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# Model Updating of different Bridge types using ambient vibration and OMA identification.

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**Abstract**

Structural Health Monitoring (SHM) to date is an increasingly crucial issue in civil engineering. Many structures like bridges and viaducts nowadays are often older than their design life and show signs of structural deterioration that can potentially affect structural safety. One of the most popular monitoring techniques is dynamic monitoring under environmental vibrations using Operational Modal Analysis (OMA) techniques.
OMA needs f.e.m. models that can simulate structural behavior under the real conditions and that can be used for interpretation of monitoring data and for possible damage detection.
The study proposes a model updating for two different bridges: a girder and box deck one. After an OMA identification using ambient vibration, through the modal parameter identified, modal updating is performed on f.e.m. models, changing only few essential parameters. All the f.e.m. models are made with beam elements.
The study highlights that even a simple f.e.m. model of the structure can describe accurately the dynamic and in general the structural behavior of a complex structure such as bridges and viaducts. The authors also show the differences between a f.e.m. model developed for the design and or verification and a f.e.m. model intended SHM. Model updating is performed through the management of a few parameters of the structure like the stiffness of bearing devices between superstructure and substructure, the deformability of foundation soil, the mechanical properties of concrete and reinforcement and the modal masses. The variation in each of these parameters can have a different role on the behavior of the structure.
A good reliability of the f.e.m. model can be achieved through the management of a few structural parameters obtaining results that are sufficient for models that have the purpose of interpreting SHM data and are used to perform damage research.



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*Keywords:* Bridge Dynamic Identification, Modal Updating, f.e.m. modelling.

## 1. Introduction

The infrastructure assets of Western Countries, for the most part, were developed between 1950 and 1980. These structures due to their age, which in many cases exceeds the design life, are now beginning to manifest critical issues Neves et al. (2005).

Over the past decade, exponential technological development and the increasingly affordable price of sensors have made it possible to monitor increasingly complex structures Bertagnoli et al. (2019), Bertagnoli et al. (2020). In this perspective, the development of f.e.m. models dedicated to the interpretation of monitoring data and the understanding of structural behavior becomes essential. These models may be also useful for the safety verification of the monitored structure.

Model updating is a widely studied problem and several studies can be found in the literature Gennaro et al. (2023), Shi et al. (2019), Lacanna et al. (2020), Polanco et.al (2016). Often, however, studies focus on techniques that are quite complex and difficult to be implemented in common analysis by professionals in the field, such as optimization parameters problems Ferrari et al. (2019), Deng et al. (2010) or automated model updating Altunişik et al. (2018).

The purpose of this study is to analyze the effect on the dynamic behavior of two bridges of few simple parameters, already discussed in literature. The parameters that will be considered are:

- the deformability of foundation soil Faraonis et al. (2015);
- the stiffness of the bearing devices between deck and substructure (piers/abutments) Hester et al. (2019);
- the stiffness of the materials Gennaro et al. (2023), Polanco et.al (2016).

Often in the literature these effects are considered individually. In the present study, for each structural type examined, all of them will be considered to evaluate their mutual effect on the structural behavior.
Two different structures are analysed:

- A highway viaduct consisting of simply supported girders and three-dimensional framed piers;
- A highway viaduct consisting of a continuous box deck on multiple piers.

Manual modal updating is performed (instead of automatic modal updating) to have full control on the effect of each change of the parameters. Thus, the study is not aimed at achieving a perfect modal updating, but at understanding the effect of each parameter on the structural behavior.

Modal updating is performed using frequencies and modal shapes reconstructed by OMA techniques. A commercial software implementing the parametric frequency domain method PolyMAX Peeters et al. (2004), Peeters and Herman (2005) is used for the dynamic identification. Control parameters like foundation stiffness, bearing device properties and material stiffness are varied until experimental and f.e.m. modal frequencies and shapes differ by a magnitude that is considered engineeringly acceptable.

## 2. Case study description

Two different structures will be considered in the following paper. Geometric details of the structure and the location of the accelerometers used for dynamic identification will be reported for each of them.

Case Study 1 is a highway viaduct built in the late 1950s. The viaduct consists of 50m and 21m prestressed reinforced concrete girder decks simple supported on 3D frame piers. Fig. 1a is a plan view and Fig. 1b is a side view of the viaduct. Red dots in the figures indicate the location of the accelerometers. All accelerometers are triaxial. Only a part of the viaduct is dynamically monitored.

Case Study 2 is a highway viaduct built in the late 1990s. The viaduct consists of a prestressed concrete continuous box shaped deck on multiple piers. the span between supports is 100 meters. Fig. 2 shows the geometry of the viaduct. Blue dots indicate the location of monoaxial accelerometers, green dots indicate the location of triaxial accelerometers. Thus, the deck is monitored only with uniaxial accelerometers that acquire in the vertical direction. Only three of the six piers are monitored with triaxial accelerometers.

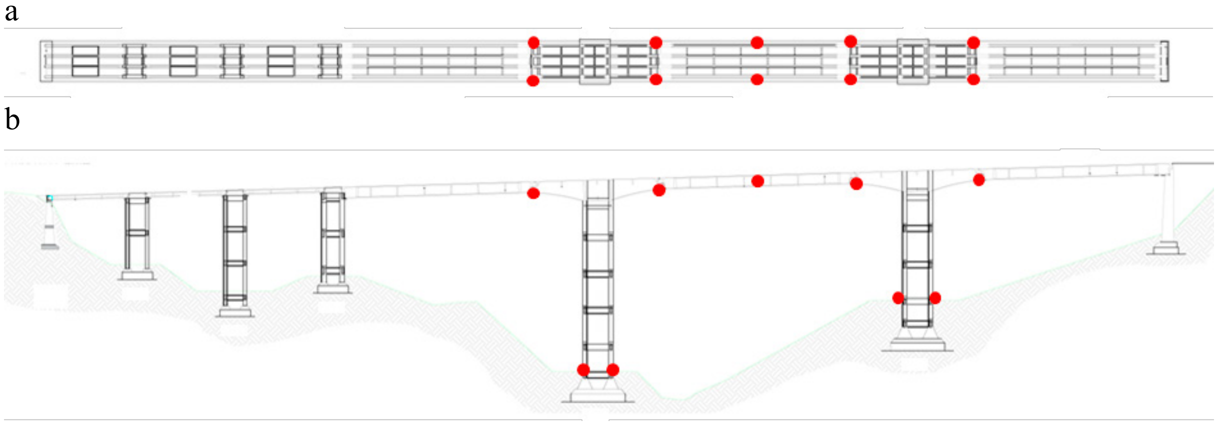


Fig. 1. Case Study 1. Geometry and sensors positions. (a) Plan view. (b) Side view.

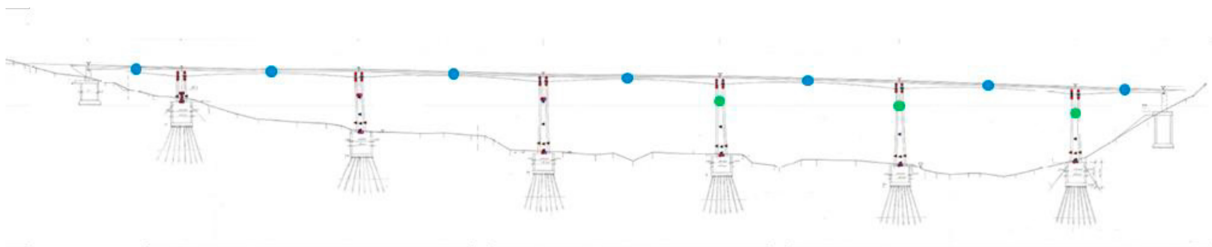

Fig. 2. Case Study 2. Geometry and sensors position.

## 3. Numerical models description

For both bridges described in the previous section, a linear elastic fem model is realized using only beam elements. Commercial f.e.m. software SAP2000 (SAP 2000: Weinheim , Germany) is used.

The f.e.m. model of Case Study 1 is shown in Fig. 3a. The bearing devices between superstructure and substructure are modelled as springs with translational stiffness in the three main directions. The stiffness of bearing devices in the starting f.e.m. model is equal to the stiffness of a new bearing device of same dimension taken from producer catalogue: in detail horizontal stiffness equal to 8.5 kN/mm in the longitudinal and transverse directions and a vertical stiffness equal to 9000 kN/mm.

Elastic modulus of concrete is obtained from the results of tests on cylindrical concrete specimens taken from the structure. Three different values are used: 35000 MPa for substructures, 37500 MPa for 50m decks and pier hammers, 35500 MPa for 21m decks.

Piers have a superficial foundation, therefore soil-foundation interaction is modelled placing vertical springs at the base of the footings of the piers; these springs are assumed to have an initial stiffness that generates a vertical displacement equal to 1mm per kg/cm$^2$ of applied load starting from the configuration of applied permanent loads and full consolidation occurred.

The f.e.m. model of Case Study 2 is shown in Fig. 3b. The box deck is modelled as a continuous beam supported on the piers. On each pier, the deck rests on two rows of supports; each one counts 3 bearing devices, the external bearings are free to slide in longitudinal and transverse directions, while the middle one is free to slide in longitudinal direction only.

The base of the piers is fully restrained as shaft foundations are present under each pier. Elastic modulus of concrete is set equal to the one declared in the calculation report of the structure as in-situ test on materials are not available; E = 32500 MPa is used for the piers and E=34500 MPa for the deck.

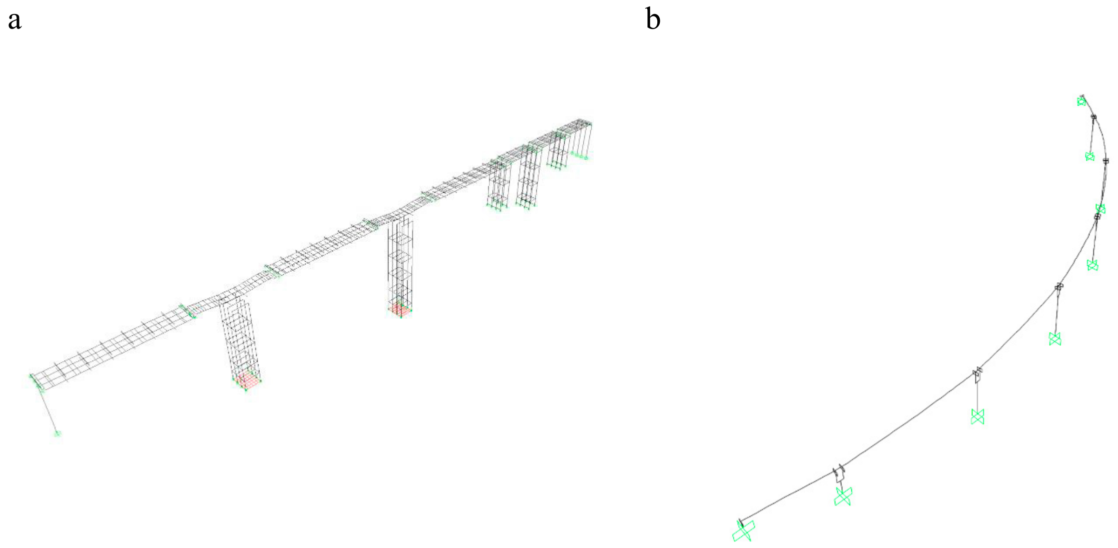


Fig. 3. Numerical model. (a) Case Study 1. (b) Case Study 2.

## 4. Methodology

The authors studied the influence of the parameters that are considered essential for model updating grounding on their design expertise. Model updating is concluded when the variations between frequencies and modal shapes identified by OMA techniques and f.e.m. model results are engineeringly acceptable.
The following parameters are changed for Case Study 1:

1. Reduction of non-structural permanent loads. Current standards suggest a road pavement weight of 3.0 kN/m$^2$ (EN 1991-2, Eurocode 1–Part 2. Traffic actions on bridges). This weight corresponds to a pavement thickness of about 13÷15 cm using common bituminous conglomerates. The pavement weight has been changed achieving convergence 2.6 kN/m2 (13% reduction from nominal value) corresponding to a pavement thickness of 12÷13cm.
2. Introduction of rotational stiffness of bearing devices. The rotational stiffness in longitudinal direction, usually neglected, has been introduced. The convergence value is 26000 kNm/rad. The value corresponds to the following condition: the average value of the reaction due to permanent loads on the 50m deck support is 1300kN; the size of the bearings in the longitudinal direction is 40cm; a realistic value of the eccentricity of this reaction with respect to the support axis is ¼ of the bearing dimension, therefore 10cm; this value is obtained from more refined analyses where the bearing is modelled with f.e.m; the bending moment corresponding to this eccentricity is 130 kNm; a deformation of the rubber pad equal to 1mm on one side and a rise of 1mm on the opposite side as a result of this moment (thus a rotation of 0.005 rad) is assumed; therefore the rotational stiffness of 130/0.005 = 26000 kNm/rad is obtained.
3. Variation of horizontal stiffness of rubber bearing devices. The stiffness in the horizontal direction of the bearing devices in the starting model was assumed to be 8.5 kN/mm which corresponds to the deformability of a new bearing device. The actual rubber bearing pads may have different mechanical characteristics due to aging and deterioration. The stiffness of the rubber bearings has thus been incremented until convergence was found for a value that is about one hundred times the original one. This value simulates almost complete interlocking of the decks. This increase also covers the actions that the decks can exchange along the expansion joints, which in theory should allow for movement but could present passive resistance due to aging and presence of debris.
4. Increase in elastic modulus to consider the homogenization of sections for the presence of reinforcement. The increase in the moment inertia properties of the section due to reinforcement is taken implicitly into account by the

increase in elastic modulus. Elastic modulus of homogenized section is increased up to 10% for decks and piers' hammerheads and up to 5% for vertical elements of the piers. This variation enables accounting for the stiffening effect of reinforcement in a simplified way in a purely elastic f.e.m. model where reinforcement bars are not present.
5. Variation of the stiffness of the foundation soil. In the starting model the springs placed under the foundation of the piers to simulate the deformability of the foundation soil were assumed to allow a displacement equal to 1 mm under a load of 10N/cm$^2$ applied after consolidation under permanent loads. At convergence this stiffness is doubled.

The following changes will be made for Case Study 2:

1. The constraint conditions between superstructure and substructure are changed with respect to the original model. In detail, the 6 supports of the deck on each pile are blocked from any possibility of sliding in the longitudinal direction. This assumption is realistic under everyday service loads to which the deck is subjected during dynamic identification, as the passive resistance of the bearings can avoid the longitudinal slip under normal traffic actions.
2. Self-weight is changed. The density of reinforced concrete (originally equal to 2500 kg/m$^3$) is reduced to 2350 kg/m$^3$ at convergence. This value is in accordance with the densities found experimentally on concrete cores extracted from existing structures of same age. This value corresponds to a reduction in masses associated with self-weight of 9.4%.
3. In this case study the value of the permanent loads (barriers, kerbs and pavement) was not varied as the values of the initial model were considered quite close to the real condition after a visual inspection of the bridge.
4. Increase in elastic modulus to consider a dual effect: homogenization of sections for the presence of reinforcement and a real small increase in elastic modulus due to concrete aging. An overall increase of 10% in the elastic modulus for the concrete of the deck is reached at convergence with respect to the value used in the starting model.
5. Variation of the full restrain at the base of the piles. Instead of a fully restrain, which allows no rotation at the base of the piles, constraints with finite rotational stiffnesses are inserted. The rotational stiffness of the constraint was calculated such that under design wind action the base of the pile has a subsidence of about 1 mm. The inclusion of a constraint with finite rotational stiffness considers the real nature of the foundation structures, which on rare occasions have infinite rotational stiffnesses, but in almost all cases have finite rotational stiffnesses and therefore non-zero rotations even under the action of normal operation.

## 6. Results and discussions

This section reports the results for the two different case studies and some considerations on them.

Table 1 shows the results in terms of frequency for Case Study 1. Fig. 4a shows variations between different f.e.m. models and identified frequencies.

The original f.e.m. model has an overall average error of -22.1% with respect to the frequencies identified; the largest error is related to horizontal modes, which amounts to -40.5%, whereas the error on vertical modes amounts to -15.9%. In general, the original f.e.m. model is more deformable than the real structure.

In f.e.m. model a, the first change with respect to the original f.e.m. model is made: non-structural permanent weights are reduced. Model b is obtained from model a by adding rotational stiffness to the elastomeric bearings. Model c is obtained from model b by tripling the horizontal stiffness of the elastomeric bearings. Model d is obtained from model c increasing the elastic modulus of concrete to consider the effect of homogenization of reinforcement and concrete aging. Model e is obtained from model d increasing the stiffness of the foundation soil. Model f is model e with the elastomeric bearings almost rigid in the horizontal direction.

Model f is considered in this study the ultimate result of model updating, nevertheless it still shows a global error from the identified frequencies of -9.4%: -17.1% for horizontal modes and -7.2% for vertical modes. It basically halved the error between original f.e.m. results and identified frequencies for both horizontal and vertical modes. It still shows an average global error just below 10% but can be considered engineeringly a fair tool for evaluating and interpreting monitoring data and performing structural safety verifications.

Table 2 shows the results in terms of frequency for Case Study 2. Fig. 4b shows variations between different f.e.m. models and with identified frequencies.

The original f.e.m. model has an overall average of the error of -19.0% with respect to the frequencies identified with OMA. The error for the horizontal modes is -12.8% and the error for vertical modes is -21.4%.

In model a bearing device between deck and piers are made perfectly rigid in the horizontal direction. Model b is obtained from model a reducing structural permanent weight. Model c is obtained from model b increasing the elastic modulus of concrete to consider the effect of homogenization of reinforcement and aging. Model d is obtained from model c changing the restraint condition at the base of the piers.

Model d is considered the ultimate model updating model. There is a global error from the identified frequencies of -6.2%; the average error of horizontal modes is -2.4% and the average error of vertical modes is -7.8%. The error between original f.e.m. results and OMA identification ones is more than halved for both horizontal and vertical modes. Model d can be considered engineeringly a good model for evaluating and interpreting monitoring data and performing structural safety verifications.

Table 1. Case Study 1. Results of model updating in terms of frequency.

<table>
<tr><th></th><th></th><th></th><th colspan="7">f.e.m. Model</th></tr>
<tr><th></th><th>Frequency identified with OMA [Hz]</th><th>Direction</th><th>Original [Hz]</th><th>a [Hz]</th><th>b [Hz]</th><th>c [Hz]</th><th>d [Hz]</th><th>e [Hz]</th><th>f [Hz]</th></tr>
<tr><td>1</td><td>0.86</td><td>Transversal</td><td>0.47</td><td>0.48</td><td>0.48</td><td>0.52</td><td>0.53</td><td>0.57</td><td>0.67</td></tr>
<tr><td>2</td><td>[-]</td><td>Transversal</td><td>0.52</td><td>0.52</td><td>0.52</td><td>0.60</td><td>0.61</td><td>0.63</td><td>0.96</td></tr>
<tr><td>3</td><td>[-]</td><td>Transversal</td><td>0.61</td><td>0.62</td><td>0.62</td><td>0.70</td><td>0.71</td><td>0.76</td><td>[-]</td></tr>
<tr><td>4</td><td>[-]</td><td>Longitudinal</td><td>0.63</td><td>0.64</td><td>0.64</td><td>0.77</td><td>0.78</td><td>0.84</td><td>1.5</td></tr>
<tr><td>5</td><td>[-]</td><td>Transversal</td><td>0.70</td><td>0.71</td><td>0.71</td><td>0.84</td><td>0.85</td><td>0.87</td><td>[-]</td></tr>
<tr><td>6</td><td>[-]</td><td>Transversal</td><td>0.92</td><td>0.93</td><td>0.94</td><td>1.02</td><td>1.03</td><td>1.14</td><td>[-]</td></tr>
<tr><td>7</td><td>[-]</td><td>Transversal</td><td>1.06</td><td>1.08</td><td>1.10</td><td>1.28</td><td>1.29</td><td>1.43</td><td>[-]</td></tr>
<tr><td>8</td><td>[-]</td><td>Longitudinal</td><td>1.10</td><td>1.11</td><td>1.12</td><td>1.51</td><td>1.52</td><td>1.57</td><td>[-]</td></tr>
<tr><td>9</td><td>[-]</td><td>Longitudinal</td><td>3.10</td><td>3.13</td><td>3.13</td><td>3.31</td><td>3.36</td><td>3.37</td><td>3.72</td></tr>
<tr><td>10</td><td>[-]</td><td>Longitudinal</td><td>4.39</td><td>4.43</td><td>4.44</td><td>4.46</td><td>4.56</td><td>4.66</td><td>5.34</td></tr>
<tr><td>11</td><td>2.06</td><td>Vertical</td><td>1.77</td><td>1.80</td><td>1.80</td><td>1.78</td><td>1.83</td><td>1.86</td><td>1.87</td></tr>
<tr><td>12</td><td>2.29</td><td>Vertical</td><td>1.90</td><td>1.93</td><td>1.94</td><td>1.93</td><td>2.01</td><td>2.04</td><td>2.03</td></tr>
<tr><td>13</td><td>2.44</td><td>Vertical</td><td>1.95</td><td>1.99</td><td>2.00</td><td>2.01</td><td>2.08</td><td>2.11</td><td>2.12</td></tr>
<tr><td>14</td><td>2.63</td><td>Torsional of the 50-meter decks</td><td>1.69</td><td>1.72</td><td>1.73</td><td>2.17</td><td>2.23</td><td>2.27</td><td>2.31</td></tr>
<tr><td>15</td><td>4.49</td><td>Vertical</td><td>3.73</td><td>3.77</td><td>3.78</td><td>3.81</td><td>3.91</td><td>3.95</td><td>4.3</td></tr>
<tr><td>16</td><td>6.37</td><td>Vertical</td><td rowspan="2">5.62</td><td rowspan="2">5.70</td><td rowspan="2">5.70</td><td rowspan="2">5.70</td><td rowspan="2">5.87</td><td rowspan="2">6.23</td><td>6.12</td></tr>
<tr><td>17</td><td>6.61</td><td>Vertical</td><td>6.28</td></tr>
<tr><td>18</td><td>7.38</td><td>Vertical</td><td>6.36</td><td>6.44</td><td>6.44</td><td>6.45</td><td>6.63</td><td>6.97</td><td>7.1</td></tr>
</table>

a) Original + Reduced non-structural permanent weight.
b) a + Rotational stiffness bearing devices.
c) b + Horizontal stiffness of rubber bearings triplicated.
d) c + Homogenization effect of ordinary and prestressing reinforcement.
e) d + Increased foundation soil stiffness.
f) e + Rigid rubber bearings in horizontal direction.

Some general considerations common to the two case studies and independent of the structural type considered can be made:

- Both original f.e.m. models, which are similar to common design models, are about 20% more deformable than the actual structure when it is subjected to environmental vibration.
- In model updating, the most important stiffening effect is achieved by varying the stiffness of the bearing devices, especially their horizontal stiffness. It is common design practice to use the horizontal stiffness of these devices provided by the producer; but due to aging, deterioration, or malfunction this stiffness may be significantly higher.

- Foundation deformability is another impactful parameter. It is common practice in design phase to consider a full restraint at the base of the piers. This modelling choice may be far from the actual behavior of the structure even in the case of deep foundations.
- Permanent loads estimation and actual stiffness of sections/materials play a less impactful but still not negligible effect. it is common practice to use parameters suggested by design standards for both parameters in the absence of in situ testing.

Table 2. Case Study 2. Results of model updating in terms of frequency.

| | Frequency identified with OMA [Hz] | Direction | f.e.m. Model<br>Original [Hz] | a [Hz] | b [Hz] | c [Hz] | d [Hz] |
|---|---|---|---|---|---|---|---|
| 1 | 0.83 | Transversal | 0.71 | 0.84 | 0.86 | 0.87 | 0.80 |
| 2 | 0.90 | Transversal | 0.80 | 0.92 | 0.94 | 0.96 | 0.89 |
| 3 | 1.21 | Vertical | 0.92 | 1.06 | 1.09 | 1.12 | 1.12 |
| 4 | 1.34 | Vertical | 1.07 | 1.17 | 1.21 | 1.24 | 1.24 |
| 5 | 1.54 | Vertical | 1.24 | 1.32 | 1.36 | 1.41 | 1.41 |
| 6 | 1.74 | Vertical | 1.42 | 1.50 | 1.54 | 1.60 | 1.60 |
| 7 | 1.98 | Vertical | 1.48 | 1.70 | 1.75 | 1.83 | 1.83 |

a) Original + Rigid bearing devices in the horizontal direction.
b) a + Reduced structural permanent weight.
c) b + Homogenization effect of ordinary and prestressing reinforcement.
d) c + Variation of the constraint at the base of the piles.

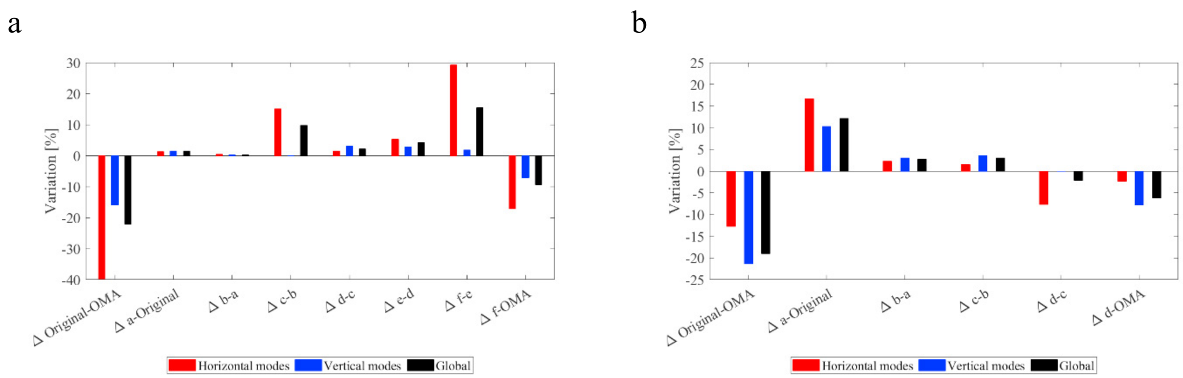


Fig. 4. Variations between different f.e.m. models and with identified frequencies. (a) Case Study 1. (b) Case Study 2.

## 7. Conclusions

In this study, the model updating of two viaducts with different structural typology is proposed: the first with girder decks on 3D medium size framed piers and the second with continuous box-shaped deck on multiple supports on high piers.

For the first structure, a more elaborate f.e.m. model is created, whereas a simpler one was used for the second structure. In this way, it is also possible to consider the effect that model complexity may have on the result of model updating.

Some parameters on which there are the greatest uncertainties were varied in model updating: structural and non-structural permanent weights, elastic modulus of concrete, horizontal stiffness of the bearing devices between superstructure and substructure, and restraint conditions at the base of the piers. The starting f.e.m. models, like the ones used in common design procedure, were both 20% less rigid than the actual structures.

The parameter that resulted to have the greatest impact on model updating is the horizontal stiffness of the bearing devices. These devices, due to aging, deterioration or malfunction, and due to the significantly low level of actions during normal operation of the structure, can be much stiffer than what assumed.

The second significantly impactful parameter is the soil deformability under foundations. Even in case of deep foundations (like shafts) no full rigid restraint can be considered, especially regarding rotations. In case of superficial foundations, the interaction between foundation soil and structure can be modelled by inserting finite vertical and rotation stiffness at the base of the piers.

Parameters that are less impactful but still have an effect are masses estimation (structural and non-structural permanent loads) and sections stiffness. Masses suggested by current standards are usually used but may overestimate the real mass. In situ tests are strongly suggested.

Both bridges presented in this study are prestressed. In prestressed structures actual section stiffness should be considered, taking into account real values of modulus of elasticity and homogenization effect coming from ordinary and prestressing reinforcement. In simply reinforced structures cracking may play the exactly opposite effect significantly reducing the stiffness of the members.

With this study, it has been shown that it is possible to perform model updating on simple f.e.m. models of structures of considerable complexity through the handling of only a few parameters and with engineering-acceptable results.

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