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Sustainable and cost-effective optimal design of steel structures by minimizing cutting trim losses

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ABSTRACT

Since the beginning of the structural optimization field, the optimal design was characterized by the least-weight configuration. In this sense, all the researchers agreed on adopting the minimum-weight optimization statement as the most promising approach to achieve an optimized employment of material. However, especially for steel structures, this approach completely fails the primary goal of encouraging standardization of pieces during the production phase. Except for rare cases, increasing diversity among structural elements leads to a dramatic increase in the financial cost as well as the environmental impact of the structure because of the material waste generated during the cutting procedure.

In this paper, a real-coded Genetic Algorithm has been adopted and the well-known one-dimensional Bin Packing Problem has been implemented within the structural optimization process. The Objective Function formulation lies in a marked change of the paradigm in which the target function is represented by the amount of steel required by the factory instead of the structural cost (e.g. weight). The proposed approach is tested on different steel structures moving from 2D truss beams to 3D domes. Addressing the optimal stock of existing elements leads to a significant waste reduction of 40% in almost all the investigated case studies.

1. Introduction

In the last decades, the scientific community has been actively engaged in addressing the imperative of reducing the costs associated with structures through the strategic management of material selection, fabrication methods, and maintenance expenses [1]. Over the past decade, within the context of structural optimization, there has been a significant focus on the minimization of material costs, with the overarching goal of creating slender structures that optimize resource utilization [2]. Conventional practice among researchers and practitioners in this field involves the optimization of structural design costs, while concurrently adhering to safety guidelines stipulated by relevant standard regulations [3]. It is noteworthy that a substantial portion of the expenditures can be attributed to material wastage resulting from the cutting process, particularly in the context of metal structures. To clarify, failing to incorporate a meticulously designed cutting strategy to reduce waste during construction can undermine the efficiency of cost optimization. In the context of the solid waste stream, construction and demolition residues would comprise approximately 23% of the total volume. This translates to an annual production of over 100 million metric tonnes [4]. Similar waste proportions have been reported in various other nations, corroborating the United States' estimates. A notable portion of this waste results from inefficient material utilization, representing an avoidable fraction of waste generation [5]. Enhanced efficiency in material usage would consequently diminish the quantity of surplus materials, reduce unnecessary workmanship, and minimize the associated costs such as waste disposal and transportation fees. Certainly, effective resource utilization serves the greater global interest, extending beyond the immediate concerns of industrialists. Improper disposal of waste generated from stock-cutting operations can potentially contribute to environmental pollution, while unchecked wastefulness poses a significant risk to the exhaustion of our planet's invaluable resources [6]. Cutting losses arise when normal steel lengths are shortened to fit the project's required lengths, frequently becoming

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the predominant source of steel waste. Indeed, a significant amount of the created steel waste according to Adham et al. ([7]) is related to cutting losses, which are mostly caused by:

- dividing an order into separate, smaller orders typically results in more waste due to fewer cutting alternatives;
- using inefficient cutting patterns in the cutting schedule results in the generation of avoidable waste that could be avoided through better stock-cutting planning;
- using the optimum cutting patterns may result in unavoidable waste that is the minimum waste generated if the optimum cutting patterns are used.

With the aim of minimizing waste, the Cutting Stock Problem (CSP) is an essential research topic to focus on to reduce waste in the construction industry.

The innovative aspects introduced in the present study are focused on an optimization procedure aimed at minimizing structural waste, and to reduce the amount of material stock produced by the factory. This approach introduces a paradigm change considering constructability issues as cutting patterns during the production phase, contrasting with the conventional minimum-weight optimization approach. Suitable applications have been developed to highlight the feasibility and benefits of the proposed approach, paving the path to more efficient and sustainable manufacturing practices.

This paper presents the following organization: In section 2, a comprehensive state-of-the-art devoted to the introduction of cutting and packing problems has been provided with a specific focus on the most promising applications in civil engineering. In sections 3 and 4, the mathematical formulation of the CSP and the structural optimization framework have been described, respectively. Finally, sections 5, 6 and 7 have been devoted to the description and discussion of the optimization outputs for each case study. In the last section, the outcomes of the research and its future developments have been summarized.

2. State of the art

2.1. Cutting and packing problems: Definition and evolution

The domain of Cutting and Packing (C&P) problems encompasses a wide array of scenarios involving the arrangement and allocation of both tangible and conceptual entities, i.e. Cutting Problems [8], Knapsack Problems, one and two-dimensional Bin Packing Problem [9,10], [11], Container and Vehicle Loading Problems [12,13], Pallet Loading [14,15], Assembly Line Balancing [16–18], Capital Budgeting [19], etc. within different disciplines [20]. Notably, despite their apparent diversity, all these problems share a common underlying logical framework, as demonstrated in Belov's seminal work [21].

One of the most prominent challenges within this domain pertains to the Bin Packing Problem (BPP), which seeks to ascertain the optimal arrangement for accommodating a maximal quantity of items within a given container (referred to as bins), while accommodating an identical inventory of goods. The BPP can be stated considering *n items*, each with an integer *weight* $w_j (j = 1\ldots, n)$ and an infinite number of identical bins with integer *capacity c*. The objective is to minimize the number of bins required for accommodating all the products, while ensuring that the cumulative weight assigned to each bin remains within the specified limit [22]. The majority of other Combinatorial Optimization Problems (COP) are either variations (e.g., the Pallet Loading Problem) or extensions (e.g., the Cutting Stock Problem) of the Basic Bin Packing Problem.

Specifically, in civil engineering, addressing the issue of minimizing waste during the steel element cutting process often involves solving the Cutting Stock Problem (CSP). In essence, it aims to determine the optimal way to cut required pieces from a stock material with minimal trim loss. From a more technical perspective, CSP can be derived from the BPP definition as follows. There are *m item* kinds, each with an integer *weight* w_j and an integer *demand* $d_j (j = 1, \ldots, m)$, as well as a huge number of identical integer *capacity c* bins. In the CSP literature, the bins are typically referred to as rolls, a word derived from early implementations in the paper industry, and "cutting" is commonly used rather than "packing". In CSP, the focus shifts from "packing" to "cutting," with the objective of producing d_j copies of each item type *j*, using the fewest possible bins, while ensuring that the total weight in each bin does not exceed its capacity [22,23].

Furthermore, the cutting stock problem can be categorized as either one-dimensional (1D) [24] or two-dimensional (2D) [25]. The 1D-CSP involves extracting a specified set of order lengths from stock rods of defined lengths, with the goal of minimizing the number of rods used. The 2D-CSP, on the other hand, aims to select the most valuable group of rectangular objects from a single rectangular plate. In the case of irregular shapes, the problem becomes known as nesting, presenting a more challenging solution [21]. These problems fall within the topic of complex combinatorial optimization, also called NP-HARD problems (i.e. non-deterministic Polynomial-time). This class of problems can be reduced in NP problems resolvable in polynomial-time [26,27].

Various linear programming, heuristic, and metaheuristic approaches have been proposed over the years. The first foray into solving C&P problems dates back to the 1930s with Kantorovich [28]. While Kantorovich's approach was limited to small-scale cases, it laid the groundwork for understanding the problem structure. Numerous heuristic approaches have since emerged, often involving the solution of the Linear Programming (LP) problem as a precursor to deriving an integer solution [29]. The problem is frequently formulated as an Integer Programming (IP) problem [30], with its LP relaxation serving as a foundation for many heuristic algorithms. The relaxation of a (mixed) integer linear program entails removing the integrality constraint from each variable, thereby allowing the IP problem to be solved as an LP problem. This relaxation technique transforms an NP-hard optimization problem (IP) into a related problem that can be solved in polynomial time. However, when dealing with scenarios where the length of an individual item is significantly smaller than the roll length, considering all practical cutting patterns that correspond to columns in the LP formulation becomes impractical. To address this issue, Gilmore and Gomory (1961) [31] introduced a creative method based on resolving the associated knapsack problem. Their column generation approach, inspired by Dantzig and Wolfe (1960) [32], systematically reduces valid patterns and incorporates them into the problem based on their impact on the Objective Function (OF). This approach rendered large-scale cutting stock problems solvable within a reasonable timeframe.

Subsequently, numerous algorithms have been developed to tackle the problem by implementing dynamic use of simply structured cutting patterns aiming to represent complex combinations of cuts [33–36]. While the most accurate approaches are based on integer linear programming, they can be computationally intensive and time-consuming. In recent times, several metaheuristic techniques, such as Genetic Algorithms (GA) [37], Simulated Annealing [38] and Tabu Search [39,40], have been implemented as more efficient alternatives [41,42].

2.2. Cutting and packing problems in civil engineering

In the context of material consumption, energy utilization, greenhouse gas emissions, and waste generation, the construction industry stands as a significant contributor coherently to the new sustainable challenges of this century [43,44]. Two primary solutions exist for this problem: minimizing waste during fabrication or reusing materials from other structures [45,46].

While the former approach has not received great attention among experts in the civil engineering field, it serves as a central focus of this research. The latter option has been extensively explored by Brütting et al. [47], who advocate for a circular economy model to reduce costs and environmental impact in construction. In a circular economy, goods

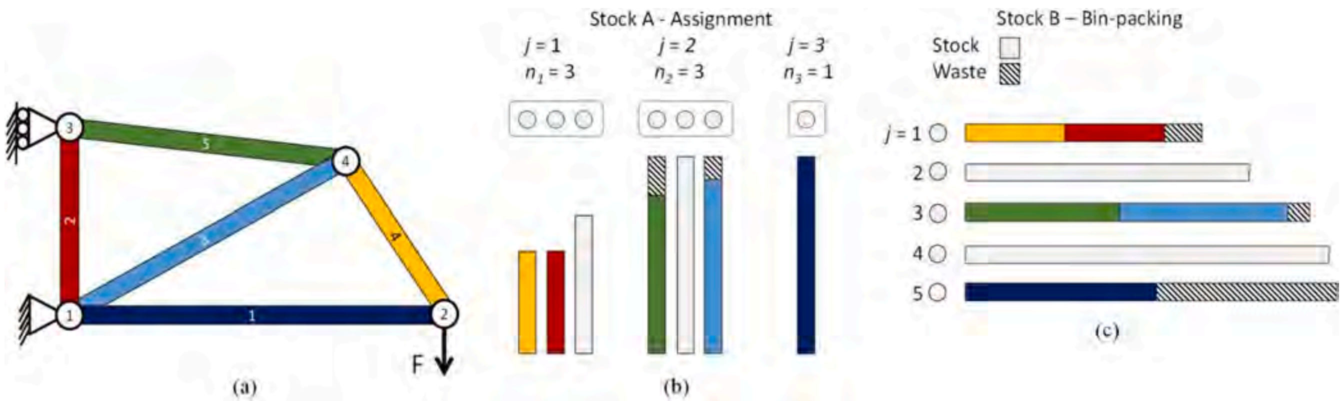

Fig. 1. (a) Cantilever truss, (b) stock A obtained by solving the assignment problem, (c) stock B obtained by solving the cutting stock problem (image inspired by [47]).

are kept in use through repair, reuse, and recycling with a particular focus on reusing materials due to its energy efficiency compared to recycling. In this study, the length and number of stock elements are the design variables of the optimization. Hence, the geometry and topology of the structure are predetermined by the mechanical and geometric properties of available elements.

In the context of this study, a specific terminology is adopted defining the *member* a positional unit or bar within a reticulated structure, *member length* the distance between nodes at that specific position. On the other hand, referring to the requested stock, the terms *element* or *item* are also used to describe the individual constituent of a stock obtained after cutting procedures on the commercial piece, called *bin*, produced into the factory. It is worth noting that the *elements* or *items* obtained as cutting patterns coincide with the *members* or *bars* composing the structure.

Fig. 1 illustrates two distinct reusing strategies for addressing the problem at hand. The first strategy involves a one-to-one assignment of elements to positions within the truss, exemplified by Stock A. The second approach adopts a bin-packing methodology, wherein multiple members can be cut from individual elements, as demonstrated by Stock B.

In the initial scenario, referred to as the 'assignment', the primary objective of the optimization is to minimize inefficiencies by reducing the overall length difference distance between individual members and the available stock elements. The second scenario focuses on determining an optimal cutting pattern that minimizes waste on a global scale. Both of these scenarios were resolved through the application of a Mixed-Integer Linear Programming (MILP) approach.

In the investigated case studies, the structures parametrically change their geometrical feature to maximize the reusing ratio of a predefined stock. No structural boundaries such as geometric compatibility, nodal displacements, deflection limits, and structural safety were considered within the problem formulation. Compared to the assignment approach, structural optimization by adopting bin-packing better reduces waste due to the lower number of cuts.

A new MILP formulation for discrete sizing and topology optimization of truss structures has been also recently proposed performing significantly better than all other formulations [48]. In their scientific endeavours, Brütting et al. also introduced a method to enhance the configurational efficiency of stock or kit-of-parts, enabling the reusing of its constituent elements across multiple structural contexts. This strategic consideration led to the proliferation of reusability within a stock of items across various structures, consequently resulting in a further reduction of waste generation.

Subsequent refinements were made to this foundational framework in the context of the assignment problem, as documented in [49]. Here, the assignment problem was integrated with topological optimization, followed by subsequent shape optimization of the truss system. Notably, an absolute 'buffer' concept was introduced, allowing for the allocation of very short items.

Additionally, in 2020, Brütting et al. continued to advance this approach in their work [50], presenting a comprehensive structural optimization methodology rooted in the aforementioned principles. In this study, a simultaneous analysis and design approach was employed, wherein structural analysis was an integral part of the optimization formulation. This involved treating member end forces, as well as nodal displacements and rotations, as continuous state variables.

Furthermore, an alternative strategy for promoting component reuse involves the custom design of a kit of parts, comprising discrete building elements pre-engineered to be adaptable for various structural configurations, serving diverse purposes.

In one of their more recent publications [51], they leveraged the assignment and Constraint Satisfaction Problem techniques to construct a kit of parts tailored for three distinct construction types. These kit-of-parts components consisted of tubular bars interconnected through spherical joints via bolts. The original topology and geometry of the structures served as input data. The optimization process unfolded in two stages. The initial stage focused on optimizing the structural geometries, as well as the length and cross-section dimensions of the kit-of-parts bars to facilitate their reuse in different structural scenarios. Subsequently, in the second stage, the optimization targeted the hole patterns for the connection details of the spherical joints, enabling each joint to be reused in multiple construction projects.

Minimization of cutting waste for real-world inspired structures has been provided by Cui and Lu in [52] who solved a rectangular two-dimensional cutting stock problem for steel bridge construction, while Araujo et al. [53] present a CSP integrated approach for the construction industry, with alternative manufacturing modes, considering different bill of materials for the same final product.

3. Proposed approach

Despite the importance of cost-saving and sustainability issues in the use of steel profiles through minimizing waste, the integration of cutting stock optimal routines within optimization frameworks still remains largely unexplored in the research literature. Moreover, the most recent adaptive metaheuristic algorithms demonstrate their efficiency in solving high-order complex combinatorial problems [54].

This work aims to overcome the limitations deriving from the optimal design based on the traditional minimum-weight approach and suggest an alternative approach reversing the structural design process. Additionally, despite previous studies, the problem formulation is not constrained to a predefined stock devoted to reusing purposes; on the contrary, the number and cross-section of bins requested for the realization of the structure dynamically change along the optimization process.

As an extension of what has already been preliminarily presented in the literature [54,55], upon which it is based, the paper includes the following additional research: the complete definition of the optimization algorithm integrating the CSP, the implementation of the proposed approach on a larger number of applications, both 2D and 3D, a consistent comparison among different algorithms, as well as between the traditional minimum-weight approach and the minimum-waste approach.

In detail, a stock-constrained structural optimization is conducted by embedding the CSP within a real-coded GA. A guided-random crossover has been developed to face the discretized nature of the investigated class of problem. The best sizing is achieved towards the identification of the optimal grouping strategy of bars for reducing waste and preserving the structural safety and the geometric compatibility of the structure. Even if, the geometric layout is fixed, the optimizer can choose the best clusters of bars to minimize the number of bins produced by the factory.

The proposed approach has been validated on numerical tests such as the 10-bar-truss case study and for more challenging applications like a 2D truss system and spatial reticular dome. For completeness reasons, a comparison between the obtained solution by the traditional minimum-weight approach and the stock-constrained optimization method is discussed.

4. Mathematical formulation of the cutting stock problem

The mathematical formulation of the CSP is herein analyzed by adopting the column generation technique to reduce the computational cost.

4.1. Bin packing problem

In order to understand how the CSP works it is necessary to exhibit the mathematical formulation of the one-dimensional bin packing problem. This problem aims to allocate a set of items into the minimum number of bins.

For initializing the problem are necessary the following parameters:

- I: set of items, indexed by i
- B: set of bins, indexed by b
- L_i: length of item i
- L: length of each bin
- $x_{i,b} \in \{0,1\}$: unitary if item i is allocated to bin b, 0 otherwise;
- $y_b \in \{0,1\}$: unitary if bin b is used, 0 otherwise

The mathematical formulation of the problem is:

$$min\sum_{b\in B} y_b \tag{1}$$

Subjected to the following constraints:

$$\sum_{b\in B} x_{i,b} = 1 \quad \forall i \in I \tag{2}$$

$$\sum_{i\in I} L_i x_{i,b} \leq L y_b \quad \forall b \in B \tag{3}$$

$$x_{i,b} \leq y_b \quad \forall i \in I, \quad b \in B \tag{4}$$

$$x_{i,b} \in \{0,1\} \quad \forall i \in I, \quad b \in B \tag{5}$$

$$y_b \in \{0,1\} \quad \forall b \in B \tag{6}$$

The principal equation of the bin packing problem (1) simply minimizes the number of bins used to obtain the requested items. The mathematical equation was subjected to some constraints. In particular, the eq. (2) means that each item must be assigned to a bin (i.e. each item should be cut from one of the paper rolls available). Additionally, the second condition (3) assures that the length of all items associated with a bin should not exceed the length of the bin and the third (4) entails that an item can be assigned to a bin if and only if that bin is used. Finally, the following two eqs. (5)–(6) express the domains of the two decision variables $x_{i,b}$ and y_b.

4.2. Column generation

The bin packing problem is a complex combinatorial problem. For simplifying this problem the Restricted Master Problem (RMP) and then the Column Generating Subproblem (CGS) formulation are used in this paper [56]. In the former, the main element composing the structure is no longer the bin. As a result, the feasible cutting pattern represents the possible arrangement of items in a bin. Since enumerating all feasible cutting patterns is prohibitively time-consuming, it generates valid patterns iteratively and adds them to the problem according to their contribution to the OF (i.e. minimization of the reduced cost).

The first step is to set up the Restrained Master Problem (RMP).

The parameters involved in this step are:

- I: set of unique items (subset of items with unique distinct lengths), indexed by i
- P: set of paths, indexed by p
- L_i: length of item i
- Q_i: quantity needed for item i
- L: length of each bin
- $M_{i,p}$: matrix whose element (i,p) defines the number of times item i is included in path p
- $x_p \in \mathbb{Z}$: number of times path p is chosen

The mathematical formulation is:

$$min\sum_{p\in P} x_p \tag{7}$$

Subjected to:

$$\sum_{p\in P} M_{i,p} x_p \geq Q_i \quad \forall i \in I \tag{8}$$

$$x_p \in \mathbb{Z} \tag{9}$$

The OF of the RMP (7) is the minimization of the number of paths used which is strictly correlated with the minimization of the number of bins. The OF is subjected to two constraints. First, it needs to select the number of paths in a way such that every unique item appears at least as many times as needed and this is what is done in (8). The second constraint defines the x_p domain.

The next step is to write the dual problem. The dual problem is a formulation correlated with the principal problem exposed above which is the primal. The dual problem is written in such a way that:

- A mostly horizontal constraint matrix becomes a mostly vertical constraint matrix (i.e. column generation process);
- A minimization problem (7) becomes a maximization problem (10);
- The objective value coefficients of the primal become constraint right-hand side values of the dual;
- The objective value coefficients of the dual are the dual values of the primal.

In particular, the dual problem is:

$$max\sum_{i\in I} Q_i \lambda_i \tag{10}$$

Subjected to the following constraints:

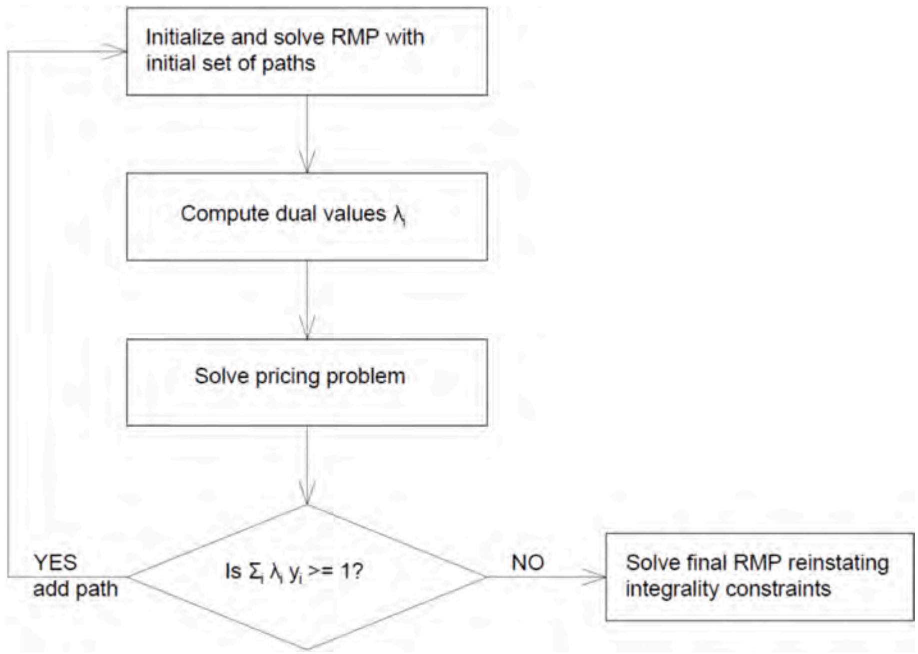

Fig. 2. Column Generation algorithm for the solution of Cutting Stock Problem.

$$\sum M_{p,i}\lambda_i \leq 1 \quad \forall p \in P \tag{11}$$

$$\lambda_i \in \mathbb{Z} \tag{12}$$

The λ_i is the dual value referred to a specific item constraint. Each dual value gives an indication of how profitable is to add the associate item to a new path.

Moreover, to determine the best path to add it is necessary to set up the pricing problem where a new decision variable $y_i \quad \forall i \in I$ represents how many times a certain item i appears in the new path. More in detail:

$$max\sum_{i\in I}\lambda_i y_i \tag{13}$$

Subjected to the following constraints:

$$\sum_{i\in I}L_i y_i \leq L \tag{14}$$

$$y_i \in \mathbb{Z} \tag{15}$$

Where the (14) ensures that the newly added path is feasible and the (15) imposes the y_i domain.

Forehead, to decide if a certain path should be added to the RMP it needs to verify the gain obtained by the addition of the new path with the following formula:

$$c - z \leq 0 \tag{16}$$

Where c is the original cost from the primal problem and z is the reduced cost computed in the pricing problem.

From the primal, it is possible to get $c = 1$ while from the pricing problem $z = \sum_{i\in I}\lambda_i y_i$. Finally by substituting:

$$1 - \sum_{i\in I}\lambda_i y_i \leq 0 \tag{17}$$

Which lastly becomes:

$$\sum_{i\in I}\lambda_i y_i \geq 1 \tag{18}$$

Until the previous condition is satisfied by the new generate path, this one was added to the primal RMP, and the procedure was iterated. To make more clear the overall process, the flowchart of the implemented algorithm is shown in Fig. 2.

5. Structural optimization procedure

When considering the cutting stock problem in the context of structural optimization, the goal is to design a structure that minimizes material usage while still satisfying the boundary conditions. This can be achieved by determining the optimal cross-section assignment of the items composing the stock which are used for the structure assembly. In the following sections, the formulation of the optimization problem and the integration of CSP within the optimization framework has been introduced.

5.1. Problem statement definition

With respect to traditional optimization approaches (i.e [57–60]) in which the OF is expressed in terms of the total weight of the structure as a sum of the mass of each element (structural mass), in this study the target function, $W(x)$, has been evaluated by computing the amount of steel requested during the production phase (stock mass). The structural cost can be expressed in the following form:

$$W(\mathbf{x}) = \rho\sum_{g=1}^{g=k} n_g A_g(\mathbf{x}) L_g \tag{19}$$

where ρ is the steel mass density assumed to be equal for all members composing the structure. n_g, A_g and L_g are the cardinality, the cross-sectional area, and the length of bins belonging to the same group g of elements with the same cross-sectional area A_g, respectively. k represents the total number of groups of elements with the same cross-sectional areas which dynamically change according to the stock output.

The design variable vector, $\mathbf{x}$, represents the set of discrete cross-sectional areas of each element such that the bins area, A_g, can be evaluated. The length of this vector as well as the grouping strategy adopted to assign the same section to different members will be declared case-by-case.

CSP has been implemented within the optimization process and it has been independently solved for all groups of elements with the same cross-sectional properties. Finally, the solutions obtained by the CSP

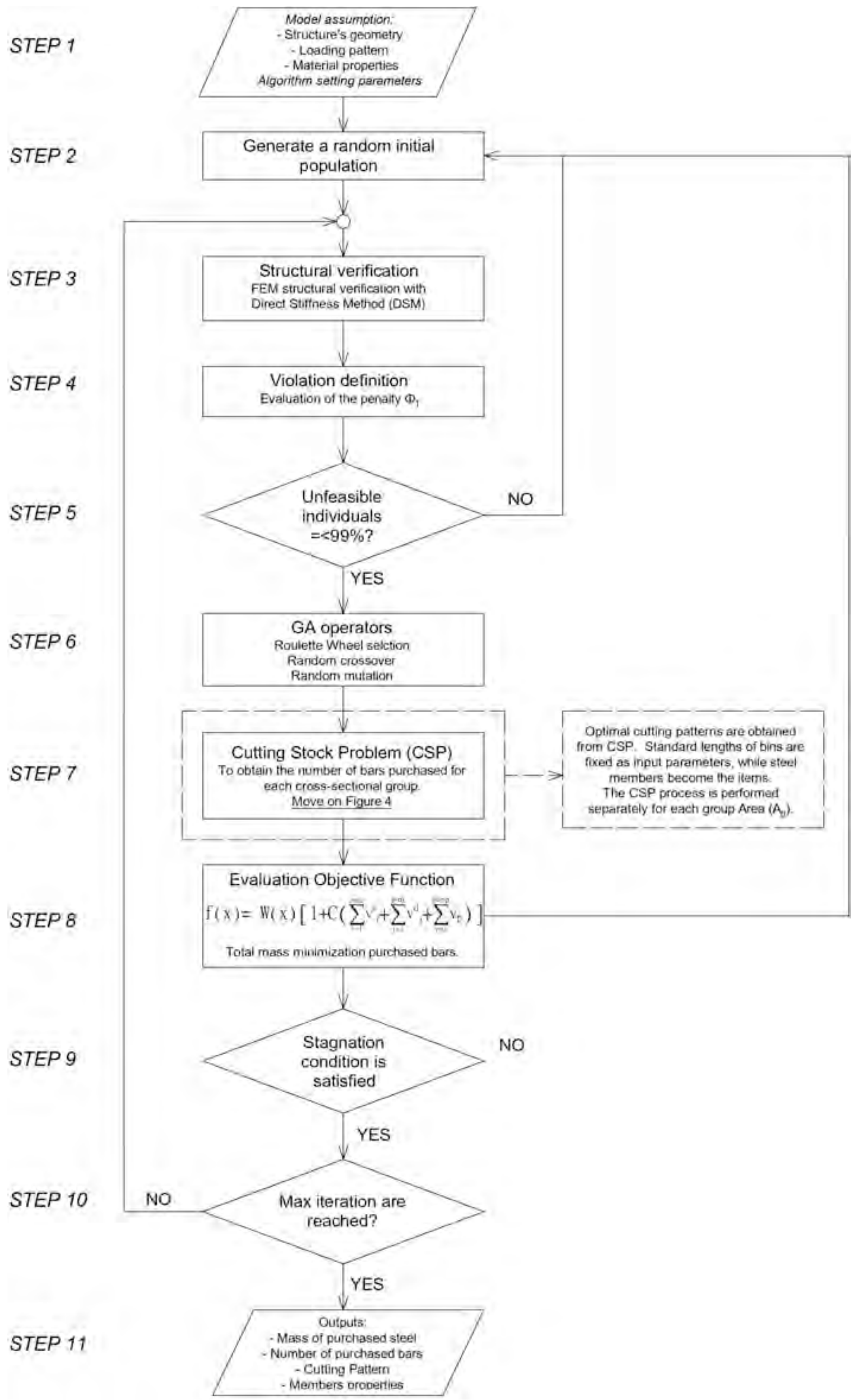

Fig. 3. Structural Optimization via CSP problem algorithm.

routine, complying with the established structural constraints, have been adopted for the evaluation of the OF fitness (i.e. number of bins).

The optimum design problem, considered in the present work, is a constrained problem. It can be transformed into an unconstrained one using a penalty function. Here, the penalty function suggested by Rajeev and Krishnamoorthy [61] has been adopted, so the OF of the problem can be computed as

$$min f(x) = W(x)\left[1 + C\left(\sum_{i=1}^{i=ne} v_i^s + \sum_{j=1}^{j=nj} v_j^d + \sum_{p=1}^{p=np} v_p\right)\right] \tag{20}$$

subjected to:

$$V_i^t = \frac{N_{i,ED}}{N_{t,RD}} - 1.0 \quad i = 1, 2, \ldots ne \tag{21}$$

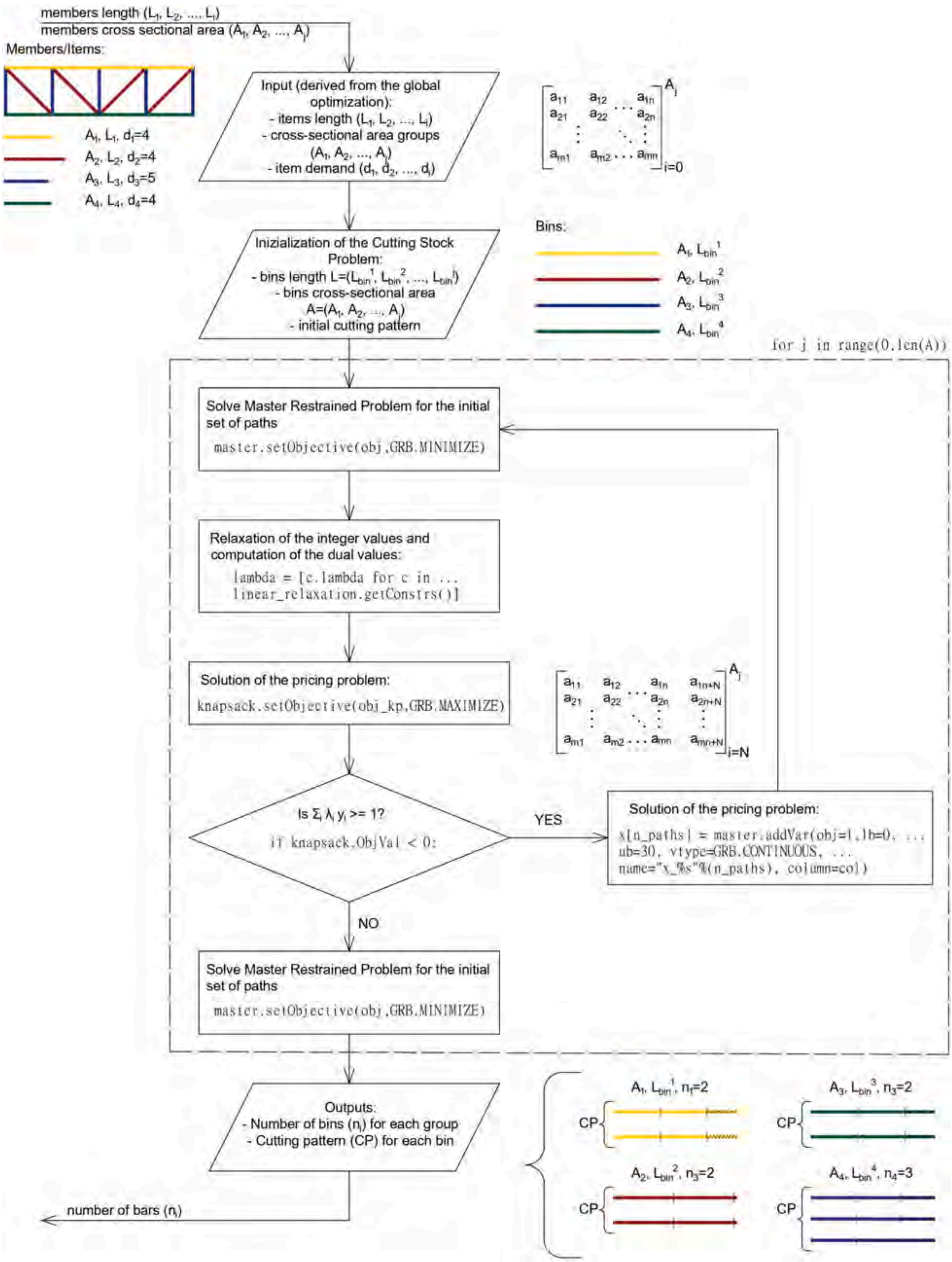

Fig. 4. Detailed CSP algorithm embedded in the structural optimization.

$$V_i^c = \frac{N_{i,ED}}{N_{c,RD}} - 1.0 \quad i = 1, 2, \ldots ne \tag{22}$$

$$V_i^b = \frac{N_{i,ED}}{N_{b,RD}} - 1.0 \quad i = 1, 2, \ldots ne \tag{23}$$

$$V_j^d = \frac{|\delta_j|}{\delta_{max,y}} - 1.0 \quad j = 1, 2, \ldots nj \tag{24}$$

$$V_p = \frac{d_p^{ch}}{d^{dg}} - 1.0 \quad p = 1, 2, \ldots np \tag{25}$$

where $N_{t,RD}$, $N_{c,RD}$, $N_{b,RD}$ are the tension strength, compression strength and buckling strength of the specific section calculated according to Eurocode 3 [62] (EN 1993-1:2005 and EN 1993–2:2006) while N_{ED} represents the stress acting to the single member. δ and $\delta_{max,y}$ are the vertical displacement experienced by the j-th node and the maximum value assumed as threshold, respectively. The higher allowable displacement depends to the specific application and has been settled case-by-case.

In Eq.20, $W(x)$ is calculated by Eq.(19); C is a penalty constant, which is equal to 10 in this work; v_i^s, v_j^d, and v_i^p are the violations of normalized stress ratio, displacement ratio and size considerations, respectively and are computed using Eq.26.

$$\left\{v_i^s, v_j^d, v_p\right\} = max\left(0, \left\{V_i^t, V_i^c, V_i^b\right\}, V_j^d, V_p\right) \tag{26}$$

In this way, the penalty function integrates information related to either magnitude of penalization's level and number of unfeasible individuals. Especially for the geometric compatibility of trusses (see Eq.25), the size constraints is verified by ensuring that the diameter of the compression and bottom chords, d^{ch}, is always greater than diagonals, d^{dg}. The adopted size adaptation among these members is functional to constructional consideration and common practice in Civil Engineering.

5.2. Flowchart and pseudocode

In this section, the step-by-step pseudocode of the employed algorithm has been described. In addition, two distinct flowcharts have been depicted for introducing the principal steps of the algorithm (Fig. 3) and for clarifying how the CSP has been implemented within the main optimization process (Fig. 4). The optimization framework has been realized in *Python* [63] as well as the Finite Element (FE) code for the structural analysis of each case study.

Accordingly to the order depicted in the graphical scheme in Fig. 3, the main steps of the optimization process are the following:

STEP 1: Geometry definition and algorithm settings of the optimization problem: truss topology and number of members, loading scenarios, cross-sections and material type (e.g. Young modulus, E, and density, ρ) and parameters tuning of the algorithm's parameters;

STEP 2: Generation of a random initial population with N individuals (each characterized by a different size variable layout). The algorithm works with discrete design variables taken from commercial standards (EN 10210).

STEP 3: Performance of the structural analysis for a given loading scenario and verification of the structural conformity according to Eq.s (21)–(25) (EC3 6.3.3.). The structural analysis is performed by a FE code, which adopts the Direct Stiffness Method (DSM) [64].

STEP 4: Evaluation of the single penalty according to Eq.s(21)–(25) and computation of the proper level of penalization coherently to Eq. (26);

STEP 5: Verifying if at least 1% of the entire population is feasible. If this condition is not satisfied the reinitialization of the entire population (return to *STEP 2*) can start with a randomic samples selection from a reduced catalogue where the 5% of sections with the lowest Area are

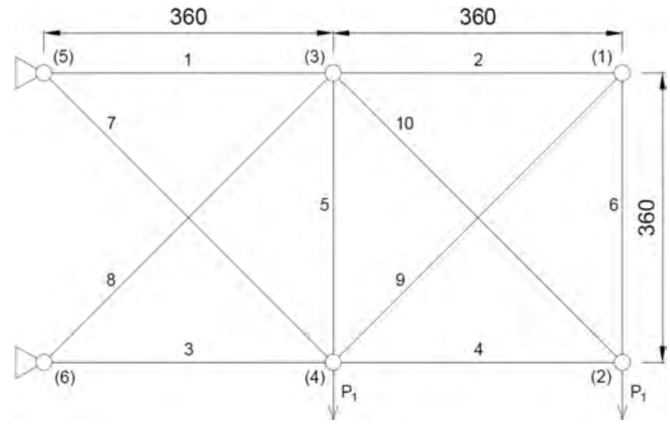

Fig. 5. Configuration of the in-plane 10-bar truss, measures are expressed in inches (in.)

removed. Once the condition is satisfied the original length of the catalogue is restored;

STEP 6: At this stage the GA operators are activated. Specifically, the roulette wheel selection has been implemented in order to guarantee that the two fittest parents are selected for the next steps. Adopting this technique, a probability to each parent is assigned and the parents with higher fitness are more likely to be chosen for crossover. From each couple of parents, 1 child has been obtained towards random crossover. Lower and upper bounds are imposed at this stage such that if only a gene of the new offspring is not ranged within the imposed interval (higher than the maximum value or lower than the minimum value of the cross-sections' catalogue), it is forced to assume maximum or minimum value, respectively. Finally, aiming to improve the exploration and exploitation ability of the algorithm, a mutation rate of 5% is assigned. In this way, new genes are introduced into the population by modifying the gene pools of parents in a random way.

STEP 7: Once the geometry layout of the truss is defined and the sectional properties are assigned, the number of standard purchased commercial bars (i.e. number of bins) and the relative cutting patterns for each cross-sectional group g are evaluated. The grouping on the elements is necessary because the items with the same cross-sectional area must be allocated in the same bin class for the evaluation of the optimal cutting. The CSP allocates the various structural members into the standard factory bars in an optimal way, in order to minimize waste. For more details, a flow chart of the CSP routine integrated into the main optimization process has been reported in Fig. 4.

STEP 8: Evaluation of the OF $W(x)$ according to Eq.(19). The calculation of fitness is computed for each solution.

STEP 9: Check of the stagnation condition. This step avoids the algorithm stuck in a local optimum, while trying to search for a global optimum. Specifically, this condition verifies if the best solution is the same for a predetermined consecutive number of iterations. Whether the response is affirmative the optimization process reinitializes the population (return to *STEP 2*). Aiming to maintain promising solutions in the population, the best 2% of the best feasible individuals obtained by the previous iteration survive to the next one. Otherwise, the optimization process continues with the following steps.

STEP 10: This condition simply checks if the stopping criteria are reached. Whether the number of the current iteration is lower the process comes to *STEP 3* otherwise continue with the plotting of the output results.

STEP 11: The outputs of the entire optimization process are identified as the overall mass of purchased steel, the number of bins and bars for each cross-sectional class as well as the relative cutting pattern relative to the optimal individual.

Table 1
Model assumption relative to the 10 bar truss.

Parameter	Value
Modulus of elasticity of steel E	10,000 ksi
Steel density ρ	0.10 lb/in^3
Loading P_1	100 $kips$
Length of purchased bars (bins) L_{bin}	1020 in
No. of design variables	10
Bounds of design variables (A_{min}, A_{max})	[1.62, 33.5] in^2
Maximum allowable stress $(N_{t,RD}, N_{c,RD})$	$\pm$ 25 ksi
Maximum allowable displacement $(\delta_{x,y})$	$\pm$ 2 in

Table 2
Discrete cross-sectional standard area within the design variables are chosen.

Discrete cross-sectional areas A $[in^2]$						
1.62	1.8	1.99	2.13	2.38	2.62	2.63
2.88	2.93	3.09	3.13	3.38	3.47	3.55
3.63	3.84	3.87	3.88	4.18	4.22	4.49
4.59	4.8	4.97	5.12	5.74	7.22	7.97
11.5	13.5	13.9	14.2	15.5	16	16.9
18.8	19.9	22	22.9	26.5	30	33.5

Table 3
Optimization algorithm parameters set by the operator.

Parameter	Value
Maximum number of iterations	200
Number of individuals per population	200
Catalogue percentage of reduction	5%
Mutations' probability	1%
Proportional children	1 child for each parent
Stagnation condition	10 iterations

6. Case study I: 10 bar-truss

In this section, a straightforward application of the CSP integrated into the structural optimization procedure is presented adopting the ten-bar truss benchmark from [65]. Despite the simplicity of the case study and the minimal assumptions made in the optimization process, it serves as an initial demonstration of the proposed approach.

6.1. Model definition and parameters' setting

Accordingly with Fig. 5, the investigated configuration constitutes a statically determined cantilever system with a truss arrangement, comprising ten steel bars. This structure is subjected to constraints by means of two pinned supports, located at nodes 5 and 6. The analysis is conducted under a singular loading scenario denoted as P_1, wherein equivalent forces of magnitude P_1 are symmetrically applied to nodes 2 and 4.

In order to provide a comprehensive overview of the numerical model assumptions pertaining to the material properties and geometric characteristics of the truss, Table 1 has been reported.

The design variables of the problem consist of the 10 cross-sectional areas associated with the truss components. This approach exclusively considers solutions that align with the available cross-sectional area options found in the provided standards list. Consequently, a specific set of 42 discrete values (as detailed in Table 2) has been used to represent the potential cross-sectional areas for each truss member according to [65]. In this numerical test, geometric compatibility has not been considered since the adopted catalogue is exclusively populated by cross-section areas without any information about geometric features of the section. The parameters' settings of the GA algorithm are summarized in Table 3.

Table 4
Result of the optimization via CSP related to Scenario (a).

	M_{stock} (OF) [lb]	M_{truss} [lb]	M_{waste} [lb]
Best	6825.84	5791.97	1033.87
μ	7320.13	6158.41	1161.73
σ	308.64	257.05	98.37

6.2. Results and discussion

In this subsection, the results of the Structural Optimization (SO) process pertaining to the 10-bar case study are outlined. Specifically, two optimization scenarios have been conducted:

- **Scenario (a)**: SO implementing CSP for the minimization of purchased steel bars;
- **Scenario (b)**: SO by adopting a traditional minimum-weight approach.

To facilitate a comparison between the two previously mentioned approaches, the CSP procedure has been performed at the end of the scenario (b) such that the number of bins has been evaluated based on the least-weight optimal design. In this way, the optimal design obtained from the two distinct approaches can be compared in terms of *structural weight* (i.e. total mass) and *waste mass*.

Scenario (a). #10 independent runs have been performed and details of the outcome of each run have been reported in Table A.13 of Appendix A. However, the best and worst results as well as the mean and the standard deviation calculated among the 10 runs are summarized in Table 4.

Even if the CSP has been implemented within the optimization process, a residual waste mass, M_{waste}, equal to almost 15% of the total truss weight, M_{truss}, can be observed. As expected, the stock mass, M_{stock}, is rather close to M_{truss} validating a full-exploitation of the optimal cutting capability of CSP. It is worth noting that the results obtained by the optimal cutting routines strongly depend to the fixed length of the bins provided by the factory. With a higher bin length, the optimizer has been encouraged to allocate more truss members into a single bin. In the next case study, specific consideration for the assumption of the bins length will be introduced coherently to the common practice in the industrial and civil engineering field.

Other interesting observations pertaining to the functioning of the CSP within the context of the SO process, can be derived by Fig.6.

In this Figure, the optimal cutting pattern has been reported aiming to give a clear graphical representation of the cross-section assignation provided by the optimizer and the residual waste for each bin.

The optimal configuration is characterized by accommodating two members per bin, which corresponds to the maximum possible allocation for a single bar. As expected, for this specific case, the optimizer was forced to assign #5 different area properties as a consequence of the optimal cutting configuration.

In other words, the optimizer operates on two primary levels: firstly, by selecting the most appropriate cutting pattern to maximize the use of the bin reducing waste, and secondly, by choosing the minimum cross-sectional area for each bar.

Scenario (b). As in the previous scenario, #10 runs have been conducted using the traditional minimum-weight SO approach. Subsequently, the mass of the stock, M_{stock}, was determined by applying the CSP to the members composing the optimal truss design. The outcomes of these 10 runs have been reported in Table A.14 of Appendix A. Table 5 presents the best solution as well as the mean and the standard deviation based on #10 runs.

From these preliminary results, it is evident how the least-weight solution lies in a significant increase of mass waste which is almost equal to total weight (i.e. structural mass). Even if the investigated case study is composed by few members resulting in limited cutting options,

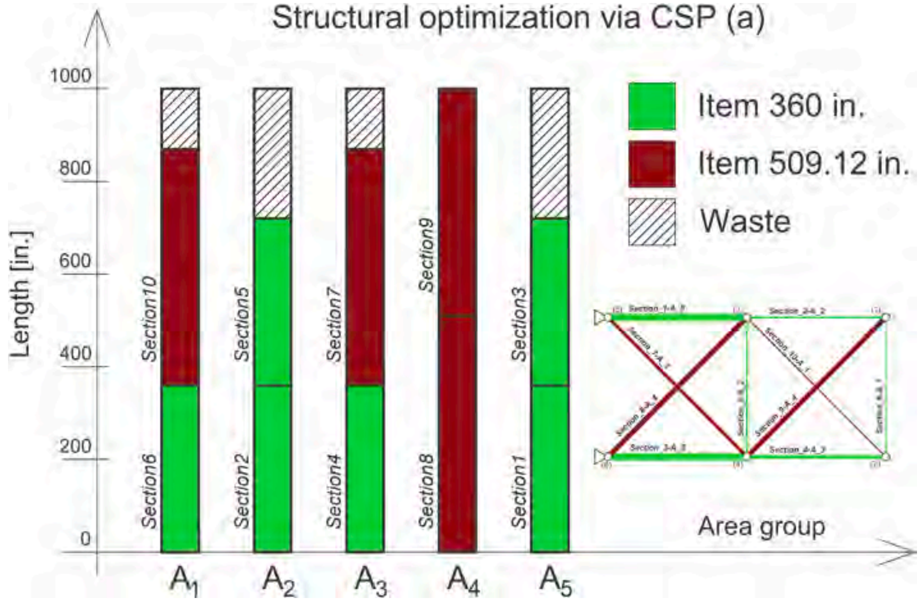

Fig. 6. Optimal cutting pattern of the 10 bar truss obtained by scenario (a). The dashed area represents the remaining bin's length deriving from the cutting process. The same colors have been used to identify the relationship between items and the position of the truss members.

Table 5
Result of the optimization via traditional approach (b).

	M_{stock} [lb]	M_{truss} (OF) [lb]	M_{waste} [lb]
Best	13078.44	5545.76	7532.68
μ	12133.92	5632.88	6501.04
σ	1343.16	64.15	1348.10

these preliminary results demonstrate the benefits of adopting the proposed approach.

In Fig. 7, a stock representation obtained by performing the CSP after the minimum-weight optimization approach is depicted. It appears evident how the optimizer preferred an optimal design with a high level of area diversity instead of reducing waste.

Final comparison: scenario (a)* vs *scenario (b). Once the results obtained by two distinct SO approaches have been introduced, the percentage ratio between the structural mass, M_{truss}, and mass waste, M_{waste}, obtained by the two approaches have been reported in Table 6.

Even if a reduction of total mass moving from scenario (a) to scenario (b) is recognized, a dramatic increase of the mass waste occurs. Specifically, the M_{waste}/M_{truss} ratio varies from 17% of scenario (a) up to 136% of scenario (b). In other words, the optimal design obtained by the minimum-weight approach leads to a mass waste that is almost 1.5 times the total mass of steel required for the realization of the steel structure.

Similar considerations can be pointed out with specific regard to the M_{waste}/M_{stock} ratio which is the index indicating the effective waste

Table 6
Performance comparison between CSP approach (scenario-a) and traditional approach (scenario b).

	M_{truss} [lb]	M_{waste} [lb]	M_{waste}/M_{truss}	M_{waste}/M_{stock}
Scenario (a)	5791.97	1033.87	17%	15%
Scenario (b)	5545.76	7532.68	136%	58%

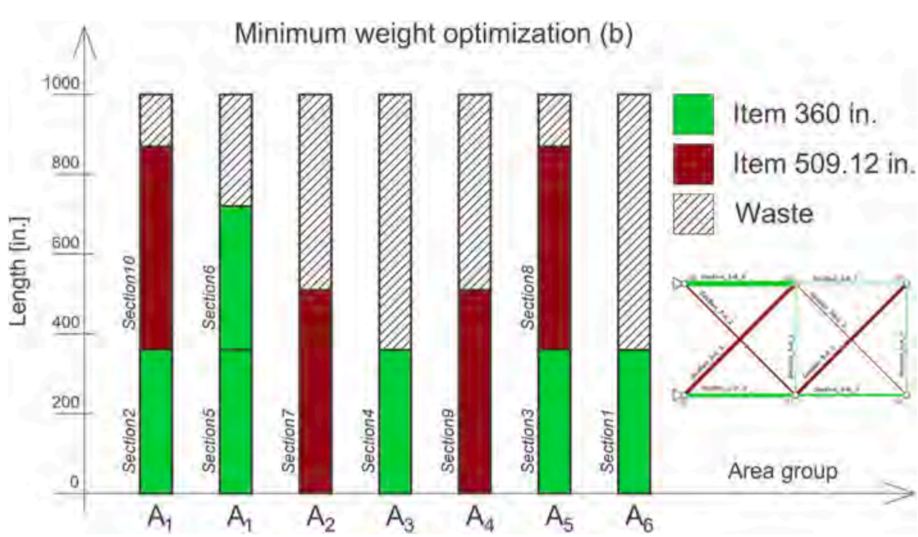

Fig. 7. Optimal cutting pattern of the 10 bar truss obtained by scenario (b). The dashed area represents the remaining bin's length deriving from the cutting process. The same colors have been used to identify the relationship between items and the position of the truss members.

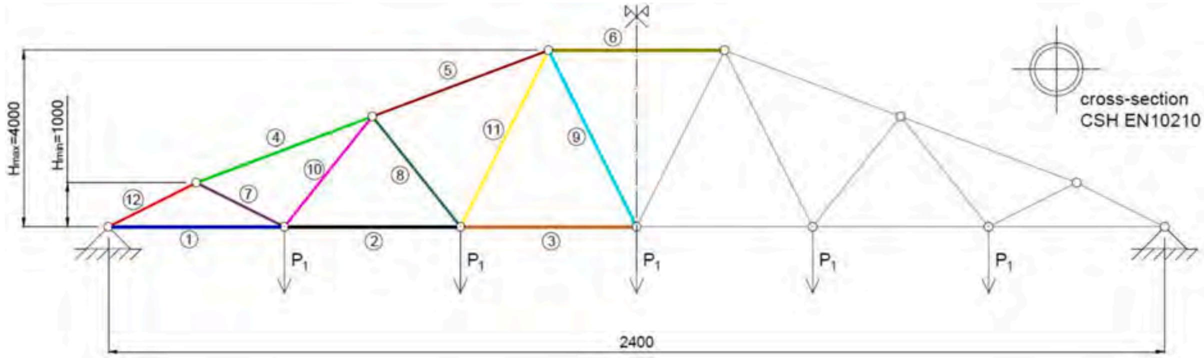

Fig. 8. Configuration of the Warren truss under analysis, the numbers indicate the design variables which are 12 for symmetry reasons. Measures are expressed in millimetres (mm).

Table 7
Model assumption relative to the symmetric Warren truss.

Parameter	Value
Modulus of elasticity of steel E	210,000 MPa
Steel density ρ	7.85 t/m^3
Loading lower chord nodes P_1	240 kN
Length of purchased bars (bins) L_{bin}	15 m
Number of design variables	4
Bounds of design variables (A_{min}, A_{max})	[137, 24,700] mm^2

Table 8
Optimization algorithm parameters set by the operator.

Parameter	Value
Maximum number of iterations	200
Number of individuals per population	300
Areas excluded if unfeasibility condition isn't satisfied	5 smaller DV
Mutations' probability	5%
Proportional children	1 child for each parent
Stagnation condition	20 iterations

resulting by the cutting procedures of the stock. The gap between the two scenarios reduces, however, scenario (b) still remains the worst approach with a waste of 58% of the total stock mass.

Generally, these findings suggest that the algorithm incorporating the CSP performs well, as it necessitates fewer purchased bars, with only a slight uptick in structural weight. As expected, the number of different cross-sections is increased from scenario (a) to scenario (b) (refers to Fig. 6–7). If in the former, the optimizer tries to group as much as possible the bars for obtaining the maximum cutting performance; in the latter, it prefers to differentiate the sectional properties of bars aiming to reduce the total structural weight of the truss.

7. Case study II: Warren truss

Within this section, a more complex application example is introduced. The structure under analysis is a Warren truss, widely adopted as support of an industrial building's roof, composed of 23 members.

7.1. Model definition and parameters' setting

As depicted in Fig. 8, the structure is subjected to symmetric loading and a symmetric geometry is assumed. This assumption approximately halves the design variables which becomes 12 cross-sectional areas.

In Tables 7, 8 the characteristics of the model properties and the setting parameters are summarized. For transportability reasons, the maximum bin's length has been adopted equal to 15 m, while the loading scenario is characterized by concentrated forces applied to the joints of the bottom chord.

Static action only has been considered in the analysis like (i) non-structural distributed loads equal to 4 kN/m^2, (ii) snow distributed loads equal to 1.5 kN/m^2, and (iii) maintenance distributed loads of 0.5 kN/m^2. To obtain the in-plane loading conditions of the truss, an influence area of 10 m is considered as the theoretical distance between two Warren trusses that support the roof system of an ideal industrial building. Finally, the mentioned action has been applied to the joint of the bottom chord as depicted in Fig. 8. The self-weight of the structure, which dynamically changes with the member sizing, has been considered separately for the FE structural analysis.

The cross-section catalogue is populated by 150 different Circular-Hollow Sections (CHS) coherently to the standard regulation EN10210 with assumed lower and upper bounds set at 137 and 24700 mm^2, respectively.

Table 9
Result of the 4 DV Warren optimization via CSP approach (a).

	M_{stock} (OF) [kg]	M_{truss} [kg]	M_{waste} [kg]
Best	2656.44	2189.96	466.48
μ	2770.02	2265.11	504.91
σ	98.30	73.80	37.38

7.2. Results and discussion

As shown in the previous case study, two scenarios have been investigated aiming to compare the two approaches. The output of the optimization results (see Tables A.15- A.16) for each scenario have been reported in Appendix A.

Scenario (a). In Table 9, details about the best solution as well as average and standard deviation, calculated by considering the 10 runs, have been reported.

Implementing CSP into the SO process leads to significant waste-saving. It can be observed that with a M_{stock} and M_{truss} equal to 2656 kg and 2190 kg, respectively, the M_{waste} is 466.8 kg. Additionally, a significant decrease of the mean, μ, and standard deviation, σ, is recognized demonstrating a good convergence capability of the algorithm.

In this case, the stock representation (see Fig. 9) shows that the rate of exploitation of the bins is the maximum that can be attained.

At first, it is interesting to notice that all the runs give as a result the areas' grouping in three bins' classes because the two external webs are simply allocated in the same group of the upper chord members. As observed for the previous case study, the optimizer prefers design solutions with a limited number of different cross sections aiming to increase the possible cutting options for bins with the same sectional properties.

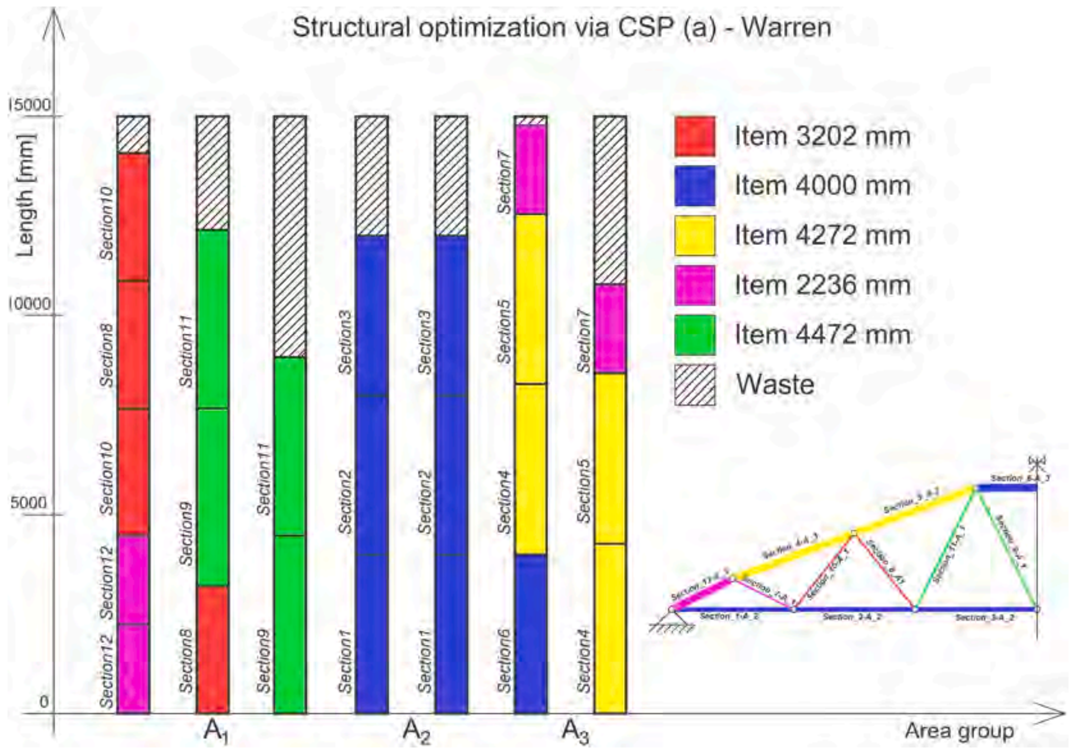

Fig. 9. Optimal cutting pattern of the Warren truss obtained by scenario (a). The dashed area represents the remaining bin's length deriving from the cutting process. The same colors have been used to identify the relationship between items and the position of the truss members.

Table 10
Result of the symmetric Warren optimization via traditional approach (b).

	M_{stock} [kg]	M_{truss} (OF) [kg]	M_{waste} [kg]
Best	4610.62	1864.7	2745.9
μ	4855.4	2049.9	2805.5
σ	222.82	26.38	215.64

It is worth noting that the grouping strategy follows perfectly the engineering practice for which truss beams are basically composed of 3 cross-sectional clusters: lower chord, upper chord and diagonals. As observed by several authors [66,67], this classification is mainly due to an almost constant level of stress for all the members belonging to the same group.

Scenario (b). In Table 10, the main feature of the best solution and the statistics based on #10 runs have been reported.

As expected, the number of different cross-sections employed by the optimizer increased to 10 (see Fig. 10). As in the previous case, the optimizer aims to distinguish as much as possible the members' areas in order to achieve the least-weight configuration. Additionally, it appears evident by the graphical representation of the cutting patterns that the mass waste increased significantly due to the cross-section variability of the members. As proof of the goodness of the optimal sizing, the final layout of the Warren truss has been depicted in Fig.10. According to common practice, from the support to the middle span, the assigned cross-section Areas decrease along the upper and bottom chords while thin and heavy section have been assigned to the tensioned and compressed diagonals, respectively.

Scenario (a)* vs *Scenario (b). In this last section, an overview of the results obtained by adopting the two approaches and a detailed discussion concerning the differences of the optimal design solution have been reported also for this case study.

In Table 11, the percentage ratio of the mass waste with respect to the structural mass (i.e. mass of the truss) and the stock mass is reported also for this case.

The trend observed in the previous case study is validated again for the current application. Specifically, if the waste produced, by adopting the CSP approach, leads to 21% of waste for the total mass of the truss equal to 2189.96 kg; the minimum-weight design produces a mass of waste that is almost 1.5 times the total mass of the truss. The gain obtained by reducing the total mass of the structure is completely loss by the mass waste production. Similar considerations can be made by calculating the ratio between the amount of mass waste (M_{waste}) and the mass of stock (M_{stock}).

As expected, the main discrepancy between the two scenarios can be addressed by the different number of sections, which are mainly related to the different problem definitions of the optimization process. Additionally, the section assignment respects the geometric compatibility imposed so the feasibility of the connections, between the compression and tension chords and the diagonals, is always respected.

A sensitivity analysis has been conducted by extending the two mentioned approaches on the same truss typologies with an increasing number of spans.

In Fig. 11(a)-(b), the trend of M_{truss}, M_{stock} and M_{waste} have been reported for each scenario by investigating the effect of the adopted approaches on the increase of the number of pieces composing the structure. Specifically, truss spans varying between 6 and 22 have been investigated.

By observing Fig. 11(a), the results obtained for the Warren truss with No.6 spans have been confirmed for all the other scenarios. More in detail, by increasing the number of spans (i.e. number of pieces) the gap between the M_{truss} and the M_{stock} increases. Especially for the configuration with the number of spans equal to 18, 20 and 22, the M_{stock} exponentially increases. With specific regard to these structures, the amount of steel required by the factory is 4 times the structural weight (M_{truss}). Additionally, even if the number of DVs increases, from the Warren configuration with No.12 up to 44 different cross-sections (for symmetry reasons), the M_{truss} still increases. Since the geometry is fixed as well as the total span length, the configuration with the maximum number of spans represents the design with the worst structural

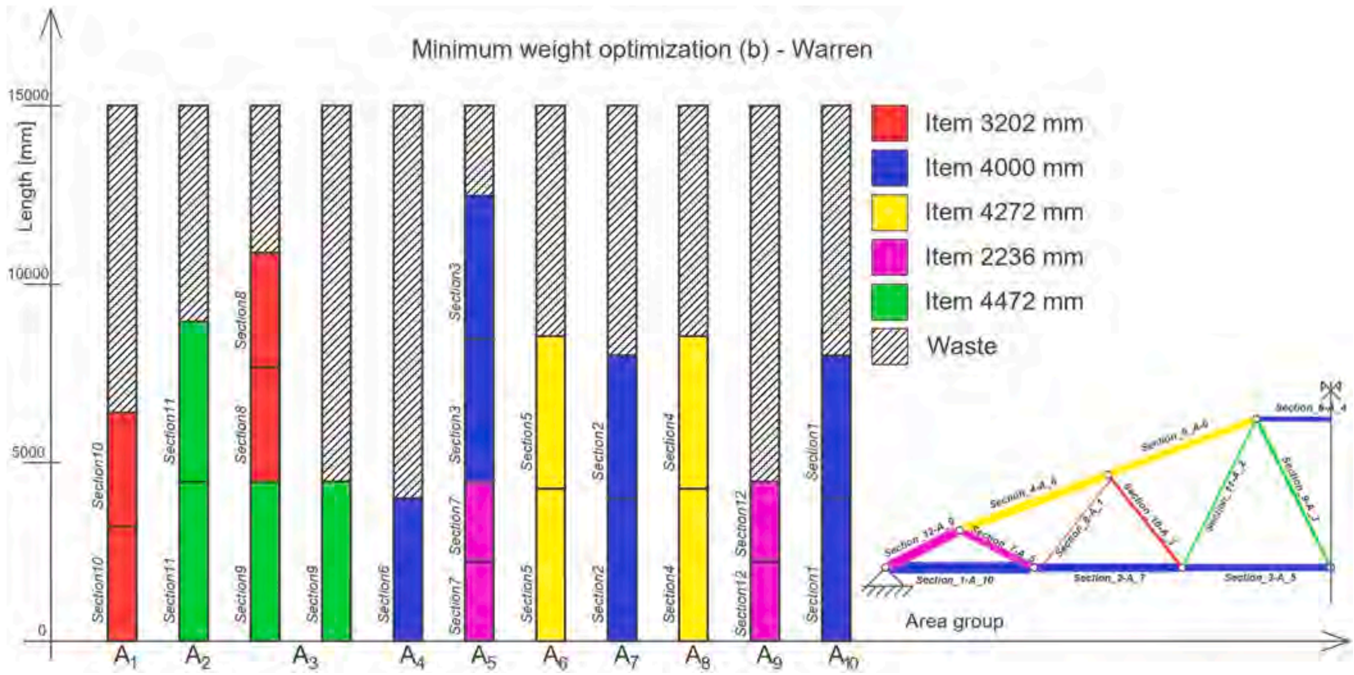

Fig. 10. Optimal cutting pattern of the Warren truss obtained by scenario (b). The dashed area represents the remaining bin's length deriving from the cutting process. The same colors have been used to identify the relationship between items and the position of the truss members.

Table 11
Performance comparison between CSP approach (scenario-a) and traditional approach (scenario b).

	M_{truss} [kg]	M_{waste} [kg]	M_{waste}/M_{truss}	M_{waste}/M_{stock}
Scenario (a)	2189.96	466.48	21%	17%
Scenario (b)	1864.7	2745.9	147%	60%

performance due to the non-efficient diagonal inclinations varying between 65 and 73 degrees. On the other hand, design configurations with no. of spans equal to 6, 8 and 10 are characterized by diagonal inclination angles close to the optimal value of 45 degrees.

The results obtained by the proposed CSP approach depicted in Fig. 11(b) show an opposite trend. Even if the M_{truss} increases for the non-efficient diagonal inclination angles, the M_{stock} values for each configuration remain close to the M_{truss} resulting in significant waste-saving. As expected, the design solutions are characterized by higher structural mass than the traditional approach. However, this apparent loss of structural weight is successfully counterbalanced by the reduction of the total M_{stock} for each scenario. Especially for the most promising design represented by the truss with no. of spans equal to 10, the M_{stock} is almost half of that one obtained by the traditional approach.

Finally, as observed in Fig. 9, the grouping strategy obtained by the CSP approach is maintained independently of the no. of spans. Two different sections have been assigned to the elements composing the compressed and tensioned chords, while the last section has been assigned to the tensioned and compressed diagonals on which the level of stress is lower than the formers. This result coherently agrees with the

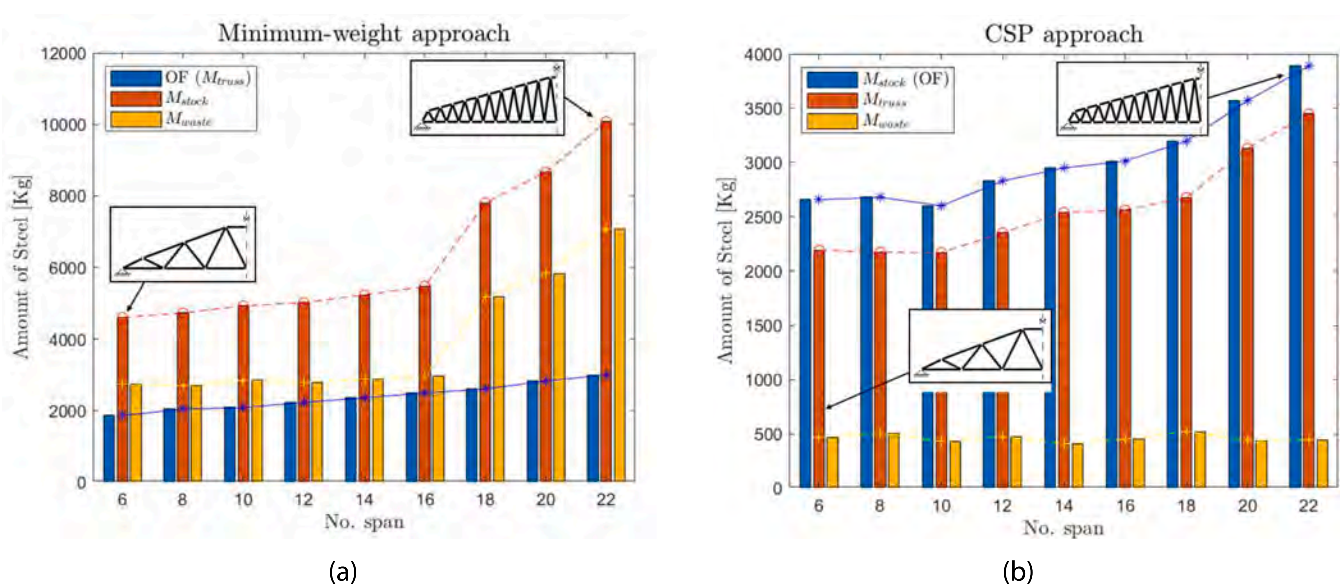

Fig. 11. Output of the optimization process following (a) the traditional minimum-weight approach and (b) the CSP approach.

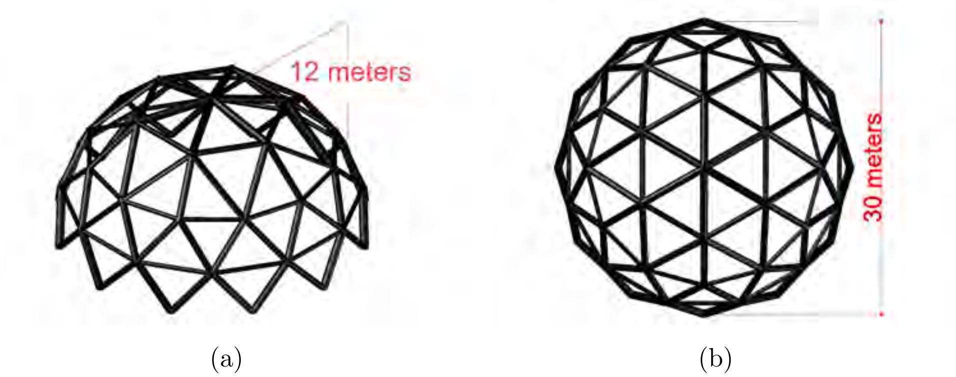

Fig. 12. Prospective (a) and top view (b) of the dome.

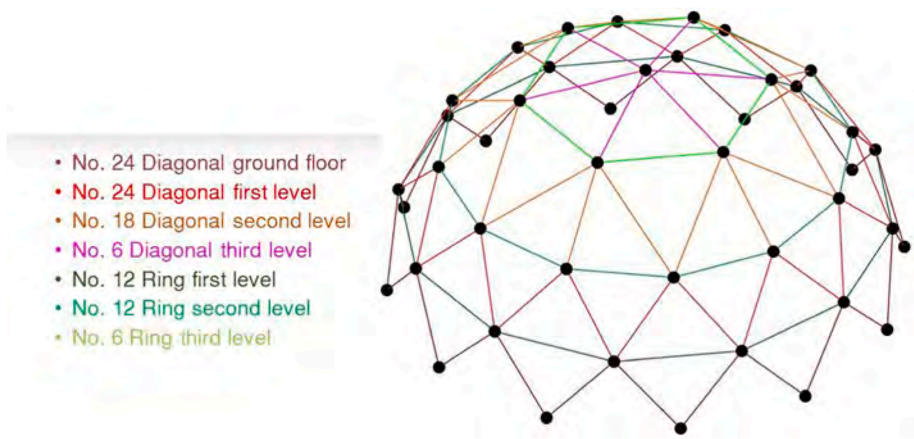

Fig. 13. Type of elements composing the dome. Members with similar structural behavior are depicted with the same color.

necessity of lowering the number of different cross-sections aiming to ensure an optimal cutting of the stock.

8. Case study III: Spatial reticular dome

In this section, the last case study will be presented. A reticular dome realized by steel members, resulting in triangular mesh, will be investigated. Due to their topology, these structural systems are allowed to cover huge spans thanks to their structural behavior and are preferred for large-span structures like stadiums or exhibition centers.

8.1. Model definition and parameters' setting

The adopted reticular dome consists of 102 bars with varying lengths and inclinations, all interconnected by 43 joints. The Structural Steel adopted is the S275, characterized by a mass density $\rho = 7850kg/m^3$ and an elastic modulus $E = 210GPa$. It is constrained by simple supports at each joint of the base.

To delve into more specific details, each level of the structure is comprised of circular horizontal hoops and diagonal bars. The first and second levels consist of 24 diagonal bars and 12 members composing each circular ring. Moving upwards, the third and fourth levels include 18 and 6 diagonal bars, respectively, interconnected by a horizontal hoop consisting of six bars.

The kit-of-parts used for this structure consists of tubular elements with a CHS profile, joined together at spherical nodes. Fig. 12 provides a perspective and top view of the structure as well as its members' arrangement.

Additionally, in Fig. 13 a graphical representation of the main groups of elements composing the structure has been reported. As illustrated in previous works (e.g., [68,69]), the optimal grouping strategy operated by the optimizer has a crucial role in reducing computational effort and achieving the best cutting criteria for minimum waste.

The discrete design variables are determined by choosing cross-sectional areas from a standard list (EN 10210) complying with the identified grouping strategy. Hence, elements with similar structural behavior and belonging to the same cluster have the same cross-sections. Lower and upper bounds of the cross-section variability range are set at 137 and 4000 mm^2, respectively. It results in a catalogue composed of 150 CHS profiles characterized by different diameters and thicknesses.

In the proposed approach, the structure experiences only gravitational actions like structural and non-structural permanent loads, snow and maintenance. Structural glass has been chosen as the skin area of the dome and a non-structural permanent load equal to 0.2 kN/m^2 has been adopted. According to the previous case, live loads like snow and maintenance have been assumed equal to 1.5 kN/m^2 and 0.5 kN/m^2, respectively. Consequently, all the loads have been applied at the level of the joints as concentrated loads according to the corresponding influence area.

8.2. Results and discussion

As shown for the previous case studies, two scenarios have been investigated aiming to compare the two approaches. The output of the optimization results (see Tables A.17-A.18) for each scenario has been reported in Appendix A.

***Scenario (a)* vs *Scenario (b)*.** The problem statement outlined in the previous section serves as the basis for optimizing the dome, and #10 runs have been performed to evaluate the method's robustness and accuracy also for this case. Aiming to fully exploit the capability of the

Table 12
Outputs of the optimization process obtained from scenario (a) and scenario (b).

	M_{truss} [kg]	M_{stock} [kg]	M_{waste} [kg]	M_{waste}/M_{truss}	M_{waste}/M_{stock}
Scenario (a)	2851.1	3329.7	478.4	16.0%	14.0%
Scenario (b)	2773.6	3683.2	909.5	32.8%	24.7%

cutting routine, the length of the bins has been fixed equal to 18 m coherently with the information obtained by specialized factories on these types of structures. The same trend of the previous case studies has been observed by analyzing separately the two scenarios. Hence, to simplify the presentation, in this section, only the comparison between the two optimization approaches has been introduced and an overview of the optimal outcomes has been reported in Table 12. As expected, scenario (b) leads to a mass saving of the M_{truss} with respect to scenario (a) even if the M_{stock} of the former still remains significantly high. Specifically, for scenario (b), the M_{truss} is equal to 75% of the M_{stock} resulting in a significant waste equal to 30% of the M_{truss}. In other words, for the realization of the dome, the total waste is almost 1/3 the muss of the dome. On the other hand, an evident improvement has been obtained following the CSP approach. Scenario (a) leads to a significant reduction of the waste equal to only 16.0% of the mass of the dome.

Examining Fig. 14(a)-(b), it becomes clear that the benefits of implementing the proposed approach are evident in terms of material saving. The optimal design of the dome, achieved through a straightforward weight minimization process (scenario b), leads to substantial material wastage, as evidenced by the non-optimal cutting pattern of bins. Coherently, as observed numerically, the total M_{stock} obtained by scenario (b) is significantly higher than one from scenario (a). As for the previous case study, the optimal stock of elements has been achieved by an optimized grouping strategy adopted by the optimizer. In scenario (b), the optimizer selects 7 distinct cross-sectional areas to enhance the structural performance of each group element to achieve the minimum weight of the structure. In contrast, in scenario (a), where the CSP is resolved during each iteration of the optimization procedure, the total number of bins required during production is significantly reduced, and the variety of different cross-sections decreases to 4.

The impact of integrating CSP with structural optimization becomes clearer when comparing the total number of bins obtained in both scenarios. Despite scenario (b) resulting in a lower structural mass compared to scenario (a), the reduction in material wastage, derived from the CSP approach, overcomes the weight loss.

As computed for the case study of the Warren truss, the sensitivity of

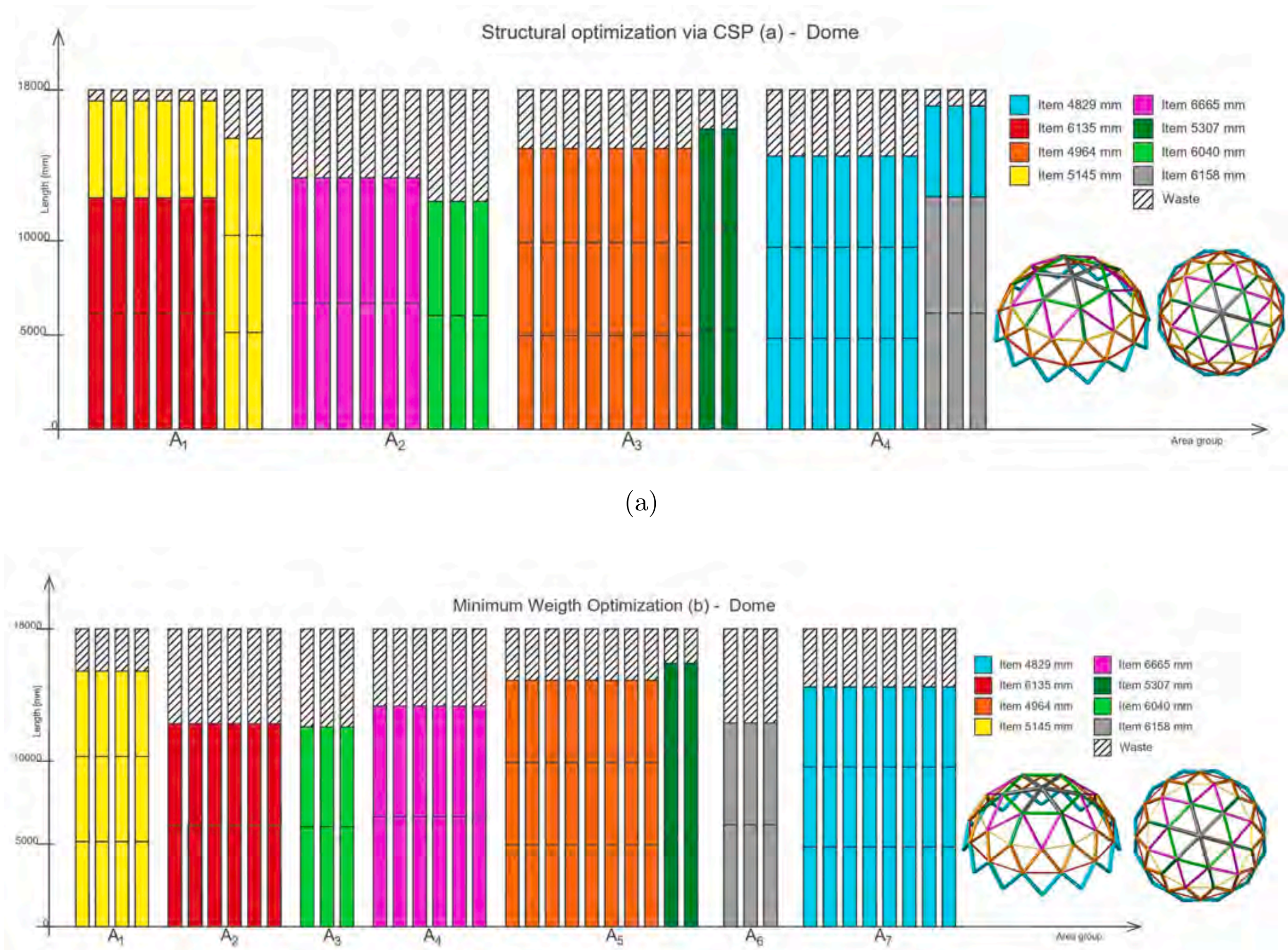

Fig. 14. Optimal cutting pattern obtained by (a) the CSP approach and (b) the minimum-weight approach for the trussed dome. The dashed area represents the remaining bin's length deriving from the cutting process. The same colors are adopted to identify the relationship between the items (i.e. cutting patterns) and their position within the structure.

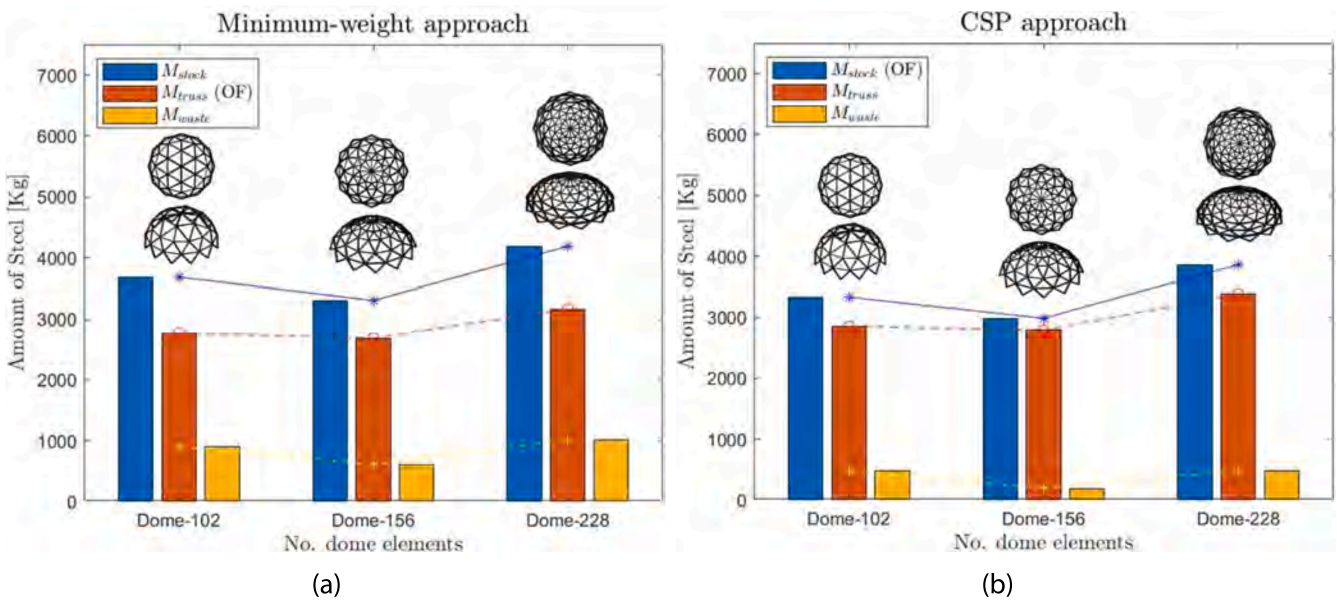

Fig. 15. Output of the optimization process following (a) the traditional minimum-weight approach and (b) the CSP approach.

the optimization problem with respect to the total number of elements has been assessed. To achieve this goal, a parametric analysis has been performed by varying the level of mesh refinement of the dome when geometry (i.e. height and radius of the dome) is fixed. For each case study, both the optimization approach has been performed and the results have been depicted in Fig. 15 (a)-(b).

In both scenarios, the optimal solution has been obtained for the dome with 156 no. of elements. Specifically, the waste obtained by the minimum-weight approach is almost 1 ton while it is negligible for the CSP approach.

Generally, with the exception of the dome with 228 no. of elements, the discrepancy between the mass of the dome calculated by both approaches remains still contained. On the other hand, the difference in terms of mass of the stock among the two approaches still remains constant to 0.5 tons for each case study.

9. Conclusions and future developments

This paper introduced an optimization procedure to minimize structural waste. The minimum amount of material stock produced by the factory has been adopted as the OF of the optimization problem. The feasibility of the procedure has been tested and the convenience of considering constructability issues as cutting patterns during the production phase has been verified by comparing it with the results obtained by the common minimum-weight approach. The strategy to adopt the cutting stock procedure as an internal routine embedded in a well-known optimization algorithm, such as the Genetic Algorithm, allows to achieve a significant reduction of the number of bins with a negligible increase in the structural weight for each of the investigated case studies. More in detail, the main outcomes of the research and the results obtained from each scenario can be summarized as follows:

- The benchmark test of the 10-bar truss has been adopted as a preliminary test for checking the goodness of the optimization approach. Though the limited number of members composing the structure and low potential combinatorial solutions, the proposed approach reveals a significant mass saving of the material stock, M_{stock}, with a negligible increase of the truss mass, M_{truss}, if compared with the weight minimization approach;
- Similar outcomes have also been observed for the Warren truss case study where the importance of an optimized grouping strategy for achieving minimum stock material has been observed. By limiting the number of different sections, the optimizer is guided to allocate more items into the same bin resulting in a reduction of M_{stock} and material waste, M_{waste}. On the contrary, the problem formulation of the minimum-weight approach naturally leads to a huge diversity of the members' cross-sectional properties due to the members' area refinement for achieving the optimal sizing of the structure (i.e. least weight). Finally, the parametric analysis by varying the number of spans (i.e. number of elements) of the Warren truss allows the identification of the best solution in terms of optimal sizing and overall layout of the truss. A global minimum of M_{stock} as well as M_{waste} has been observed for Warren with no. of span equal to 10;
- An application of a real-world reticular dome has been provided. In this case study, the efficiency of the proposed approach has been proved again. Even if the discrepancy between the M_{stock} obtained with the proposed approach and the one derived by the minimum weight approach is less marked, the reduction in terms of M_{waste} with respect to the total mass of the dome and total amount of stock remains significant. As for the previous case study, a sensitivity analysis has been performed by varying the number of elements composing the mesh of the dome. The dome with 156 no. of elements represents the best design for both approaches. With specific regard to the CSP approach, it efficiently reduces the total mass of the stock resulting in a negligible cutting waste.

In future works, several improvements could be introduced by adding constructability criteria, such as the total number of pieces employed for the minimization of the structural complexity during the construction process. LCA analysis could be also integrated into the optimization process considering environmental parameters for the sustainability assessment of the design.

Limitations of the current work, mainly related to the investigated application case studies, will be overcome by considering different types of structures like steel-concrete mixed bridges and/or box bridges, where the sensitivity of the optimal design, varying the connection typologies (e.g. welded/bolded), could be investigated. Additionally, based on the promising approach obtained by the current research, the limitation deriving from solving a single-objective size optimization problem could be overcome by including changes in the layout arrangement through the nodal coordinates relaxation method. Including such a new level of complexity will allow to address the potentiality of the proposed method with respect to the traditional minimum-weight approach.

CRediT authorship contribution statement

Raffaele Cucuzza: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Majid Movahedi Rad:** Writing – review & editing, Writing – original draft, Visualization, Validation, Data curation. **Marco Domaneschi:** Writing – review & editing, Writing – original draft, Supervision, Software, Investigation. **Giuseppe C. Marano:** Visualization, Validation, Supervision, Methodology, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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Appendix A

In this section, the results obtained from the optimization conducted for scenario (a) and scenario (b) have been reported. The optimal cutting and corresponding steel waste produced via CSP and without it as well as the values of cross-section areas have been provided for all the 10 runs of each case study. Optimal results have been highlighted in green.

A.1. Case No.1: 10 Bar-truss

Table A.13
Structural Optimization via CSP results of 10 runs for 10 bar truss - Scenario (a).

Attempt	Cross-sectional areas [in^2]	Number of bins	Structural Mass [lb]	Stock Mass [lb]	Waste Mass [lb]
1	[30.0 1.99 22.0 22.0 1.99 4.18 13.9 13.9 30.0 4.18]	[1, 1, 1, 1, 1]	6113.27	7351.14	1237.87
2	[18.8 4.22 26.5 18.8 1.8 1.8 26.5 22.0 22.0 4.22]	[1, 1, 1, 1, 1]	6393.24	7478.64	1085.40
3	[30.0 1.8 30.0 13.5 1.8 1.62 13.5 22.0 22.0 1.62]	[1, 1, 1, 1, 1]	5843.82	7029.84	1186.02
4	[22.0 1.62 22.0 22.0 1.99 1.62 22.9 22.0 22.9 1.99]	[2, 1, 1, 1]	6117.41	7192.02	1074.61
5	[30.0 1.99 30.0 13.5 1.99 1.99 13.5 22.0 22.0 1.99]	[1, 2, 1, 1]	5889.66	7086.96	1197.30
6	[30.0 1.62 22.0 11.5 1.62 1.8 11.5 22.0 30.0 1.8]	[1, 1, 1, 1, 1]	5791.97	6825.84	1033.87
7	[30.0 3.13 22.9 15.5 3.13 2.88 15.5 30.0 22.9 2.88]	[1, 1, 1, 1, 1]	6420.43	7589.82	1169.39
8	[26.5 4.49 26.5 13.9 4.49 15.5 15.5 16.9 16.9 13.9]	[1, 1, 1, 1, 1]	6507.30	7883.58	1376.28
9	[26.5 3.87 26.5 13.9 2.88 2.88 26.5 26.5 13.9 3.87]	[2, 1, 1, 1]	6358.10	7512.30	1154.20
10	[26.5 1.99 22.0 18.8 1.99 1.8 26.5 18.8 22.0 1.8]	[1, 1, 1, 1, 1]	6148.88	7251.18	1102.30
		μ	6158.41	7320.13	1161.72
		σ	257.05	308.64	98.37

Table A.14
Traditional Structural Optimization results of 10 runs for 10 bar truss - Scenario (b).

Attempt	Cross-sectional areas [in^2]	Number of bins	Structural Mass [lb]	Stock Mass [lb]	Waste Mass [lb]
1	[30. 1.8 26.5 14.2 1.62 1.62 13.5 18.8 22. 1.62]	[1 1 1 1 2 1 1 1]	5573.62	13264.08	7690.46
2	[30. 1.8 26.5 16. 1.62 1.62 11.5 18.8 22. 1.8]	[1 1 1 1 1 1 1 1]	5580.42	10323.42	4743.00
3	[22.9 1.62 30. 13.9 1.62 1.62 15.5 22. 22.9 1.8]	[1 2 1 1 1 1 1]	5746.47	11152.68	5406.21
4	[30. 1.62 26.5 11.5 1.62 1.62 7.97 26.5 22. 1.62]	[1 2 1 1 1 1]	5545.76	13078.44	7532.68
5	[30. 3.38 26.5 13.5 1.62 1.8 15.5 16.9 22. 3.47]	[1 1 1 1 1 1 1 1 1 1]	5711.06	13736.34	8025.28
6	[26.5 1.62 22.9 15.5 1.62 1.62 13.5 22.9 22.9 1.62]	[1 2 2 1 1]	5612.90	10663.08	5050.18
7	[30. 1.62 26.5 13.9 1.62 1.62 13.5 22. 18.8 2.38]	[1 2 1 1 1 1 1 1]	5595.03	13292.64	7697.61
8	[26.5 1.8 30. 14.2 1.62 1.62 13.9 22.9 18.8 1.99]	[1 1 1 1 1 1 1 1 1]	5658.64	13434.42	7775.78
9	[30. 1.8 26.5 15.5 1.8 2.13 16. 18.8 18.8 2.88]	[1 1 1 1 1 1 1 1]	5673.77	11588.22	5914.45
10	[26.5 2.13 26.5 16. 1.62 1.8 14.2 18.8 22.9 1.99]	[1 1 1 1 1 1 1 1 1]	5631.08	10805.88	5174.80
		μ	5632.89	12.133.92	6501.04
		σ	64.17	1343.16	1348.10

A.2. Case No.2: Warren Truss

Table A.15
Structural Optimization via CSP results of 10 runs for Warren truss - Scenario (a).

Attempt	Cross-sectional areas $[mm^2]$	Number of bins	Structural Mass [kg]	Stock Mass [kg]	Waste Mass [kg]
1	[4210, 4210, 4210, 5310, 5310, 5310 1370, 1370, 1370, 1370, 1370, 5310]	[2, 2, 3]	2236.80	2725.91	489.11
2	[4970, 4970, 4970, 5280, 5280, 5280 906, 906, 906, 906, 906, 4970]	[2, 2, 3]	2234.99	2733.92	498.93
3	[4210, 4210, 4210, 5940, 5940, 5940 1120, 1120, 1120, 1120, 1120, 5940]	[2, 2, 3]	2294.20	2785.96	491.77
4	[4500, 4500, 4500, 4710, 4710, 4710 2060, 2060, 2060, 2060, 2060, 4710]	[2, 2, 3]	2361.53	2896.65	535.12
5	[4030, 4030, 4030, 6120, 6120, 6120 906, 906, 906, 906, 906, 4030]	[2, 2, 3]	2163.95	2710.37	546.42
6	[3710, 3710, 3710, 5890, 5890, 5890, 1120, 1120, 1120, 1120, 1120, 5890]	[2, 2, 3]	2189.96	2656.44	466.48
7	[4210, 4210, 4210, 6120, 6120, 6120 680, 680, 680, 680, 680, 6120]	[2, 2, 3]	2208.85	2672.93	464.08
8	[4500, 4500, 4500, 5990, 5990, 5990 1390, 1390, 1390, 1390, 1390, 4500]	[2, 2, 3]	2381.09	2961.41	580.32
9	[4020, 4020, 4020, 5890, 5890, 5890 1120, 1120, 1120, 1120, 1120, 5890]	[2, 2, 3]	2248.37	2729.44	481.08
10	[5280, 5280, 5280, 4670, 4670, 4670 1370, 1370, 1370, 1370, 1370, 5280]	[2, 2, 3]	2331.39	2827.18	495.79
		μ	5632.89	12.133.92	6501.04
		σ	64.17	1343.16	1348.10

Table A.16
Traditional Structural Optimization results of 10 runs for Warren truss - Scenario (b).

Attempt	Cross-sectional areas $[mm^2]$	Number of bins	Structural Mass [kg]	Stock Mass [kg]	Waste Mass [kg]
1	[3710, 8110, 4710, 7370, 8770, 12500, 506, 2140, 360, 906, 1070, 1630]	[1 1 1 1 1 1 1 1 1 1 1 1]	2238.33	5097.33	2859.33
2	[4210, 7370, 3310, 6910, 4500, 6120, 1540, 5770, 1120, 1660, 360, 10000]	[1 1 1 1 1 1 1 1 1 1 1 1]	2300.97	5225.44	2924.47
3	[2640, 4030, 7940, 5010, 13000, 8110, 578, 820, 679, 1560, 906, 2960]	[1 1 1 1 1 1 1 1 1 1 1 1]	2048.27	4879.44	2831.17
3	[2640, 4030, 7940, 5010, 13000, 8110, 578, 820, 679, 1560, 906, 2960]	[1 1 1 1 1 1 1 1 1 1 1 1]	2048.27	4879.44	2831.17
5	[2060, 5990, 4670, 7920, 8740, 15500, 965, 238, 454, 680, 574, 5890]	[1 1 1 1 1 1 1 1 1 1 1 1]	1904.00	4628.47	2724.47
6	[2060, 5990, 4670, 7920, 8740, 15500, 965, 238, 454, 680, 574, 5890]	[1 1 1 1 1 1 1 1 1 1 1 1]	1904.00	4628.47	2724.47
7	[2060, 5990, 4670, 7920, 8740, 15500, 965, 238, 454, 680, 574, 5890]	[1 1 1 1 1 1 1 1 1 1 1 1]	1904.00	4628.47	2724.47
8	[2060, 5990, 4670, 7720, 8740, 15500, 965, 238, 454, 680, 574, 5890]	[1 1 1 1 1 1 1 2 1 1]	1864.7	4610.62	2745.9
9	[2640, 4030, 7940, 5010, 13000, 8110, 578, 820, 679, 1560, 906, 2960]	[1 1 1 1 1 1 1 1 1 1 1 1]	2048.27	4879.44	2831.17
10	[3710, 8110, 4710, 7370, 8770, 12500, 506, 2140, 360, 906, 1070, 1630]	[1 1 1 1 1 1 1 1 1 1 1 1]	2238.33	5097.33	2859.33
		μ	2049.91	4855.46	2805.60
		σ	160.63	228.55	70.85

A.3. Case No. 3: Spatial Reticular Dome

Table A.17
Structural Optimization via CSP results of 10 runs for reticular Dome - Scenario (a).

Attempt	Cross-sectional areas $[mm^2]$	Number of bins	Structural Mass [kg]	Stock Mass [kg]	Waste Mass [kg]
1	$[A_1 = 307,\ A_2 = 574,\ A_3 = 733,\ A_4 = 862]$	[8, 9, 10, 10]	2851.1	3329.7	478.4
2	$[A_1 = 357,\ A_2 = 574,\ A_3 = 733,\ A_4 = 862]$	[8, 9, 10, 10]	2871.1	3429.7	558.4
3	$[A_1 = 307,\ A_2 = 574,\ A_3 = 733,\ A_4 = 862]$	[8, 9, 10, 10]	2851.1	3329.7	478.4
4	$[A_1 = 357,\ A_2 = 574,\ A_3 = 733,\ A_4 = 862]$	[8, 9, 10, 10]	2871.1	3429.7	558.4
5	$[A_1 = 307,\ A_2 = 574,\ A_3 = 733,\ A_4 = 862]$	[8, 9, 10, 10]	2851.1	3329.7	478.4
6	$[A_1 = 307,\ A_2 = 574,\ A_3 = 733,\ A_4 = 862]$	[8, 9, 10, 10]	2851.1	3329.7	478.4
7	$[A_1 = 307,\ A_2 = 574,\ A_3 = 733,\ A_4 = 862]$	[8, 9, 10, 10]	2851.1	3329.7	478.4
8	$[A_1 = 357,\ A_2 = 574,\ A_3 = 733,\ A_4 = 862]$	[8, 9, 10, 10]	2871.1	3429.7	558.4
9	$[A_1 = 357,\ A_2 = 587,\ A_3 = 733,\ A_4 = 862]$	[8, 9, 10, 10]	3071.1	3529.7	458.6
10	$[A_1 = 307,\ A_2 = 574,\ A_3 = 733,\ A_4 = 862]$	[8, 9, 10, 10]	2851.1	3329.7	478.4
		μ	2879.1	3379.7	500.42
		σ	68.12	70.71	40.47

Table A.18
Traditional Structural Optimization results of 10 runs for reticular dome - Scenario (b).

Attempt	Cross-sectional areas $[mm^2]$	Number of bins	Structural Mass [kg]	Stock Mass [kg]	Waste Mass [kg]
1	$[A_1 = 182,\ A_2 = 307,\ A_3 = 574,\ A_4 = 707,\ A_5 = 733,\ A_6 = 862,\ A_7 = 906]$	[4, 5, 3, 6, 10, 3, 8]	2773.6	3683.2	909.5
2	$[A_1 = 182,\ A_2 = 307,\ A_3 = 574,\ A_4 = 707,\ A_5 = 733,\ A_6 = 862,\ A_7 = 906]$	[4, 5, 3, 6, 10, 3, 8]	2773.6	3683.2	909.5
3	$[A_1 = 182,\ A_2 = 307,\ A_3 = 574,\ A_4 = 707,\ A_5 = 733,\ A_6 = 862,\ A_7 = 906]$	[4, 5, 3, 6, 10, 3, 8]	2773.6	3683.2	909.5
3	$[A_1 = 182,\ A_2 = 307,\ A_3 = 574,\ A_4 = 707,\ A_5 = 733,\ A_6 = 862,\ A_7 = 906]$	[4, 5, 3, 6, 10, 3, 8]	2773.6	3683.2	909.5
5	$[A_1 = 197,\ A_2 = 307,\ A_3 = 574,\ A_4 = 707,\ A_5 = 733,\ A_6 = 862,\ A_7 = 906]$	[4, 5, 3, 6, 10, 3, 8]	2873.6	3783.2	1009.5
6	$[A_1 = 197,\ A_2 = 307,\ A_3 = 574,\ A_4 = 707,\ A_5 = 733,\ A_6 = 862,\ A_7 = 906]$	[4, 5, 3, 6, 10, 3, 8]	2873.6	3783.2	1009.5
7	$[A_1 = 197,\ A_2 = 357,\ A_3 = 574,\ A_4 = 707,\ A_5 = 733,\ A_6 = 862,\ A_7 = 906]$	[4, 5, 3, 6, 10, 3, 8]	3073.6	3983.2	910.4
8	$[A_1 = 182,\ A_2 = 307,\ A_3 = 574,\ A_4 = 707,\ A_5 = 733,\ A_6 = 862,\ A_7 = 906]$	[4, 5, 3, 6, 10, 3, 8]	2773.6	3683.2	909.5
9	$[A_1 = 182,\ A_2 = 307,\ A_3 = 574,\ A_4 = 707,\ A_5 = 733,\ A_6 = 862,\ A_7 = 906]$	[4, 5, 3, 6, 10, 3, 8]	2773.6	3683.2	909.5
10	$[A_1 = 182,\ A_2 = 307,\ A_3 = 574,\ A_4 = 707,\ A_5 = 733,\ A_6 = 862,\ A_7 = 906]$	[4, 5, 3, 6, 10, 3, 8]	2773.6	3683.2	909.5
		μ	2823.6	3733.2	929.59
		σ	97.18	97.18	42.12

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