Stabilization of the Integral Equation for Modelling Metasurfaces via Impedance Boundary Conditions

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Stabilization of the Integral Equation for modelling Metasurfaces via Impedance Boundary Conditions

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***Abstract*—Impedance Boundary Condition (IBC) is a widely used approximation in the analysis of metasurfaces, and it greatly simplifies the design process. However, for some ranges of impedance values of practical interest in metasurface applications, the Integral Equation formulation has shown instabilities. This contribution proposes a way to improve that shortcoming; the method is based on the property of the involved operators and the nature of the IBC approximation.**

***Index Terms*—Metasurfaces, Integral Equations, Impedance Boundary Conditions**

I. Introduction

Surface Impedance Boundary conditions (IBC) are a widespread tool for the design of metasurfaces of various types. The associate analysis problem can be cast in terms of Integral Equations (IE) and, for thin structures, this leads to the Electric Field Integral Equation (EFIE) which is extended to incorporate the IBC term (EFIE-IBC).

However, the IBC addition to an IE yields an ill-conditioned matrix for certain ranges of impedance values [2]. In [1] is proposed a stabilized CFIE-IBC formulation without drawback: unfortunately, this is only applied on closed structures, leaving the possible ill-conditioned EFIE-IBC formulation for thin flat structures. A solution presented in [2] considers both sides of the IBC surface instead of the exterior side only when the model allows it. This paper will investigate a solution to the one-side EFIE-IBC instabilities, restrained to open planar surfaces.

II. Background and Notation

The IBC (1) relates the electric to the magnetic fields,

$$\hat{n} \times \mathbf{E} = \hat{n} \times \left(\underline{\underline{\mathcal{Z}_s}} (\hat{n} \times \mathbf{H}) \right) \tag{1}$$

$$\mathbf{J} = \hat{n} \times \mathbf{H} \qquad \qquad \mathbf{M} = -\hat{n} \times \mathbf{E} \tag{2}$$

In this work, we will concentrate on isotropic surfaces, and constant impedance: this allows us to reduce the complexity while retaining the problem and allowing us to test solutions to it. The restriction to isotropic IBC allows rewriting the tensorial surface impedance $\underline{\underline{Z_s}}$ as the dyadic identity scaled by the IBC value $Z_s \underline{\underline{\mathcal{I}}}$.

The surface current is discretized as a linear combination of Rao-Wilton-Glisson (RWG) functions [3] denoted $\mathbf{\Lambda}_n$, according to the Method of Moments with classical Galerkin test method, to give a numerical solution of the EFIE-IBC. For planar structures, the resulting matrix problem is written as a summation of the discretized EFIE and the Gram matrix $\mathbf{G}^{\mathbf{\Lambda}}$ relative to RWG functions.

$$\left[\mathbf{L} - Z_s \mathbf{G}^{\mathbf{\Lambda}} \right] \cdot \mathbf{J}^{\mathbf{\Lambda}} = V^E \tag{3}$$

where, V^E denotes the right-hand vector of the linear system.

The presence of the IBC term may significantly deteriorate the conditioning of the EFIE-IBC system [2] for reactive IBC of the ”wrong” sign; this is exemplified in Fig. 1.

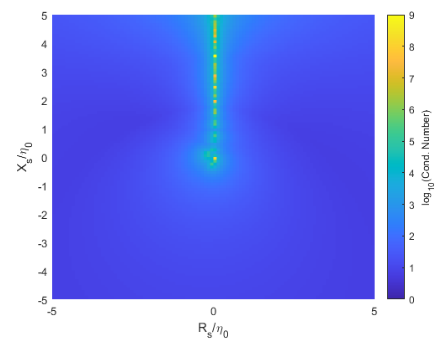

Fig. 1: Condition number of the EFIE-IBC according to $Z_s = R_s - iX_s$ for a rectangular patch $0.5\lambda_0 \times 0.45\lambda_0$ with $N = 153$ RWG functions

Unfortunately, this prevents the use of the EFIE-IBC in many practical metasurface problems; partial solutions have been addressed in [2] by shifting from one-sided (”opaque”) IBC to two-sided (”transparent”) IBC. That enlarges the range of applicability of the EFIE-IBC but does not solve the problem for all cases. The issue was also addressed in [1], but the solution is not directly applicable to thin structures, as the metasurfaces are.

In this communication, we propose an approach that significantly enlarges the applicability range of the integral equation approach to metasurfaces.

III. A filtered EFIE-IBC formulation

The approach presented here is based on the analysis of the problem that arises by adding the IBC identity to the EFIE operator. The issue was addressed first in [2] in terms of matrix properties, which is briefly summarized here. The identity term adds to the diagonal (and whereabouts), being weighted by the impedance value, and may add or subtract to the EFIE diagonal, which is dominantly imaginary; the presence of the IBC term clearly can result in rendering the diagonal entries smaller than the off-diagonal terms, with ensuing effect on the matrix conditioning. This easily leads to totally unstable solutions.

We address here the issue of avoiding this ill-conditioning when the sign of the impedance (reactance) cannot be modified. We recall that an integral equation with a non-singular (compact) operator (like in inverse problems) is an ill-posed problem. The "prototype" of a well-conditioned problem is a second-order equation, like the Magnetic Field Integral Equation (MFIE), in which there is an identity plus a non-singular operator (i.e. a compact one). If the EFIE is not, it essentially behaves similarly, because the EFIE operator is singular, which guarantees the well-posedness of the first-kind EFIE integral equation. Hence, the addition of an identity term clearly interferes with the posedness of the problem.

We also recall that the IBC arises as the homogenization approximation of the real structure composed of unit cells; in many cases, the value of the approximating impedance depends on the incidence angle, which would not lead to the IBC, which instead does not retain this spatial dispersion: the plane-wave response of an identity operator is clearly a constant for all wave-vectors.

Thus, we identify the problem of the IBC approximation as the total spectral flatness of the associated term. We, therefore, propose to limit the spectral range in which we consider the EFIE-IBC, where "spectral" here means plane-wave spectrum (2D Fourier transform). Hence, we aim at restricting the EFIE-IBC to a pre-defined region of the wavenumber domain.

In this study, this is addressed in the simplest way, by using basis functions with predefined spectral occupation; for this reason, we study a rectangular domain and employ suitable waveguide eigenfunctions. In order to maintain the solution scalable to non-rectangular domains, we express the rectangular modes in terms of RWG basis functions; this allows us to interpret the use of the former as a matrix basis change. More importantly, the same operations can be performed via FFT, thus compatible with fast methods; this implementation is beyond the scope of the present work.

We use standard waveguide vector modes (which guarantee div-conforming properties needed to discretize the EFIE).

We then have two representation of the current,

$$\mathbf{J}^{\mathbf{\Lambda}} = \sum_{n=1}^{N} \alpha_n \mathbf{\Lambda}_n \qquad \mathbf{J}^{\mathbf{\Phi}} = \sum_{k=1}^{K} \gamma_k \mathbf{\Phi}_k \tag{4}$$

We represent each $\mathbf{\Phi}_k$ in terms of RWG functions (5),

$$\mathbf{\Phi}_k(\mathbf{r}) = \sum_{n=1}^{N} \psi_{kn} \mathbf{\Lambda}_n(\mathbf{r}), \tag{5}$$

leading to the eigenfunction filtering EFIE-IBC matrix problem of (3),

$$\psi^T \cdot \left[\mathbf{L} - Z_s \mathbf{G}^{\mathbf{\Lambda}}\right] \cdot \psi \cdot \mathbf{J}^{\mathbf{\Lambda}} = \psi^T \cdot V^E \tag{6}$$

This spectral basis spans a subspace with prescribed wavenumber range; by choosing the number of modes one determines the spectral range of the solution, thus limiting the "infinite band" of the identity introduced by the IBC approximation

IV. Numerical Results

An XY rectangular patch of length $L = 0.5\lambda$ and width $W = 0.45\lambda$ at frequency $f = 2.4$ GHz and meshed with a $\lambda/10$ edge length provides an example to observe the eigenfunction filtering interest. An incident plane wave $\mathbf{E}^{inc}$, coming from the top, illuminates the surface. Waveguide eigenfunctions [4] are kept in order to represent spatial variations up to $(k^x_{max}, k^y_{max}) = (3k_0, 3k_0)$.

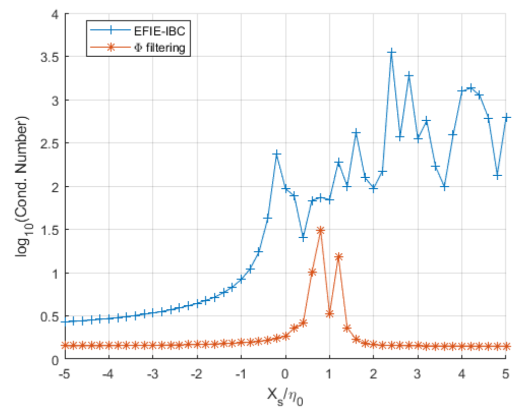

Fig. 2: Impact of the eigenfunction filtering on EFIE-IBC's condition number according to reactive isotropic $Z_s = -iX_s$ values, $N = 153$, $K = 9$

Figure 2 shows both matrix problems (3) and (6) conditioning, for reactive impedance surface values Z_s normalized by the intrinsic impedance in free space η_0. The eigenfunction filtering controls the condition number increase that appears for specific inductive values. As the EFIE-IBC cannot be solved for the unstable values, we verified instead that the filtered solution was accurate with respect to the RWG one for the stable impedance values; the error between the filtered solution and the standard one stayed between 10^{-5} and 10^{-4} for capacitive values $X_s = [-5Z_0; -2Z_0]$.

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