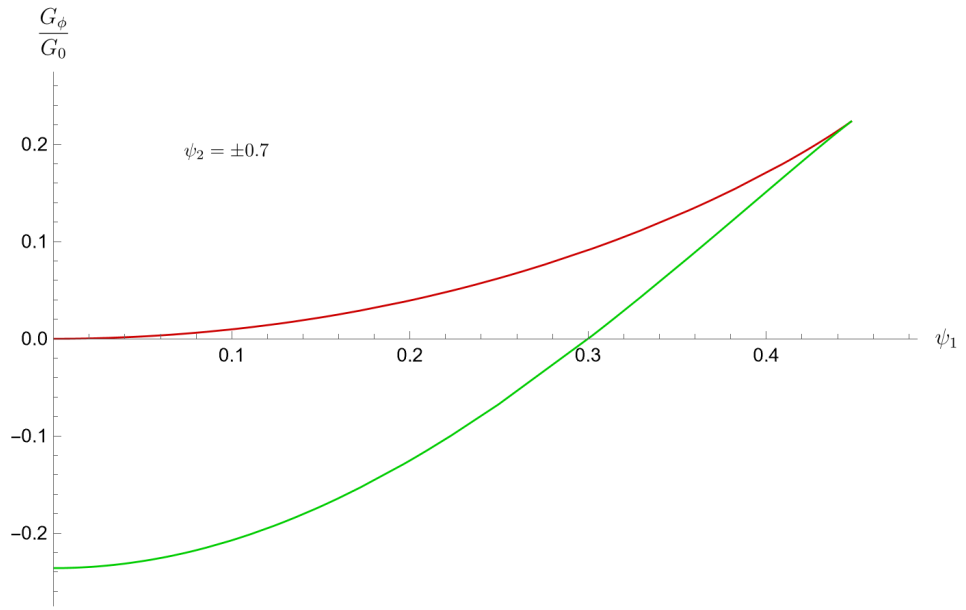


Figure 4. Rescaled free energy $\frac{G_\phi}{|G_0|}$ as a function of ψ_1, for $\psi_2 = \pm 0.7$. The $x \gtrless 1$ branches of the solution are represented by different hues. The shift in color occurs when one of the spacetimes, pertaining to a configuration, transits into another (see section 4).

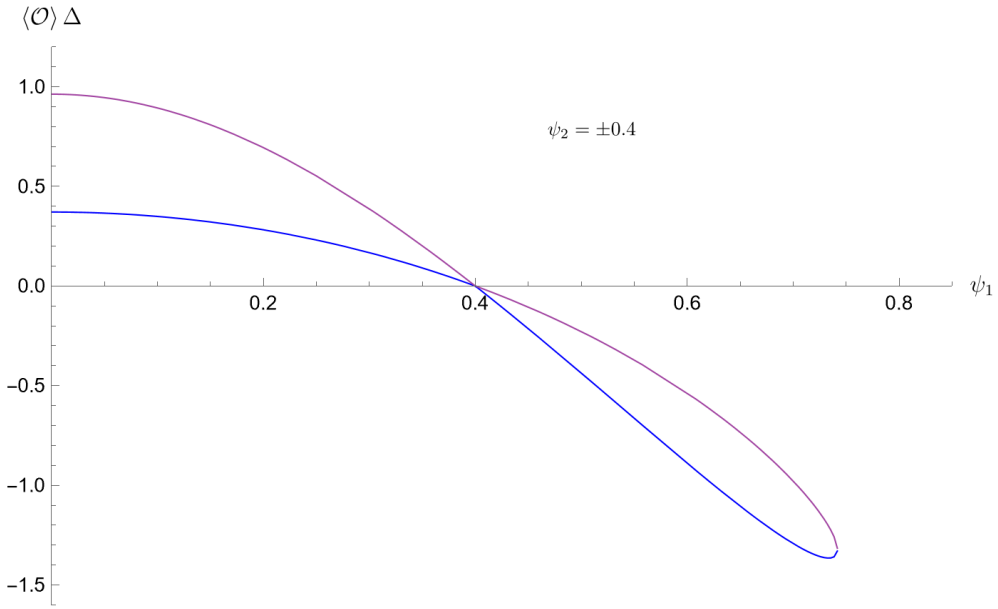


Figure 5. Rescaled vev $\langle \mathcal{O} \rangle \Delta$ as a function of ψ_1, for $\psi_2 = \pm 0.4$. Different colours are used to represent different branches of the solution.

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6.3 Fixed charges q_1 and q_2

In the fixed charges boundary framework, the free energy of the solution is found by exploiting the Legendre transform of the Euclidean action

$$\begin{aligned}\frac{F_\phi}{V} &= \frac{S_{\rm E}}{V} - \langle J^\nu_\Lambda \rangle\, A^\Lambda_\nu\Big|_{x\to 1} \\ &= -\frac{\mu}{2\,L^2\,\kappa} - \langle J^\varphi_1 \rangle\, Q_1\,(1-x_0) - \langle J^\varphi_2 \rangle\, Q_2\left(-1+x_0^{-1}\right) \\ &= \mp\frac{(Q_1^2-Q_2^2)}{2\,\eta\,\kappa} \pm \frac{Q_1^2}{\eta\,\kappa}\,(1-x_0) \pm \frac{Q_2^2}{\eta\,\kappa}\left(-1+x_0^{-1}\right),\end{aligned} \tag{6.10}$$

the signs depending on whether the solution falls within the range $x<1$ or $x>1$.

Supersymmetric solutions. In section 5.2 we have seen how this fixed charges framework gives rise to the possibility of finding both susy hairy and scalar-free EM soliton configurations satisfying the same boundary conditions. This will lead to amazing implications on the (expected) stability of the supersymmetric configurations.

Supersymmetric hairy solutions satisfy the condition (5.13), $q_1 = q_2$. The rescaled charges (4.22) are related to the single charge of the EM configuration [5] as

$$q_1 = \frac{\Delta^2}{4\pi^2 L}\,\frac{Q}{L^2}\,, \tag{6.11}$$

having used the relations of section 4.2.

The free energy of the Einstein-Maxwell configuration [5] is given by

$$\frac{F_{\rm EM}}{\Delta\,\Delta_z} = -\frac{\mu_0}{2\,\kappa\,L^2} + \frac{2\,Q^2}{\kappa\,L^2\,r_0}\,, \qquad \mu_0 = \frac{r_0^4 - L^2\,Q^2}{r_0\,L^2}\,, \tag{6.12}$$

with $\Delta = \frac{4\pi L^2\,r_0^3}{3\,r_0^4+L^2Q^2}$. Using relation (6.11), we can rewrite the above EM free energy as

$$\frac{F_{\rm EM}}{\Delta\,\Delta_z} = \frac{2\pi^3 L^2}{\Delta^3\,\kappa}\,X^2\,(5-4\,X)\,, \tag{6.13}$$

with

$$q_1^2 = \frac{1}{16}\,X^3\,(4-3\,X)\,, \qquad X = \frac{\Delta\,r_0}{\pi L^2}\,. \tag{6.14}$$

For the supersymmetric hairy solutions we find

$$\frac{F_\phi}{|G_0|} = \frac{27}{4}\,|q_1|\,, \tag{6.15}$$

having used (4.22), (4.23), (5.13) and (5.15) in (6.10).

In order to compare the susy hairy solution and the non-susy EM configuration (featuring the same boundary conditions) we plot their free energies in figure 6 as functions of q_1. We can see that both the Einstein-Maxwell and the hairy solitons exist for values of q_1 such that $q_1 \leq \frac{1}{4}$. In the extreme value $q_1 = \frac{1}{4}$ the different configurations merge yielding a supersymmetric soliton.

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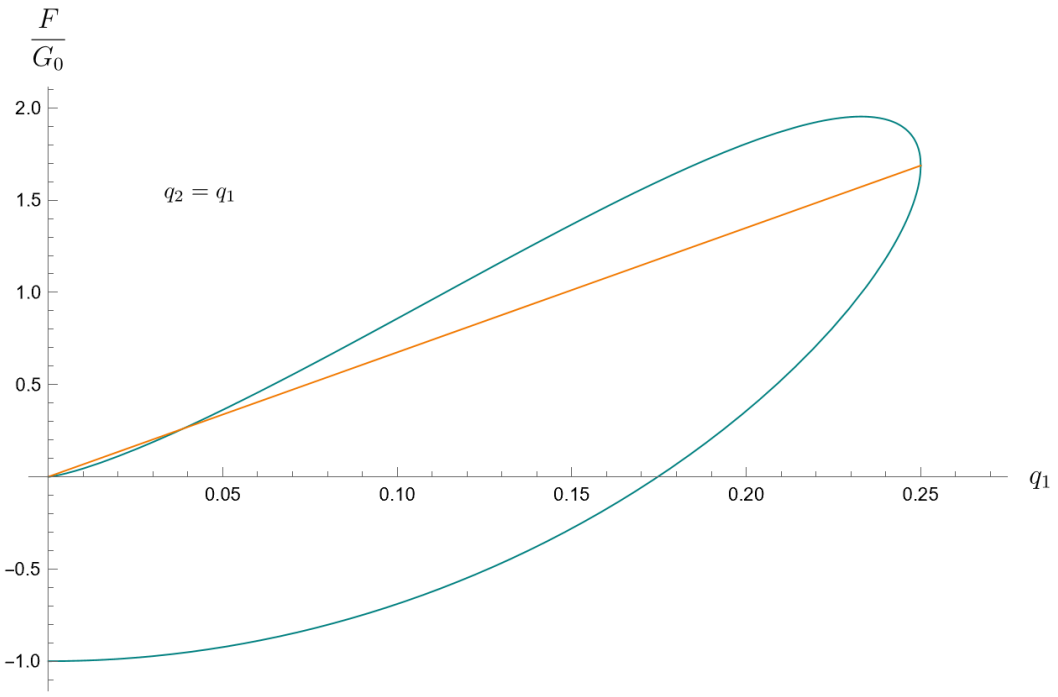


Figure 6. Rescaled free energy $\frac{F}{|G_0|}$ as a function of q_1 for supersymmetric condition $q_1 = q_2$. The orange line represents the hairy supersymmetric solitons (6.15), while the non-supersymmetric pure Einstein-Maxwell solitons (6.13) are shown in blue.

The phase diagram illustrates the existence of a branch of non-susy Einstein-Maxwell solutions having lower free energy than the hairy supersymmetric solution. This may come as a surprise, as susy solutions satisfy a BPS bound that would be expected to identify them as the lowest energy configurations. This unconventional result, however, does not conflict with the positive energy theorem [41, 42]. The latter implies that the energy of a susy configuration is the lowest of the class featuring the same boundary conditions, but a necessary condition for the theorem to apply is the existence, for the non-susy configuration, of an asymptotic Killing spinor coinciding, up to $O(1/r^2)$ terms at infinity, with the Killing spinor of the susy one. The key point in the latter prescription lies in the definition of asymptotic Killing spinors arising in non-SUSY configurations. These latter spinors satisfy some Killing spinor equation only at the asymptotic region of the spacetime, not necessarily holding true for the entire space (as it happens for Killing spinors of a susy solutions). These properties are discussed more in detail in [6].

Since our susy hairy solutions have antiperiodic boundary conditions at infinity, the positive energy theorem applies only for non-susy solutions with an asymptotic Killing spinor with the same properties [6, 7]. This only happens for values of the charges at infinity where the susy solution's free energy is less than the non-supersymmetric one, as one would assume, preventing contradictions with the positive energy theorem implications. The presented situation is then remarkable, as an example of framework in which an instability of the supersymmetric solutions under quantum phase transitions can occur [21].

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