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Review Article

Mathematical Models for the Assessment of an Environmental System in Landscape Ecology

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Abstract

In the framework of the theory of Landscape Ecology, a review of Lotka-Volterrra type models is proposed. Such models can be considered useful tools in order to represent and evaluate the dynamical behavior and the ecological stability of an environmental system which, as known, is subjected during time to several transformations. At this purpose, after such a review and presentation of different models, an application to an important wine region in France is performed using a model recently introduced in literature.

Keywords: `Lotka-Volterrra Models, Nonlinear Ordinary Differential Equations, Landscaspe Ecology`

AMS subject classification: `34D20, 91C99`

1. Introduction

Impacts due to human behaviors show that our planet undergoes severe changes in its health. Pollution, rapid growth of towns, depopulation of rural areas are some of the drivers of loss of ecosystems, soil degradation, damages to landscapes and decrease of resilience of environmental systems.

The need to integrate the necessity of environmental protection into planning, evaluation and management policies of the ecological health of a territory has now consolidated inside the institutionalized reasons that refer to measures developed since the early 70s at the international level.

It is worth mentioning the attention of the international community towards the problem of environmental protection was confirmed by the so-called Brundtland Report of 1987. More recently, United Nations introduced through the 2030 Global Agenda the 17 Sustainable Development Goals (SDGs) and related targets to deal with the threefold dimension of sustainability paradigm [1]. Among the SDGs, there is the 11th one "Building more sustainable cities and community" that has direct relationships with the spatial planning, as well as to the protection and enhancement of environmental goods and services. The purpose is to guarantee a better life quality for both present and future generations.

The attention on the SDGs achievement increased in the latest years due to the rise of global warming [2], the related increase of natural disasters, and consequently the growing awareness of world communities in the thinning of world security margins [3].

Effective environmental protection involves the need to promote new planning tools for the protection of ecological stability, expanding the concept of the environment itself. This must not be identified only with the characteristics of its physical components (air, water, or soil, among others) but must consider the environment itself as a complex and heterogeneous system, dynamic and capable of evolving [4]. This system, as we will see, can be defined by the combination of different portions of the territory [5], characterized by exchanges of matter and biological energy, stored in the organic compounds. Biological energy is the amount per $\mathrm{Mcal/m^2}$ per year produced by each Landscape Unit that constitutes the environmental system under investigation. For example, homogeneous forestry areas can produce high value of biological energy, by contrast built-up areas are not able to produce it.

This is an approach at the basis of the theoretical principles and operating methods of the so-called

Landscape Ecology [6–8]. This discipline, through a critical rethinking of the principles of environmental protection, establishes that the planning choices of a territory cannot be conditioned only by the environmental characteristics of the individual portions of the territory itself, but must consider the system, including also the landscape, in its entirely complexity, identifying the laws that govern the evolutionary dynamics.

According to Landscape Ecology, the functioning of an environmental system must be assessed in relation to the connection between the various territorial portions that make it up (energy connectivity) [9] and, ultimately, to its stability. The relationship between ecological stability and energy connectivity, as it will be seen, plays a central role in the study of the transformation processes of the territory towards scenarios characterized by different levels of environmental quality and bio-diversity.

In this perspective, therefore, the attention of those who work in the field of spatial planning and management must be aimed at mitigating all the processes that to some extent cause the loss of environmental connectivity, processes among which fragmentation plays a fundamental role of the territory. Fragmentation, in fact, constitutes a degenerative process on which the loss of quality of the environment and landscape [10] largely depends. This phenomenon indicates a state of structural alteration of the landscape mosaic which results in greater isolation of populations and a progressive reduction in the levels of ecological diversity [11–13].

In the context of Landscape Ecology, a quantitative tool that evaluates the ecological health of an environmental system very effectively, highlighting the more or less high level of fragmentation of the territory, is the so-called ecological graph. This tool was introduced in 2007 by Fabbri [14] and consists, as will be seen, in the construction of a graph that takes into account a large number of data of the territory under observation, summarized by a series of indicators that allow to evaluate the overall ecological status of the territory itself and, thanks to the determination of its energy connectivity, the degree of fragmentation as well.

However, the ecological graph provides a static photograph of the environmental system taken into consideration, without showing what possible transformations it may be subject to. In fact, an environmental system, and its landscape, is in a meta-stable equilibrium that only under limited perturbations remains in its state, otherwise it evolves towards scenarios that show how the system has undergone significant environmental changes.

These observations were the starting point to propose some mathematical models which, using the data provided by the graph as initial conditions, can show the evolution over time of the transformations to which the system is subjected. Therefore, starting from 2010, with a first simple [15] model, this objective has been pursued. Subsequently, other models were developed which took increasingly into account the characteristics of the territory. The purpose of this work, in fact, is to present these models, discussing their properties.

The article is organized as follows: in the next Section the ecological graph will be presented and the problem of the dynamic evaluation of an environmental system will be formulated. In Section 3, the so-called PANDORA models (Procedure for mAthematical aNalysis of lanDscape evOlution and equilibRium scenarios Assessment) will be presented in the three different versions [16–18].

In Section 4 another mathematical model [19], recently published, related to the general theory of neural networks will be discussed. Such a model has a mathematical structure rather different from that of PANDORA models, so that it shows a rather complex dynamical behavior with a stability analysis which exhibits several bifurcations, as outlined in the above paper. An application of this model in an important and famous wine region of France [20] is then presented in Section 5.

Finally, possible developments and new perspectives in the context of the use of mathematical models in Landscape Ecology are discussed in the last Section 6.

2. The ecological graph and an approach for the assessment of an environmental system

As already mentioned in the previous Section, in 2007 Fabbri [14] has proposed a new tool, named ecological graph, that can be useful for the assessment of an ecological system. The main idea of the proposed procedure consists in the determination of the biological energy of the system and of its transmission into the various portions of the region (see the article by Brown et al. [21], together with the contributions included in the volume [22] edited by Fabbri itself).

For this purpose the environment is defined as an isolated system distributed in n landscape units (LU), sometimes cited in literature as Ecological Sectors. The LUs are defined by their borders constituted by anthrop or natural barriers, like roads, highways, railways, rivers, channels, hills, among others. Each barrier is characterized, with respect to the passage of bio-energy, by a permeability index $p \in [0,1]$, where 0 indicates complete impermeability and 1 total permeability (see for the values of p the classification given in [16]).

Each LU, at its turn, is distributed in $m_i,\ i=1,\dots,n$, biotopes that are ground portions of the same type of land cover. Of course in order to represent the environment under observation in such a way it is necessary to use the Geographic Information System (GIS) which is capable to give to the users any information about the land cover, the barriers, the built up surfaces and other morphological characters of the territory itself, like perimeters or areas of particular portions.

Each biotope, following its specific nature, is characterized by the bio-energy produced, measured by the index of Bio-potential Territorial Capacity [7] (BTC index). Such an index, for the j-th biotope belonging to the i-th LU, $j=1,\dots,m_i$, will be indicated hereinafter by the symbol b_{ji} assuming values in the range $[0,6.5]$ $Mcal/(m^2 \cdot year)$, where 0 corresponds to a ground without any vegetation whereas the value $b_{max}=6.5$ is that assigned to a ground covered by an oaks wood, which generally, at our latitudes, is considered that producing the maximum value of bio-energy. Again for the values of the BTC index b_{ji} the reader is addressed to the article [16].

Generally the BTC index considers 5 distinct ecological classes from A to E, i.e.

$$\mathrm{A}=[0,0.4),\ \mathrm{B}=[0.4,1.2],\ \mathrm{C}=(1.2,2.4],\ \mathrm{D}=(2.4,4.0],\ \mathrm{E}=(4.0,6.5]. \tag{1}$$

The total BTC in $Mcal/year$ for every LU is given by

$$\mathcal{B}_i=\sum_{j=1}^{m_i} b_{ji}s_{ji}, \tag{2}$$

where s_{ji} is the area of the ji-biotope. Obviously it is possible to define the maximum value $\mathcal{B}_i^{max}$ of BTC for each LU by the formula

$$\mathcal{B}_i^{max}=b_{max}\,S_i \tag{3}$$

equivalent to the BTC of a LU, of area S_i, which is entirely covered by a wood of oaks.

In order to build an ecological graph it is necessary to introduce other quantities. The first, hereinafter named as biological power, is defined in the following way [22]

$$\mathcal{M}_i=(1+K_i)\mathcal{B}_i, \tag{4}$$

where the parameters $K_i \in [0,1]$ depend on the physical and morphological properties both of the LUs and of the biotopes. In the mathematical models presented in the next section these parameters depend, for the LUs, on the morphology of their borders, by their permeability, by the index of landscape diversity (Shannon index [23]), whereas, for the biotopes, by the length of the ecotone bands (i.e, the borders between biotopes), by the solar exposition and by the humidity. For their computation the reader is addressed to the papers [16–18,24]. Such physical and morphological properties of the LUs and of the biotopes are responsible of the transmission and diffusion of bio-energy through the territory, so that $\mathcal{M}_i$ may be considered as a quantity that increases the value of $\mathcal{B}_i$ up to its double. By using the definition of $\mathcal{B}_i^{max}$ in (3), the maximum value of $\mathcal{M}_i$ for each LU is defined by

$$\mathcal{M}_i^{max}=2\mathcal{B}_i^{max}=2b_{max}\,S_i \tag{5}$$

Finally, the second quantity necessary for the design of an ecological graph is the biological energy flux between two LUs i and k, defined by the formula [22]

$$\Phi_{ik} = \frac{\mathcal{M}_i + \mathcal{M}_k}{2(P_i + P_k)} \frac{H_{ik}}{L_{ik}} \tag{6}$$

where the constant H_{ik} depends on the permeability of the s various sectors constituting the border between the LUs, L_{ik} being the length of their border and P_i and P_k the corresponding perimeters; in formula

$$H_{ik} = \sum_{r=1}^{s} L_{ik}^r p^r \tag{7}$$

where L_{ik}^r is the portion r of the border with permeability $p^r \in [0, 1]$. Let us note that the determination of the fluxes between LUs allows to compute the connectivity index that, as we shall see, is mostly important in stating the ecological health of the environment thanks to the energy exchanges between the LUs theirselves.

Once, for every LU, the values of $\mathcal{M}_i$ and of the fluxes Φ_{ik} are determined, it is possible to build the ecological graph, indicating with a circle the center of each LU and with an arch the connection between the LUs. Of course the dimension of each circle is proportional to the values of $\mathcal{M}_i$, whereas the width of the arches is again proportional to the values of the fluxes. An example is given by the picture reported in Fig. 1 used for the environmental evaluation of a municipality in Piedmont (see paper [25]). Such a picture gives, for the region under observation, a qualitative image capable to distinguish where bio-energy production is higher and which is the state of connectivity between all the LUs of the system.

As already discussed the GIS, beside all the data necessary to build the ecological graph, is capable to furnish any information useful for the assessment of an environmental system (see at this purpose the article [26]). Between these data, one with a role of relevant importance, beside that of bio-energy $\mathcal{B}_i$, is the ground total area $\mathcal{V}_i$ of each LU which presents a green of high ecological quality, say with a land cover possessing a BTC index greater than 2.4 $Mcal/(m^2 \cdot year)$, i.e. green belonging to ecological classes D and E.

On the other hand, as anticipated in the Introduction, the ecological graph is only a static picture of the state of the system when the territorial data are recovered. Therefore a new assessment procedure involving mathematical models has been developed in order to describe a possible evolution of the system transformations towards new stable equilibria, starting from the initial data given by the GIS. Such a procedure involves the study of the mathematical model as a dynamical system looking for the asymptotic analysis of its equilibrium solutions that can be viewed as possible stable scenarios of the environment.

A first approach to these ideas has been proposed by a first very simple model [15], that despite some simplifications and drawbacks, has given a rather satisfactory evaluation of the ecological state of the municipality of Rivolta d'Adda in the province of Cremona (Lombard country, Italy), showing as well the crucial role played by the connectivity index.

On the other hand the main drawback of this model is due to the mathematical structure of the model itself that does not admit as an equilibrium solution, and therefore as a scenario, a territory completely fragmented so that energy does not flow from one LU to another.

The development and modifications of such a model (see the discussion carried on in the paper [17]) have allowed to build a family of new models, named PANDORA, which in several applications performed in regions of Italy have shown that, unfortunately, several environments present a strong fragmentation.

By concluding the present section, let us comment that the use of such models allows not only significant evaluations of the ecological health of the territories under observation. In fact these models can be used as well in a comparative way examining several different designs of the considered region in order to furnish to the planner which design results to be the best one to preserve the ecological state of the system.

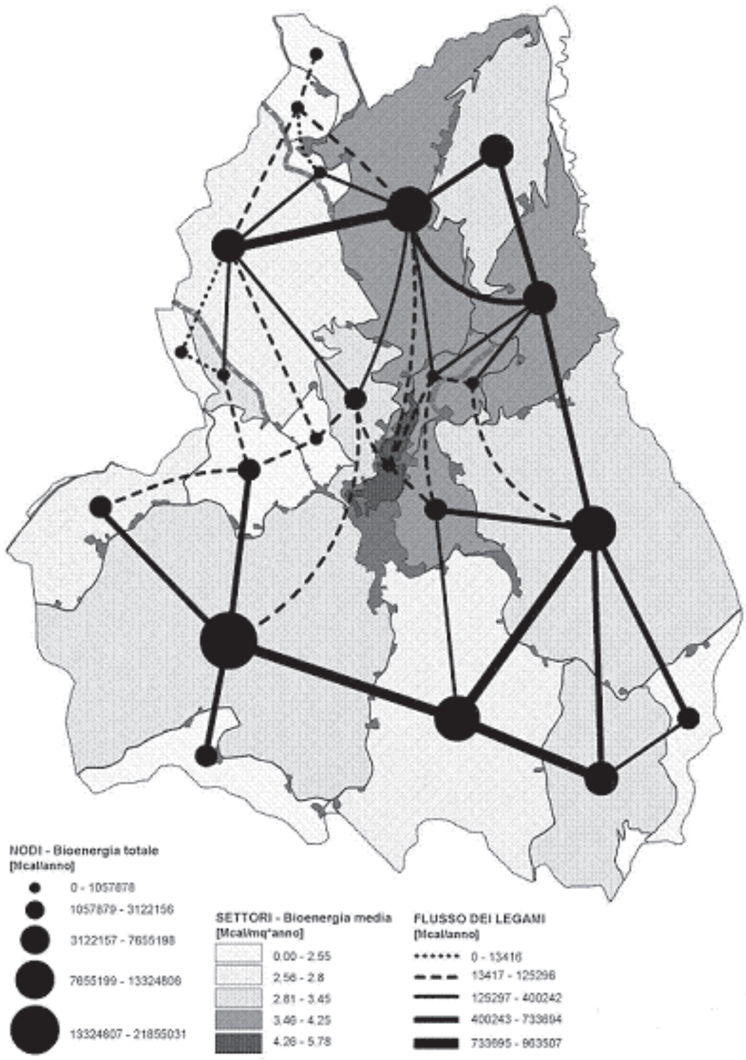

Figure 1. Ecological graph of the municipality of Monforte d'Alba (Piedmont, Italy), [25].

3. The PANDORA models

The models here reviewed have all the mathematical structure of Lotka–Volterra cooperative type systems of differenial equations [27], with terms that contribute to increase the state variables and other which contrast this growth since they take into account the environmental impacts. It is worthwhile important to underline that these models admit always at least one stable equilibrium solution [28]. However for any detail about these models, their derivation and stability analysis the reader is addressed to the papers [16–18,24,29].

Anyway, starting from the ideas and the experience made using the aforementioned work [15], in 2010 a first version of PANDORA models was proposed [16]. This model considers the environmental system distributed in LUs and biotopes. Nevertheless the outputs are defined at the scale of the entire system and not for every LU. The first equation is relative to the normalized variable V defined as the percentage of the surface of the whole system that is covered by vegetation of high ecological quality, i.e. that of classes D and E. In formula

$$V = \frac{1}{S} \sum_{i=1}^{n} \mathcal{V}_i \in [0, 1], \tag{8}$$

where S is the total area of the environmental system.

The second state variable M, again in normalized form, takes into account all the biological powers $\mathcal{M}_i$ of the LUs. Is defined as follows

$$M = \frac{1}{\mathcal{M}^{max}} \sum_{i=1}^{n} \mathcal{M}_i \in [0,1], \qquad \mathcal{M}^{max} = \sum_{i=1}^{n} \mathcal{M}_i^{max} = 2\sum_{i=1}^{n} \mathcal{B}_i^{max} = 2b_{max}S. \tag{9}$$

The model PANDORA 1.0 is then defined by the following two ordinary differential equations

$$\begin{cases} V'(t) = bV(t)[1-V(t)] - hUV(t) \\ M'(t) = cM(t)[1-M(t)] - k[1-V(t)]M(t), \end{cases} \tag{10}$$

where the first right hand side terms are positive and thus determine the growth of the variables V e M, whereas the other terms are negative representing the environmental impact. As already said it is easy to recognize that the form of the equations are those of the Lotka–Volterra cooperative type system [27,28]. The choice of such a structure of the equations and of the gain and loss terms is coherent with what suggested and largely adopted in literature [30,31] to model in a mathematical fashion the phenomenology that we have described. It is worthwhile noting that the mathematical structure of the version 1.0 of PANDORA models is rather simple since the first equation is uncoupled by the other, so that it is possible to find, as we shall see, the general solution and the stability conditions of equilibria in analytical form; on the other hand such a peculiarity is not present in the further versions of PANDORA models.

The index b that appears in the first equation is defined as follows

$$b = \frac{1}{\mathcal{B}^{max}} \sum_{i=1}^{n} \mathcal{B}_i, \qquad \mathcal{B}^{max} = b_{max}\, S, \tag{11}$$

with the $\mathcal{B}_i$ deduced by the ecological graph, and represents the production rate of bio–energy of the whole system, whereas the parameter h is given by the ratio between the sum of all the perimeters of the build-up areas and the total perimeter P of the system; in such a way h measures the dispersion of the build-up areas in the system itself. In a similar way the other parameter k is given by the ratio between the sum of all perimeters of the impermeable barriers present in the whole territory and the perimeter P. Moreover U measures the intensity of build-up areas; thus it is defined by the ratio between the sum of all the surfaces of the build-up areas present in the territory and the total area S of the environmental system. Finally the connectivity index between all the LUs, following [16,25], is given by

$$c = \frac{1}{3(n-2)} \sum_{i=1}^{n} \frac{\Phi_i}{\max_i\{\Phi_i\}}, \qquad \Phi_i = \sum_{k\in I_i} \Phi_{ik}, \tag{12}$$

where I_i is the set of all the LUs that have a border with the i–th LU. Note that all the quantities V, M, b, U admit values in the range $[0,1]$, whereas h a k are positive definite and can assume values greater than one. All the parameters present in the equations and the initial data of V and M are of course obtained by the GIS and by the ecological graph.

As it has been said, from the simple form of the model equations (10) and it is easy to show the existence of four equilibrium solutions (V_e, M_e) that may be interpreted as four possible scenarios for the environmental system under observation. Of course, since these equilibria are reached asymptotically by the general solution of the equations, the trend to such scenarios must be intended as a long time behavior. The equilibrium solutions are the following

$$(V_e = 0, M_e = 0), \qquad (V_e = (b-hU)/b, M_e = 0) \tag{13}$$

$$(V_e = 0, M_e = (bc-hkU)/bc), \quad (V_e = (b-hU)/b, M_e = (bc-hkU)/bc). \tag{14}$$

The first is relative to a system that tends to loose completely the ecological quality because of the complete fragmentation of its territory. The second may be interpretable as an environment that exhibits a general fragmentation ($M_e \to 0$) but with some islands of good quality green ($V_e \neq 0$). Opposite to this one, the third equilibrium could represent a territory where bio-energy is preserved ($M_e \neq 0$) but green of ecological quality is absent. Such a territory can be referred to a typical scenario of a land characterized by agricultural production, where BTC has low values ($V_e \to 0$) but bio-energy is transmitted around the territory because impermeable barriers are not present. Finally, the last equilibrium shows an environment with a good ecological health in terms of a high bio-energy production in presence of a significant quality of green.

It is also simple to show that the asymptotic stability of each equilibrium solution implies the instability or non-existence of the other three. Moreover, in order to control to which scenario the environmental system tends, it is not necessary to determine the general solution of the differential system: it is sufficient to verify the stability conditions of each equilibrium which are given by simple inequalities on the parameters of the model computed directly from the ecological graph. These inequalities are:

for the first scenario $c < k$ and $b < hU$,
for the second $bc < hkU$ and $b > hU$,
for the third $c > k$ and $b < hU$, and, finally,
for the last $bc > hkU$ and $b > hU$.

Even if the model PANDORA 1.0 exhibits a very simple mathematical structure, when applied to real systems, has given satisfactory results for the assessment of their ecological health, at least at the level of the whole environment. Examples can be found in paper [16] for the river basin of Marta and Traponzo in Province of Viterbo (Lazio region), or in the article [25] for the municipality of Monforte d'Alba in Province of Cuneo (Piedmont region).

The subsequent version 2.0 of the model (see [17]) introduces two important novelties. The first consists in considering as state variables of the model $V_i(t) = \mathcal{V}_i(t)/S_i$ and $M_i(t) = \mathcal{M}_i(t)/\mathcal{M}_i^{max}$, $i = 1, \dots, n$ defined for each LU of the system, so that now the variables of the model are $2n$. The second novelty is the definition of the parameters b_i e c_i as functions of the state variables. Therefore such dependence implies that all the equations are then completely coupled and that the parameters themselves become time–dependent. Thus the model is defined as follows

$$(15) \qquad \begin{cases} V_i'(t) = b_i(t)V_i(t)[1 - V_i(t)] - h_i U_i V_i(t) \\ M_i'(t) = c_i(t)M_i(t)[1 - M_i(t)] - k_i[1 - V_i(t)]M_i(t), \end{cases}$$

where the coefficients h_i, U_i and k_i are defined in the same way of those of version 1.0 but now for each LU, thus referring to the surfaces S_i and perimeters P_i; on the other hand the time–dependent parameters are given by

$$(16) \qquad b_i(t) = \frac{\mathcal{M}_i(t)}{\mathcal{M}_i^{max}}, \qquad c_i(t) = \sum_{k \in I_i} \frac{\mathcal{M}_i(t) + \mathcal{M}_k(t)}{\mathcal{M}_i^{max} + \mathcal{M}_k^{max}} H_{ik}.$$

Obviously all the initial data and the constant parameters are computed by the GIS and the ecological graph. Again all the quantities appearing in the model are defined in $[0, 1]$ so that they can be expressed in percentage, except h_i and k_i that may assume values greater than one.

This version of the model has been used in paper [17] for the assessment of the ecological impact of the highway connecting in region Lazio the speedways passing through Orte and Civitavecchia. Moreover the model has been also proposed in the European project RURBANCE for the evaluation of the district of Ciriè in the Turin Province (see [32]).

Moreover an accurate stability analysis of the model, with an application in an actual environmental system, has been proposed in paper [24], testing the influence of connectivity on the generation of bifurcations.

Finally, a further version of the model (PANDORA 3.0) has been developed in paper [18] where the state variables are defined at the level of biotopes. Therefore these are only the values of BTC for each biotope included in the system, since there has no sense to define the quantities M and V at this level.

Consequently the variables are $B_{ji}(t) = b_{ji}(t)/b_{max}$, $j = 1, \dots, m_i$, $i = 1, \dots, n$ and the equations are given by

$$B'_{ji}(t) = c_i(t)B_{ji}(t)[1 - B_{ji}(t)] - h_i U_i B_{ji}(t), \tag{17}$$

where the parameters $c_i(t)$ and the constants h_i e U_i are defined as in the version of PANDORA 2.0.

Of course this model presents a huge number of equations, since the number of biotopes in an environmental system can be of several thousands. Therefore in the paper quoted above an approximated method of solution has been proposed in order to overcome the direct integration of the system. This model has been used in paper [33] for the determination of the parameters of the so-called Ecosystem Services [34,35]; starting from the definition of the ecological graph, a special software has been produced which solves the model equations (17) and compute directly the above parameters. The software is employed through an application performed in the metropolitan area of the city of Bari (see [36]).

4. The Network Landscape Model

The research activity in this field of mathematical models capable to evaluate the ecological state of an environment has been addressed to the proposition of new more sophisticated models. In this sense a possible perspective is that of building differential systems which present a structure where the connectivity between LUs is analogous to that of neural networks models.

A first example can be found in a recent paper [19] where a new model named Network Landscape Model (NLM) was presented. This model includes in their equations connectivity terms similar to those of electrical coupling in neural networks. The state variables for each LU, analogously to those of PANDORA models, are $V_i(t)$ and $B_i(t) = \mathcal{B}_i(t)/\mathcal{B}_i^{max}$, and the equations are given by

$$\begin{cases} V_i'(t) = d_i V_i(t)[1 - V_i(t)]B_i(t) - h_i U_i V_i(t) \\ B_i'(t) = a_i B_i(t)[1 - B_i(t)] - k_i[1 - V_i(t)]B_i(t) + \sum_{k \in I_i} c_{ik}[B_k(t) - B_i(t)], \end{cases} \tag{18}$$

where the new coefficients a_i and d_i depend on the solar exposition and the relative humidity of the biotopes of each LU, whereas the last term of the second equation is that relevant to the connectivity between the LUs themselves, being $c_{ik} = H_{ik}/L_{ik}$.

The qualitative analysis of such a model has presented some promising results on the ground of equilibria variety and for the presence of bifurcations depending on connectivity index. In paper [19] some simulations have been also performed in the territorial area mentioned for the RURBANCE project. We resume here the main results concerning equilibria and stability conditions, addressing the reader to the paper for computational details.

The equilibrium solutions of the model for each i–LU, $i = 1, \dots, n$, are

$$E_0 = (0, 0), \quad E_1 = (0, 1 - \alpha_i), \quad E_2^{\pm} = \left(1 - \frac{1 \mp \sqrt{1 - 4\alpha_i r_i}}{2\alpha_i}, \frac{1 \pm \sqrt{1 - 4\alpha_i r_i}}{2}\right), \tag{19}$$

where $\alpha_i = k_i/a_i, r_i = h_i U_i/d_i$.

According to the discussion made about the meaning of the equilibria, it is straightforward recognizing that E_0 corresponds to a scenario where the system tends to loose its ecological quality and presents a strong landscape fragmentation. The second equilibrium E_1 represents a scenario with no high quality vegetation, but with some production of bio-energy, thus typical of territories where agricultural production is predominant. Finally, the coexistence equilibria $E_2^{\pm}$ show a good level of bio-energy production in presence of a certain amount of high ecological quality green areas.

Of course since the state variables (V_i, B_i) must belong to the set $Q = [0, 1] \times [0, 1]$, there are some conditions of existence for the above equilibria; they are

$$E_1 \in Q \ \text{ iff } \ \alpha_i < 1; \tag{20}$$

(21) $$E_2^+ \in Q \;\text{ iff }\; r_i \leq \begin{cases} 1-\alpha_i & \text{when } \alpha_i < 1/2 \\ 1/(4\alpha_i) & \text{when } \alpha_i \geq 1/2 \end{cases};$$

(22) $$E_2^- \in Q \;\text{ iff }\; \begin{cases} \alpha_i \geq 1/2 \\ 1-\alpha_i \leq r_l \leq 1/(4\alpha_i) \end{cases}.$$

Concerning stability, rather standard calculations show the following situation:
(i) E_0 is asymptotically stable if and only if $\alpha_i > 1$;
(ii) E_1 is asymptotically stable if and only if $r_i > 1-\alpha_i$;
(iii) E_2^+ is always asymptotically stable (when it is admissible);
(iv) E_2^-, when it is admissible, is generally asymptotically unstable (since it is a saddle).

Concerning point (iv), where the equilibrium is mathematically a saddle, let us comment that E_2^- represents a scenario of moderate ecological quality. Therefore, except for some very special conditions where the scenario becomes stable, generally the system, when passes through this equilibrium, has then a transition towards the scenario E_2^+ of high ecological state.

5. An application of the neural nets model

This section is devoted to the illustration of the functioning of the network model in the landscape of Champagne-Ardenne in France.

5.1. *Case study: the Champagne-Ardenne (France)*

The Champagne-Ardenne is located in the North-eastern part of France, touching the administrative boundaries of Belgium and Luxembourg. It extends for more than 26,742 km^2 and a population density of 49.86 inhabitants/km^2 (INSEE, 2016). Reims is the most populated pole and it is far from the French capital about 140 km. This case study has been chosen for its landscape and ecological characteristics. From a landscape point of view, this case study is characterized by the presence of the UNESCO World Heritage List (WHL) *Coteaux, Maisons et Caves de Champagne*, thanks to the presence of vineyards and rural characteristics related to the sparkling wine-making process that express the Outstanding Universal Value (OUV) [37]. From an ecological point of view, several protected areas are present in this wine region. To employ the mathematical model, the Champagne-Ardenne has been been intended as an environmental system constituted by 15 Landscape Units (LUs) that correspond to the administrative boundaries of the *arrondissements*, according to the more recent French administrative reform (2016). The choice of adaptation of the Landscape Units to this type of administrative boundaries is due to the fact that this territory consists of 4 *departments*, 14 *arrondissements*, 146 *cantons* and 1947 *communes*. The *arrondissement* scale has been retained the most suitable to investigate the ecological connectivity and predict future scenarios. The Landscape Units are reported in Table 1.

The main geo-databases have been considered to obtain the value of the ecological parameters at the state of the art to be included in the network model. The COPERNICUS project provides the more recent Corine Land Cover map (CLC) which is fundamental to calculate most of the model parameters. The CLC vector map can provide the type of the LU biotopes and calculate the surface. The visualization and extraction of the meaningful data is performed through Geographic Information Systems (GIS). This application has used the open software QGIS and integrated by OpenStreetMap plugin (OSM) that revealed helpful to measure those parameters closely related to the ecological connectivity. Table 2 illustrates the ecological parameters obtained through GIS elaborations and the formulas provided in section 2.

Table 1. Main information of the case study (Elab. from [20])

LUs	Description	Area (km^2)	Population density (ab/km^2)	Population (2016)
LU1	Charleville-Mézières	1.834	86.14	158.005
LU2	Rethel	1.202	31.09	37.384
LU3	Sedan	792	73.41	58.136
LU4	Vouziers	1.414	15.44	21.846
LU5	Bar-sur-Aube	1.198	24.02	28.759
LU6	Nogent-sur-Seine	1.284	42.11	54.067
LU7	Troyes	3.545	63.77	226.084
LU8	Chaumont	2.494	25.72	64.148
LU9	Langres	2.175	20.20	43.943
LU10	Saint-Dizier	1.588	44.09	69.993
LU11	Châlons-en-Champagne	2.803	39.21	109.916
LU12	Épernay	2.340	51.40	120.269
LU13	Reims	1.529	192.72	294.674
LU14	Vitry-le-François	1.024	30.30	46.024
	Total	26.742	49.86	1.333.248

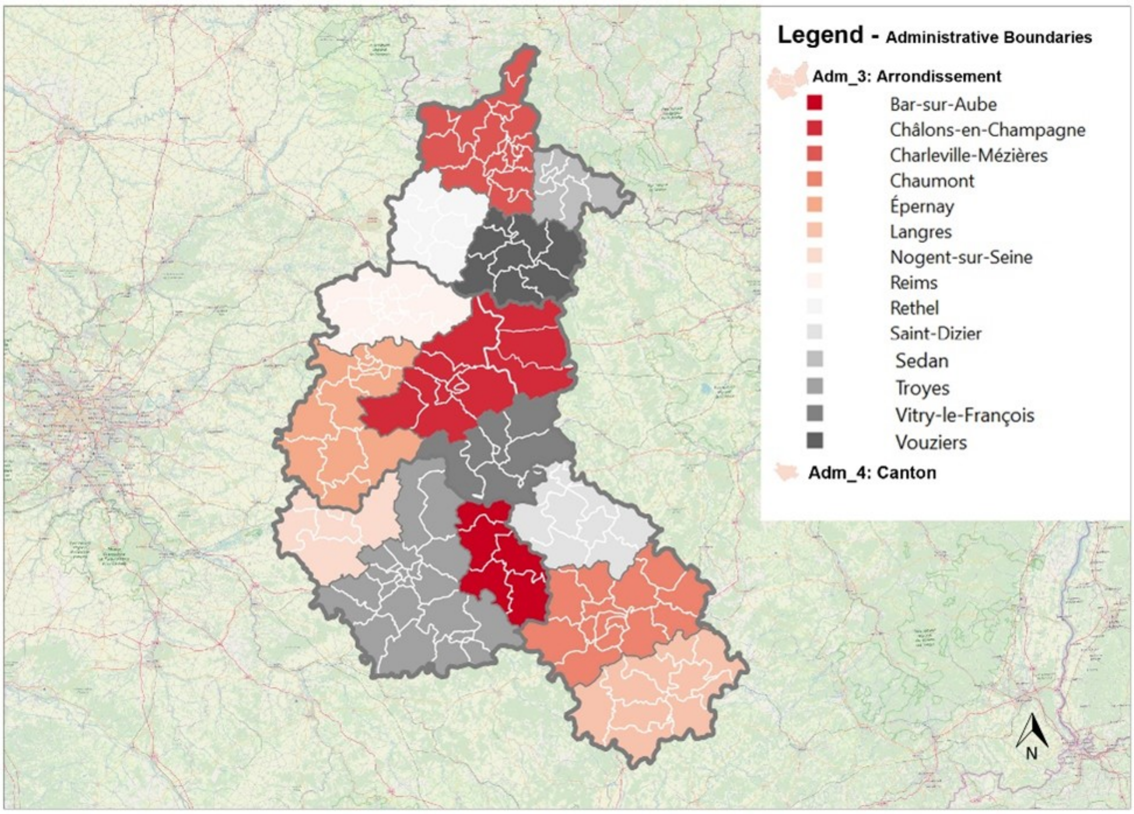

Figure 2. Territorial localization of the case study [20].

5.2. *The Network Ecological Model: a new extension*

An extension of the network mathematical model described in Section 4 is here presented. The main motivation related to the extension of the model is related to the introduction of an element of novelty to assess complex phenomena like resilience, so that it closely refers to the parameters meaning [20]. The extended version has been renamed Network Ecological Model (NEM) and Single Ecological Model (SEM) (see Eq.s 23 and 24). The NEM model considers the ecological connectivity of the i-th LU, whereas the

Table 2. Set of ecological parameters of the Champagne-Ardenne [20]

LUs	S_i	P_i	State variables		Ecological parameters						
			v_i	b_i	k_i^{ev}	k_i^{hu}	k_i^{ec}	k_i	l_i	h_i	U_i
LU1	1834	258	0,466	0,527	0,222	0,206	0,968	0,465	0,055	3,112	0,286
LU2	1202	180	0,068	0,188	0,200	0,062	0,946	0,403	0,031	1,828	0,157
LU3	792	159	0,339	0,420	0,186	0,187	0,957	0,444	0,051	1,858	0,262
LU4	1414	200	0,207	0,309	0,194	0,124	0,962	0,427	0,017	0,998	0,088
LU5	1198	200	0,328	0,406	0,199	0,113	0,955	0,423	0,032	1,634	0,169
LU6	1284	233	0,127	0,224	0,272	0,046	0,922	0,414	0,043	1,998	0,221
LU7	3545	426	0,269	0,353	0,231	0,098	0,964	0,431	0,045	2,923	0,231
LU8	2494	318	0,436	0,498	0,184	0,152	0,969	0,435	0,024	1,556	0,122
LU9	2175	282	0,342	0,430	0,192	0,175	0,975	0,447	0,025	1,896	0,128
LU10	1588	234	0,420	0,480	0,196	0,148	0,966	0,437	0,037	1,892	0,186
LU11	2803	452	0,144	0,216	0,261	0,081	0,921	0,421	0,035	1,546	0,181
LU12	2340	353	0,225	0,413	0,222	0,526	0,950	0,566	0,048	2,555	0,238
LU13	1529	232	0,189	0,284	0,231	0,055	0,951	0,412	0,074	3,042	0,383
LU14	1519	245	0,203	0,291	0,228	0,104	0,948	0,427	0,033	1,744	0,169

SEM model allows to exclude the ecological connectivity, putting c_{ki}=0, thus investigating the behaviors of the individual LUs without it:

(23)
$$\begin{cases} b_i'(t) = k_i^{ev} b_i(t)[1-b_i(t)] - l_i[1-v_i(t)]b_i(t) + \sum_{k\in I_i} c_{ki}[b_k(t)-b_i(t)] \\ v_i'(t) = k_i v_i(t)[1-v_i(t)]b_i(t) - h_i U_i v_i(t), \qquad i=1,...,n \end{cases}$$

(24)
$$\begin{cases} b_i'(t) = k_i^{ev} b_i(t)[1-b_i(t)] - l_i[1-v_i(t)]b_i(t) \\ v_i'(t) = k_i v_i(t)[1-v_i(t)]b_i(t) - h_i U_i v_i(t), \qquad i=1,\dots,n \end{cases}$$

where:
b_i is the normalized biological energy provided by each LU;
v_i represents the green areas of high ecological quality of LU_i;
b_i' and v_i' are the time derivative of the model state variables;
k_i^{ev} is the Shannon Evenness index that measures the ecological variety of the biotopes belonging to LU_i;
l_i is the incidence of impermeable barriers with respect to the LU_i area;
k_i considers the Evenness index, the relative humidity of biotopes, and the length of ecotones bands of each LU;
h_i is the incidence of the built-up perimeter with respect to the LU_i perimeter;
U_i is the incidence of the built-up areas with respect to the LU_i area;
c_{ki} is the ecological connectivity of the LU_i with its neighboring k-th LUs.

It should be noticed that in this extension the ecological parameter a_i "Solar exposure of the biotopes" is replaced by k_i^{ev} "Shannon Evenness index", so that the formula that calculates the parameter d_i replaces a_i with k_i^{ev}. For more information about the Shannon Evennes index applied to landscape ecological studies please see [25,26].

Three possible ecological scenarios are considered in the performed stability analysis. In detail:

- E_0 labels a poor ecological equilibrium that may cause over time the LU_i fragmentation. The solution is stable when E_0=(0,0);

- E_1 is a medium ecological equilibrium characterized by a dominance of agriculture areas and therefore low bioenergy production. This equilibrium is stable when E_1=(b_1, 0)=(1 - α, 0);

- $E_*^{\pm}$ refers to a good ecological equilibrium characterized by the presence of green areas of high ecological quality and therefore exchange of bioenergy. The equilibrium is reached when E_*^{+} is stable, while E_*^- is a saddle

5.3. *Results*

The NEM and SEM models have been solved through the Mathematica software by Wolfram that provided as model outputs evolution scenarios for the 14 LUs. The scenarios prediction is here commented by classifying the time of observation with the purpose of aiding Decision Makers in the understanding of the final model outputs and support the orientation of the time horizon of specific policies and actions (see Table 3) [20]. Some meaningful examples of time diagrams and phase diagrams are reported below.

Table 3. Scenarios classification according to the observation time [20]

Observation time	Scenario horizon
40	Short term
60	Medium term
80	Medium-long term
150	Long term

Before focusing on the comment of the individual results, all the LUs of the Champagne-Ardenne represent a territory devoted to agriculture activity, thus generally demonstrating a positive trend evolution [20]. Figure 3 illustrates some meaningful examples of time diagrams: LU1 (Charleville-Mézièreres) evolves and reaches the equilibrium in the short term, since the presence of a positive ecological connectivity with its neighbors. LU4 (Vouziers) records at the state of the art a high ecological value and reaches the equilibrium in the medium term, even if it negatively acts on the ecological connectivity with its neighbors. LU6 (Nogent-sur-Seine) is a typical LU with agriculture vocation, similarly to LU1 and reaches the equilibrium in the short term. LU11 (Châlons-en-Champagne) is considered the most critical among the 14 LUs. This is due to the fact that LU11 is very extended in terms of surface and and it has not a meaningful role in the ecological connectivity. Figure 4 reports the phase diagrams of the same examined LUs. In detail, LU1 comes to E^+ as the final equilibrium state and confirms the agriculture vocation. LU4 reaches as final equilibrium E^+, thus moving far from the equilibrium E_1 typical of agriculture systems. LU6 records a similar evolution to LU1, in fact it comes to E_1 as final equilibrium state. LU11 flux lines tends to reach the equilibrium E_1, even if comes to the equilibrium E^+ thanks to presence of minor green areas.
Considering the NEM model simulations of the same LUs, it is possible to state that their evolution trend is not influenced by the ecological connectivity. Table 4 describes the results obtained for the 14 LUs by considering the NEM simulations and the connectivity effect.

6. Perspectives and conclusions

The paper has illustrated the role of mathematical modeling in supporting the assessment and monitoring of environmental systems in the field of environmental protection, planning and management. The reviewed models have demonstrated their ability of representation and assessment of the dynamical behavior and ecological stability of the examined environmental systems. The application of the network model and its extension to the Champagne-Ardenne landscape produced surely an advance in the field that deserves to be deeply explored. In particular, the proposed mathematical models may carry effective results especially if related to Ecosystem Services, [38], land take and soil sealing, resilience and sustainability, which are ever more weaved in the planning and assessment of plans, programmes and projects. In particular, the mathematical model allowed on the one hand to know the wine region under investigation by considering specific ecological parameters at the state of the art, thanks also to the support of GIS methods, and on the other hand to predict the possible ecological scenarios by fixing them at a definite

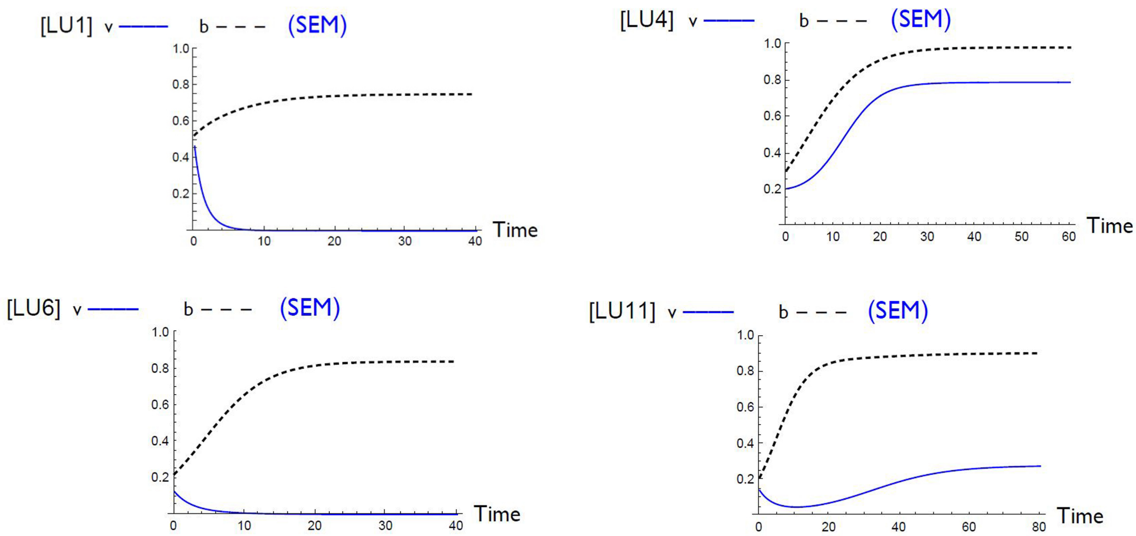

Figure 3. Time diagrams of the LUs 1, 4, 6 and 11 [20].

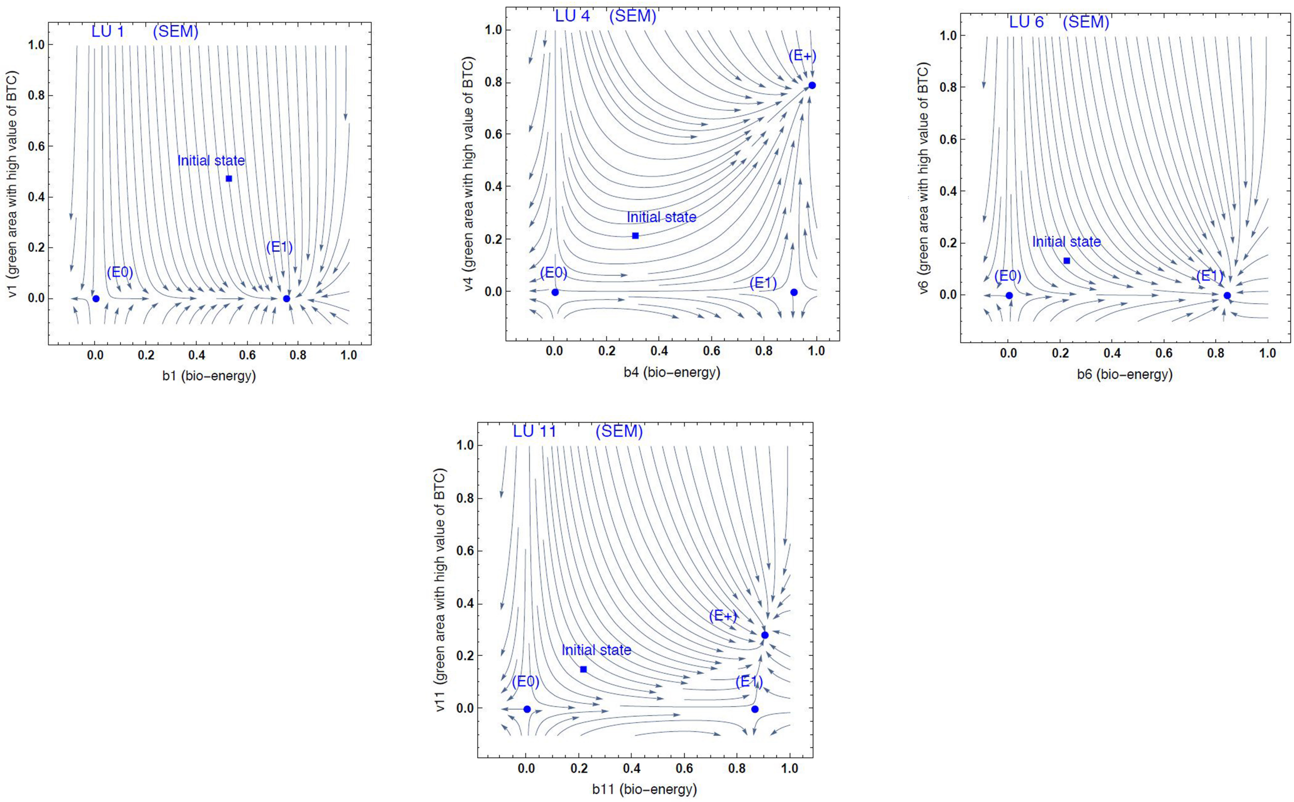

Figure 4. Phase diagrams of the LUs 1, 4, 6 and 11 [20].

arbitrary time. In fact, the possibility gained by the model to compare the state of the art with potential ecological evolutions may support the Decision Makers to identify the most valuable and the most critical areas in ecological terms and to prioritize specific interventions thus arresting the fragmentation and the consequent loss of Ecosystem Services. This mathematical model may provide a comprehensive overview about the ecological health of the system under investigation to planners and Decision Makers, thus aiding the design of policies and actions, as well as looking for possible implications on social,

Table 4. Results of the Network Ecological Model (Elab. from [20]).

LUs	NEM	Connectivity effect
LU1	E_1	Good
LU2	E_1	Poor
LU3	E_1	Good
LU4	E^+	Poor
LU5	E_1	Poor
LU6	E_1	Medium
LU7	E_1	Good
LU8	E_1, E^+	Poor
LU9	E_1, E^+	Poor
LU10	E_1, E^+	Medium
LU11	E_1	Poor
LU12	E_1	Good
LU13	E_1	Good
LU14	E_1	Poor

environmental and economic spheres [39]. The application presented in this paper has raised a number of future perspectives. The first future perspective aims to replicate the network mathematical model in similar environmental systems, characterized by rural characteristics. The intent will be the exploration of the sensitivity of the model in delivering the final outputs. In this sense, it will be interesting to focus on environmental systems with similar characteristics but located in different countries.
The second future perspective will consider the refinement of data collection, in particular referred to historical series. The ecological parameters will be calculated by considering a range of historical series in order to launch the mathematical model in the past. It should be noticed that these family of mathematical models have an arbitrary scale, so that it will be interesting to translate this time variable in a real time scale [40].
A third future perspective will the integration of the economic monetary valuation within Lotka-Volterra cooperative system, such as the Hedonic Price methods [41] and the potential integration of model outputs with other dynamical models, as such as the Agent-Based Models (ABM) and System Dynamics Models (SDM) [42,43]. Indeed, the matching of parameters representing the ecological connectivity [44], the human behavior [45] and resilience policy performances [46] could aid Decision Makers to better orient transformation scenarios.

In conclusion, it is worth mentioning a further perspective for the development of new models. This resides in the connections shown in the works [33,36] for the determination of the Ecosystem Services. These services, such as, for example, water supply, air purification, soil formation, pollination and other natural regulatory mechanisms, can be quantified by appropriate time-dependent indicators, which in turn depend on physical quantities of an environmental system [38]. It is therefore certainly of interest to develop new mathematical models of evolution for these physical quantities, in order to check the possible services offered by the territory under observation, as already shown in the aforementioned works.

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