

ENHANCING TURBULENCE MODELING WITH DATA-DRIVEN APPROACHES: A FOCUS ON THE FIELD INVERSION AND MACHINE LEARNING PARADIGM

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Abstract. The work is focused on the Field Inversion and Machine Learning (FIML) paradigm and on its potential for improving Reynolds-Averaged Navier-Stokes (RANS) models. The study explores the generalization properties of FIML by evaluating its effectiveness when multiple experimental results are available for a single test case and investigating how different objective functions can affect the correction field generated by the inverse problem. The study suggests using a weighted interpolation of different correction fields and optimizing both the source terms of the Spalart-Allmaras equation and the Reynolds stress tensor to improve generalization.

1 INTRODUCTION

Recently, there has been a growing trend in turbulence modeling that employs data-driven approaches, aiming to improve the accuracy of current models using machine learning techniques. Several reviews have been conducted [1] [2] [3], detailing the current approaches used in the CFD community and how these new algorithms can minimize costs and increase accuracy. The use of data-driven techniques in turbulence modeling has the potential to enhance the prediction of turbulent flows and to augment RANS models using additional data from experiments or simulations. Various machine learning techniques, such as Artificial Neural Networks (ANNs), Support Vector Machines (SVMs), Gaussian Process Regression (GPR), and Genetic Expression Programming (GEP), have been utilized in different applications to improve turbulence models. According to [4], there are three approaches to use data-driven algorithms. In the first approach, these algorithms are

used to estimate parameters in the turbulence model, which are typically fine-tuned based on experimental results. Noteworthy works in this category include those that operate on the Reynolds stress tensor and the Boussinesq hypothesis by integrating the work of Pope [5]. To provide additional details, this approach consists on the incorporation of Galilean invariance into the neural network and allows for greater flexibility and adaptability in handling fluid flow problems where the simple linear relation between the Reynolds stress tensor and the mean rate-of-strain fails [6] [7] [8] [9]. The second approach involves replacing the closure model with classical fully connected neural networks (FCNN) [10] [11] or neural networks with gated recurrent units (GRU) [12] to overcome long-term stability issues. Finally, the third approach consists of replacing the fully PDEs model with the use of autoencoders [13] or the Physical Informed Neural Networks (PINNs) method [14] [15] [16].

One Data-Driven approach that showed promising results in improving RANS models is the Field Inversion and Machine Learning (FIML) paradigm [17, 18, 19]. FIML is an optimization procedure that solves an inverse problem using measurements to obtain an accurate representation of the underlying physical field. By solving this inverse problem for different test cases, a data set for machine learning algorithms can be built to find a general expression that correlates the correction field with fluid-dynamics variables. In this way FIML can improve the prediction of turbulent flows by incorporating additional information from experiments or simulations.

Various studies have demonstrated the effectiveness of using FIML to enhance the accuracy of turbulence models. For instance, Fidkowski [20] utilized FIML in a gradient-based shape optimization by incorporating the corrected RANS model instead of more complex turbulent models. Yang and Xiao [21] improved the accuracy of the four-equation $k - \omega - \gamma - Ar$ transition model using FIML, while Ferrero et al. [22, 23] utilized it for data-augmented RANS modeling in turbomachinery flows. Finally, Brenner et al. [24] employed FIML in incompressible turbulent flow, where the correction field was used to directly correct the turbulent eddy viscosity instead of the source terms of the model equation.

To summarize, the FIML technique has great potential for improving the accuracy of turbulence models and enhancing fluid dynamics simulations in various scenarios. This study aims to further explore this technique by analyzing its generalization properties. The study evaluates the effectiveness of FIML when multiple experimental results are available for a single test case and investigates how different objective functions can affect the correction field generated by the inverse problem. In this work, the use of a weighted interpolation of different correction fields (obtained by different objective functions) is investigated in order to generalize the correction field. Finally, another approach is investigated by optimizing both the source terms of the Spalart-Allmaras equation and the closure of the Reynolds stress tensor, including nonlinear contributions, as demonstrated in a previous study [25] for the $k - \epsilon$ model.

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