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Adaptative Hyperparameters Selection for Modal Tracking Algorithm in Structural Health Monitoring of Masonry Monumental Structures

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Abstract. In the context of Structural Health Monitoring (SHM), a key step in the monitoring process is the extraction of dynamic features of the system. Once the parameters have been estimated through dynamic identification techniques, the following step, as well as one of the most challenging, consists in progressively assigning the identified modal properties of the different identification tasks performed over time to a particular vibration mode. This last procedure in the framework of SHM is referred to as Mode Tracking. It is commonly performed by the comparison between the estimated natural frequencies and associated mode shapes to the modal properties of a set of reference modes. This process could be easier and immediate in the case of simple structures such as multistorey structural systems, while in the case of monumental masonry buildings some more difficulties arise. These difficulties may be due to multiple factors including the structural complexity, the number of available sensors that make up the monitoring system, the possible overlap between the ranges of natural frequency values and the presence of operational and environmental variation. For these reasons, it could be very complicated to be able to track frequencies manually. In fact, if the latter could still work in the case of many sensors and frequency ranges that are not close together, in the case of a few sensors and overlapping ranges, this method would fail, leading to unsatisfactory results. In this study, an adaptative method to obtain the correct parameters for mode tracking is presented. It is applied to a case study of masonry monumental building equipped with long-term dynamic monitoring system.

Keywords: Structural Health Monitoring, Long-Term Monitoring, Dynamic Monitoring, Masonry Monumental Structures, Modal Tracking

1 Introduction

In recent years, Structural Health Monitoring (SHM) techniques have taken on a fundamental role in the observation and protection of structures and infrastructures. Ambitious infrastructures or historical buildings considered fragile are advantageous targets of SHM procedures that allow their evolving conditions to be analysed, with the

aim of intercepting any damage and degenerative processes early and, therefore, promptly implementing protective measures and interventions [1–3]. In particular, in the field of architectural heritage, these techniques are appreciated and sought after for their non-invasive and reversible nature. Indeed, they allow to monitor a structure without huge and extensive operations; thus, they are in full agreement with the principle of minimum intervention, such as that introduced in the guidelines of the International Council on Monuments and Sites (ICOMOS), [4].

In the context of SHM, the concept of damage state plays a central role. A structure is defined as "damaged" when, although it can still work satisfactorily and safely, it no longer performs in its ideal condition, configuring a non-optimal situation [5–7]. In order to evaluate the state of a structure and, therefore, a possible onset of damage, it is necessary to identify diagnostic parameters [8] that depend on the health state of the structure, such as the natural frequencies and the vibration modes of the system. Since the frequencies are related to the stiffness of the structure, this allows to have a general overview of the health state as a variation in the frequencies could be a symptom of other hidden problems. SHM data-driven methods are usually applied in the case of data coming from in situ long-term or permanent monitoring systems since a lot of samples are available to implement a reliable and deepened damage detection process. However, these types of data can be characterized by variations triggered by changes in the external environment or by noise, these should be isolated and removed in order to evaluate the actual health state without confounding effects [9, 10]. An important problem in the case of structural monitoring with data drive methods lies in the tracking of the frequency historical series. It is referred to as Mode Tracking (MT) problem and it consists in connecting the results of different acquisitions, i.e. associating each identified mode with those previously identified, building step by step well-separated time series of frequencies. This process is important because in the context of dynamic monitoring the diagnosis of a structure is not based only on the absolute values of the parameters obtained. Indeed, it cannot be said whether a structure is healthy or not based on its frequencies obtained from one-time acquisition, but rather on an established comparison between the data acquired at the moment in which the structure is considered healthy and all the data acquired subsequently which should be to compose the time series. In the literature, there are some examples of MT that present different methodologies applied for tracking the time series of frequency [9, 11–13]. Surely, the most immediate strategy for carrying out MT is to rely on a similarity parameter between modes, for example the Modal Assurance Criterion (MAC), and associate the modes that present the greatest value, beyond a certain minimum threshold. However, in particular cases of complex structures and structures with modes that are difficult to identify, the MT process based on manual operations or process with selection of only one parameter can be not satisfactory; indeed, it could lead to wrong results. For example, when using the Modal Assurance Criterion (MAC) to associate the modes that present the greatest value of this parameter, beyond a certain minimum threshold, difficulties

could arise because, considering that not all identifications contain the same vibration modes (especially in Operational Modal Analysis, OMA, where environmental arousal may not amplify all modalities, making some difficult to identify), this strategy could lead to many incorrect associations. Thus, there are many cases where the manual procedure fails and it is necessary to implement some algorithms that allow to find the right parameters (not one, but multiple parameters) through an adaptive process.
The paper is divided as follows: in Section 2, the problem is presented in order to contextualize it in the field of SHM; Section 3 shows the different methods used; then, in Section 4, the tracking procedure is applied to the case of the Church of Santa Caterina and in this section the structure and its long-term monitoring system are shown; Section 5 presents the result of the methods and a comparison between them. Finally, the conclusions are illustrated in Section 7, together with the problems encountered in the applied procedure and future developments.

2 Background

In the context of SHM, especially in the case of the analysis of frequencies through the detection of vibrations of a structure (Vibration Based Structural Health Monitoring, VB-SHM), an important step is to attribute the result of each identification over time to a specific mode. This step is consequent to the acquisition of data, pre-processing phase and identification phase that can be carried out with different algorithms such as Stochastic Subspace Identification (SSI) [14]. Thus, once the dynamic parameters of the structure are estimated through automated Operational Modal Analysis (OMA) approaches, the MT process allows to progressively assign the identified modal properties of different identification tasks performed over time to previously estimated vibration modes, also due to the presence of Environmental and Operational Variations (EOVs) which could affect the structural response of the building and consequently the recorded signals. The MT is related to the concept that during the phase of identification, it is not clear which mode each identified frequency belongs to. Therefore, the usefulness of MT lies precisely in being able to couple each frequency to its corresponding mode (mode coupling) and also understand which are the real modes of the structure and which are spurious modes. The MT is commonly achieved by comparing estimated natural frequencies and their associated mode shapes with the modal properties of a set of reference modes [9]. The MT process could be immediate and easy in the case of simple structures and materials, such as steel bridges or even concrete bridges. However, in the case of complex and masonry structures the situation could become very complicated. In these cases, increasing the number of sensors on the structure could represent a simplification to the problem, however this does not represent an exhaustive solution. Indeed, the procedure often needs to be adapted to the single case. It would be possible to opt for a manual choice of thresholds and rely on a single parameter such as

the MAC; or implement an automatic process based on one or more parameters, for example MAC and frequencies. Some examples of MT can be found in [12, 15, 16].
In this study, a comparison between two different types of selection of parameters for MT is shown. Both methods used are automatic, but they use different parameters to establish which values to keep and which to reject. They are based on the MAC error (MAC_{error}) and Frequency error (Fre_{err}), however the thresholds for these parameters are defined through different conditions. They are better explained in the following sections. The methods illustrated in the paper are applied to the Church of Santa Caterina in Casale Monferrato in Italy, a monumental architecture instrumented by a long-term dynamic monitoring system.

3 Methods

In order to carry out the coupling between frequencies and modes, the first step of MT is to define a baseline of modal parameters, which represent the reference mode to be based on. This allows to understand each frequency identified which mode of the structure it corresponds to. The reference mode can be defined in different ways:

- It can be obtained from an identification task in which most of the structural modes are identified.
- It can be derived from one-time monitoring.
- It can be defined as mean value of the frequency of each mode (or cluster).

Once the reference mode has been chosen, the objective of MT is to define the parameters with which to establish thresholds that allow frequencies to be attributed to its modes. When dealing with complex structures such as masonry buildings or masonry monumental architecture, the use of multiple parameters to track the time series of frequencies may be more efficient. Moreover, the use of automatic and adaptive procedures can be useful to find the optimal values of the parameters in order to track the correct trend of the frequencies of the structure over time. In this paper, the parameters used are MAC_{error} and Fre_{err}. The automatic procedure consists of a calibration of the threshold parameters based on an optimization algorithm that aims to optimize different quantities. In the following sections, two different methods for MT are illustrated.

3.1 First method for automatic procedure of Modal Tracking

In the first case, the reference mode is chosen among the identifications of the two experimental campaign of 2022 and 2023, in particular a identification of December 22, 2022 was chosen. In this case, in order to define the best threshold of MAC_{error} and Fre_{err} the algorithm has as main goal the following points:

1. Minimize the Standard Deviation (σ);
2. Minimize the number of missing value (NaN).

These allow to exclude values far from the mean values and in the same time it guarantees the least loss of data. However, some problems may arise when trying to trace the modes of the structure and in particular when trying to trace them simultaneously. Indeed, if a mode by its nature has a high variance that does not fall within the ranges selected by the algorithm, in that case the frequency values are rejected even though they are real values. For this reason it has been necessary to search other methods that allow to track the modes without loss of data.

3.2 Second method for automatic procedure of Modal Tracking

In the second case, the reference mode has been chosen as the mean value of modal characteristics of each cluster for each mode; indeed, the MT phase was preceded by a manual cluster phase. The automatic procedure consists of a calibration of the threshold parameters based on an optimization algorithm; the objective of the algorithm is to find the minimum optimal value x based on a pattern search method [17]. The optimization algorithm has the following goals:

I. Minimize the number of modes, i.e. the most frequent value in a set of data, so the peaks of the Probability Distribution Function (PDF) [18];
II. Minimize the number of missing value (NaN).

Thus, as first step a supervised classification analysis was carried out in order to realize a preliminary manual clustering. In this phase all the identified mode shapes are premultiplied for their associated frequency to get a final space representation that gives information about the value of the identified frequency and simultaneously highlights the main direction of motion of the mode analysed (e.g., mainly longitudinal, or mainly transversal). Thus, it is also possible to define the reference mode as mean value of the cluster. Following the optimization procedure, the signals obtained could be characterized by the presence of outliers. For this reason, it could be necessary to carry out an *Outlier Rejection (OR)* process that consists in a manual definition of thresholds for the computation of MAC_{error} and Fre_{err}. Once these margin points are marked, the algorithm was run again and this led to having more populated time series as the definition of the thresholds allows data loss to be reduced.

4 Applications: The Church of Santa Caterina in Casale Monferrato (AL)



(a) (b)

Fig. 1. The Church of Santa Caterina in Casale Monferrato (AL)

The Church of Santa Caterina located in Casale Monferrato was built in the first half of the 18th century and it is one of the most significant manifestations of the Baroque in Piedmont and in particular in Casale Monferrato. The church is part of the architectural ensemble that includes the church itself and the choir attached to it in the part behind the presbytery. The structure is characterized by a Greek cross plan and the central hall is covered by an elliptical dome set on an eight-segment windowed drum. Above the dome, a lantern was created with eight windows surmounted by an elliptical dome. The main façade, facing Piazza Castello, has an overall vertical development of approximately 19 metres, however, starting from the level of the drum shutter which is located at 13 m, the façade continues cantilevered into the tympanum for approximately 6 metres. The drum is located at a height of 13 meters and rises approximately 7 metres; it has an elliptical plan whose major axis is approximately 14 meters long and the minor one approximately 10 meters. The external surface is marked by eight pilasters supporting the dome-lantern system. The oval-shaped dome is placed on the drum and is approximately 5 meters tall. There are eight ribs that connect to the drum pilasters, the ribs are interspersed with masonry levels. The dome is covered with a thin layer of copper plates directly fixed to the external masonry.

4.1 Data of dynamic long-term monitoring system

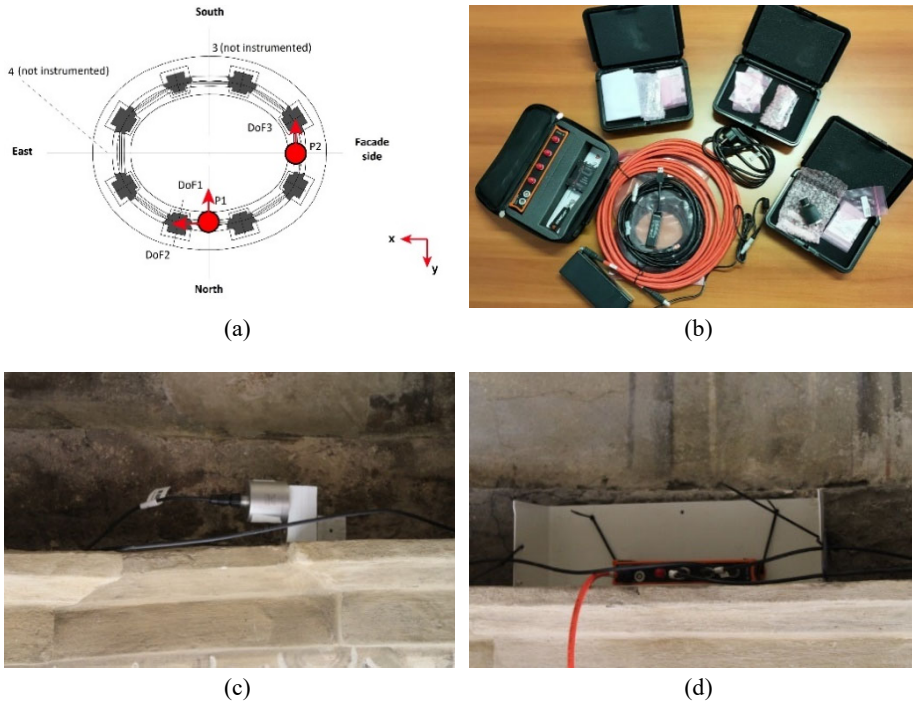


Fig. 2. Long-term monitoring system of the Church of Santa Caterina: (a) Sensors layout,(b) Components of monitoring system,(c) Accelerometer and (d) Acquisition system

The experimental setup of the monitoring system was characterized by three dynamic sensors, i.e. accelerometers (high sensitivity seismic accelerometer, ceramic shear ICP® 393B12 model, 10 V/g, 0.15 to 1k Hz, 2-pin top connection) located on top of the lantern in order to capture the diaphragmatic vibrations of the top of this structural component. Two accelerometers were placed to record the accelerations in the transversal direction and the other one was installed to acquire the acceleration in the longitudinal direction of the lantern.

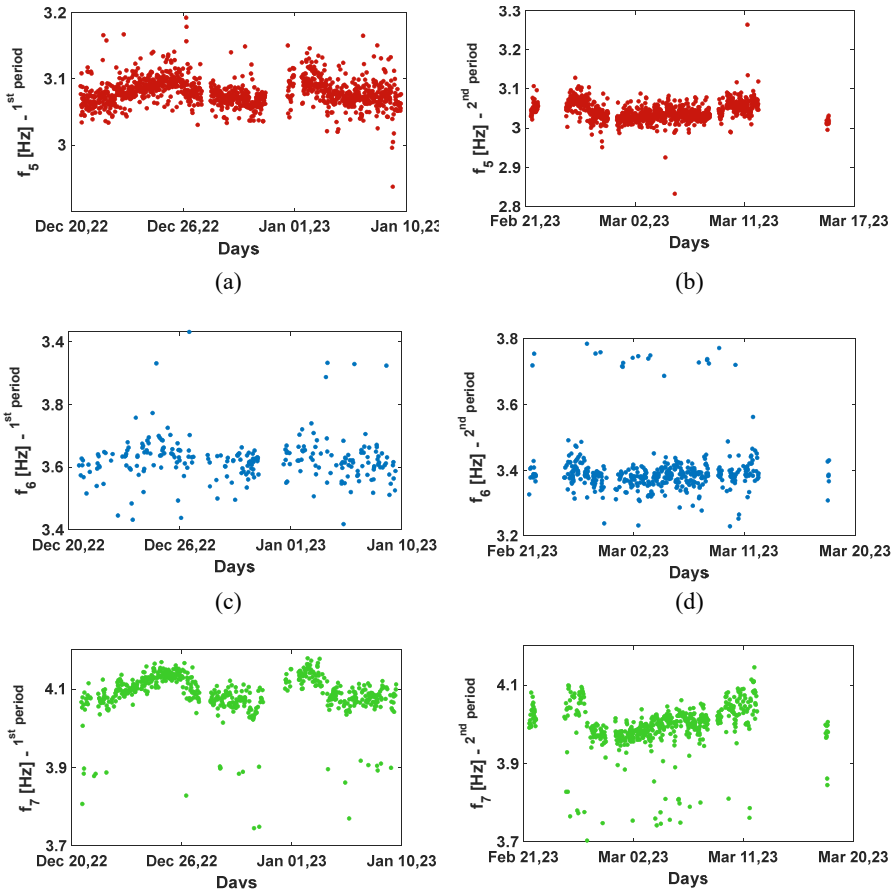
4.2 Analysis with first method

As reported in section 3.1, in the case of the first method the MT procedure involves the definition of thresholds of MAC_{err} and $Freq_{err}$ based on the σ values and number of NaNs. Thus, having found the minimum values of σ and the minimum number of NaNs, the algorithm returns the values of Frequency Weight (Wei_{Fre}), MAC_{err} and $Freq_{err}$ and on the basis of these it traces the historical series of frequency. The following figure shows the results of the first analysis for which it was possible to obtain

the first six modes of the structure. Note that the modes are indicated starting from the fifth as the first four modes do not appear to be of the structure.

Table 1. Values of MAC, MAC_{err} and $Freq_{err}$ (Running 1) with $Weig_{freq} = 1.0000$

	MAC	MAC_{err}	$Freq_{err}$
f_5	0.7475	0.2525	0.0858
f_6	0.4480	0.5520	0.1128
f_7	0.2529	0.7471	0.0846
f_8	0.8853	0.1147	0.1151
f_9	0.1664	0.8336	0.0772
f_{10}	0.1094	0.8906	0.1241



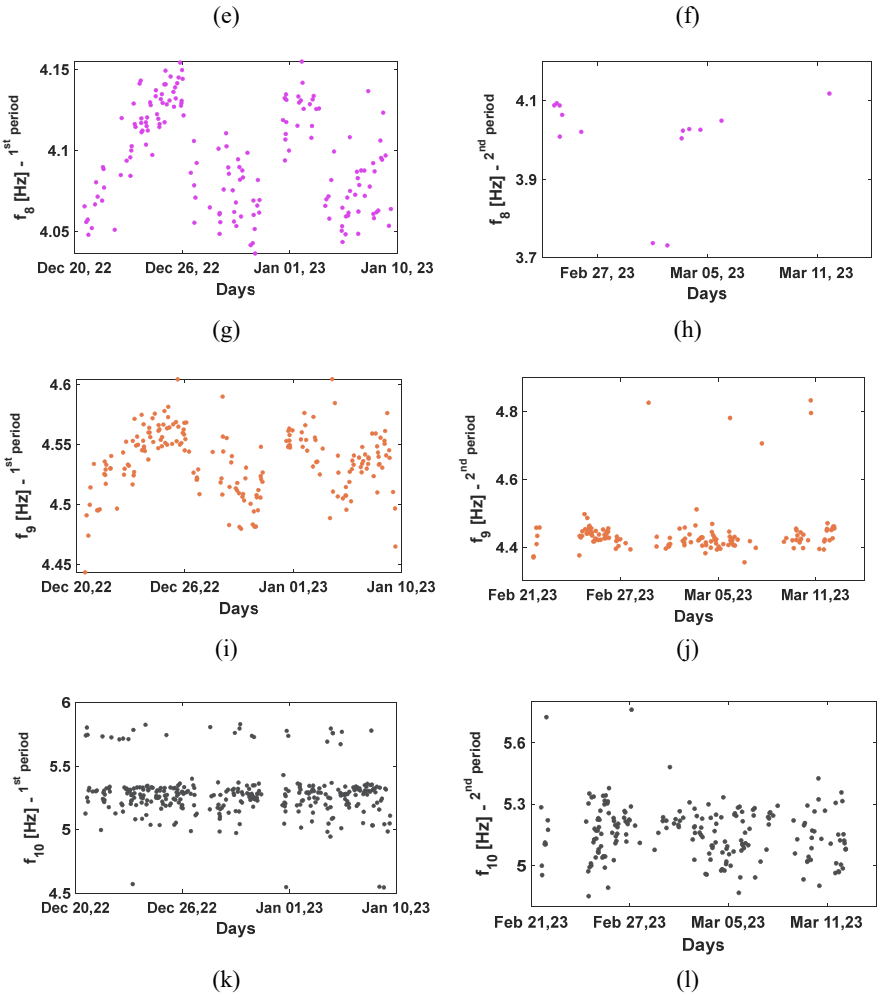
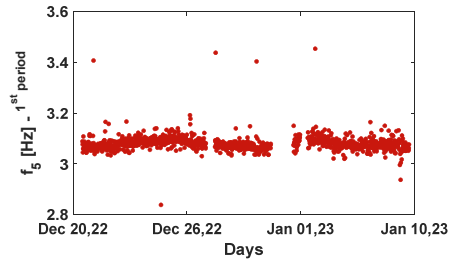


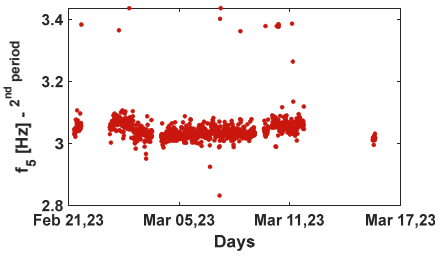
Fig. 3. Analysis with first method (Running 1): (a) Frequency 5, 1st campaign, (b) Frequency 5, 2nd campaign, (c) Frequency 6, 1st campaign (d) Frequency 6, 2nd campaign (e) Frequency 7, 1st campaign (f) Frequency 7, 2nd campaign (g) Frequency 8, 1st campaign (h) Frequency 8, 2nd campaign (i) Frequency 9, 1st campaign (j) Frequency 9, 2nd campaign (k) Frequency 10, 1st campaign (l) Frequency 10, 2nd campaign

From the analysis of the previous graphs (Fig. **3**), it is possible to note that the time series in almost all cases are characterized by the presence of many outliers due to classification problems, i.e. the algorithm is unable to correctly divide the individual modes but it confuses and therefore overlaps them. This can be observed by the presence of different and far frequency values in the graphs which represent different modes of the structure. This problem is reduced in the case of modes 5 and 9 that are the global modes of the structure, however it is very evident in the case of 6, 7 and 10.

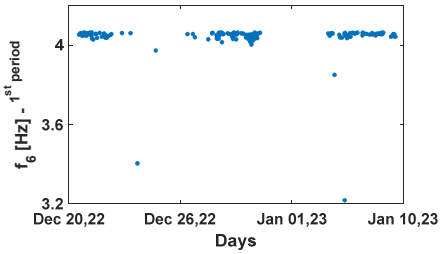
Furthermore, since $Weig_{freq}$ is equal to 1 (Table 1), that it is as if the analysis was carried out only in terms of frequency and not also in terms of MAC. Finally, from this analysis it is not possible to obtain the trend of frequency 11.
For these reasons, the analysis was repeated focusing attention on modes 5,6,8 and 9. This is possible because the algorithm allows to select the modes you want to track and study.



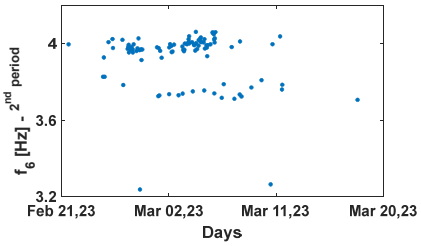
(a)



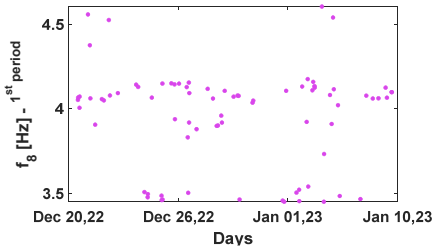
(b)



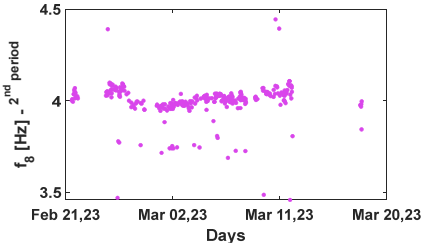
(c)



(d)



(e)



(f)

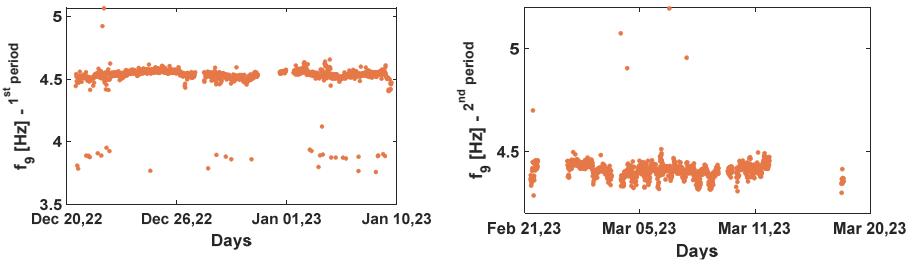
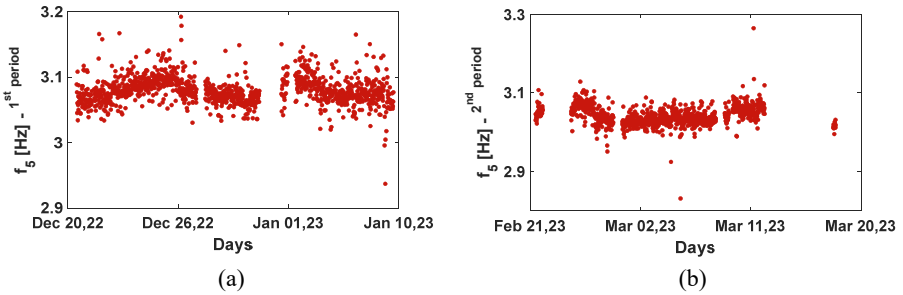


Fig. 4. Analysis with first method (Running 2): (a) Frequency 5, 1st campaign, (b) Frequency 5, 2nd campaign, (c) Frequency 6, 1st campaign (d) Frequency 6, 2nd campaign (e) Frequency 8, 1st campaign (g) Frequency 8, 2nd campaign (g) Frequency 9, 1st campaign (h) Frequency 9, 2nd campaign

Table 2. Values of MAC, MAC_{err} and $Freq_{err}$ (Running 2) with $Weig_{freq} = 0.06927$

	MAC	MAC_{err}	$Freq_{err}$
f_5	0.03324	0.9668	0.15668
f_6	0.86695	0.1330	0.17610
f_8	0.20511	0.7949	0.15113
f_9	0.13037	0.8696	0.16408

Fig. **4** shows the results of the second analysis with the first method. It possible to note that mode 5 is characterized by more concentrated values with the exception of some outliers, in the case of modes 8 and 9 they are characterized by a lower number of NaNs compared to the first analysis. However, in the case of mode 9 some classification problems emerge. For mode 6 many values are lost and the results cannot be considered acceptable. For these reasons, in the two subsequent analyses, modes 5, 8 and 9 (Fig. **5**) were selected first and then only mode 5 (Fig. 6), i.e. the global transversal mode.



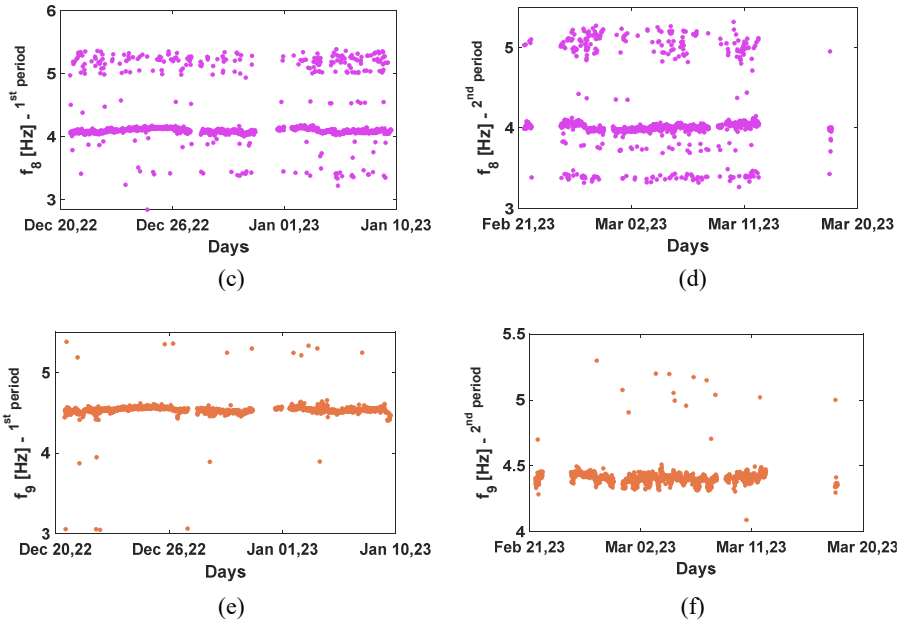
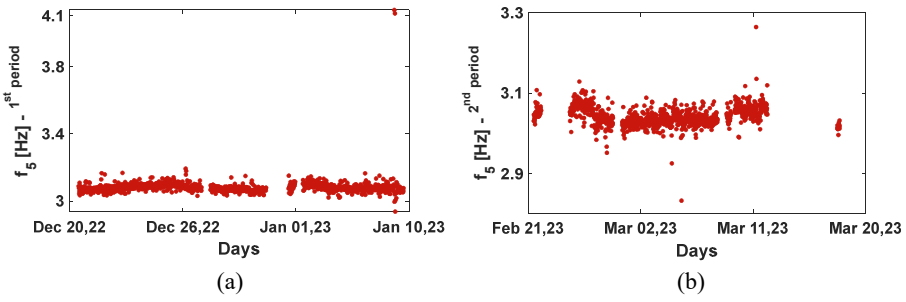
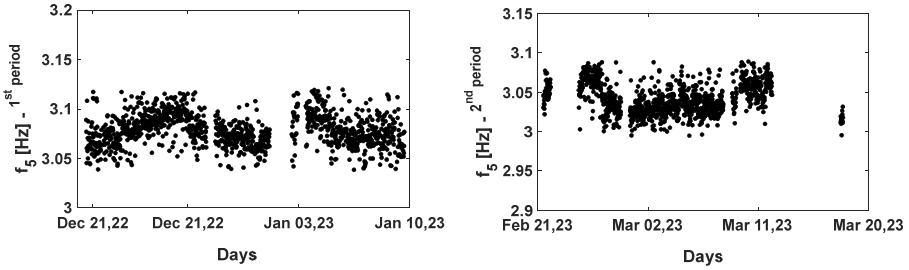


Fig. 5. Analysis with first method (Running 3): (a) Frequency 5, 1st campaign, (b) Frequency 5, 2nd campaign, (c) Frequency 8, 1st campaign (d) Frequency 8, 2nd campaign (e) Frequency 9, 1st campaign (f) Frequency 9, 2nd campaign

Table 3. Values of MAC, MAC_{err} and $Freq_{err}$ (Running 3) with $Weig_{freq} = 0.6365$

	MAC	MAC_{err}	$Freq_{err}$
f_5	0.3320	0.6680	0.3342
f_8	0.0219	0.9781	0.3313
f_9	0.3490	0.6510	0.3291





(c) (d)

Fig. 6. Analysis with first method (Running 4): (a) Frequency 5, 1st campaign, (b) Frequency 5, 2nd campaign (c) Frequency 5, 1st campaign after filtering outliers and (d) Frequency 5, 2nd campaign after filtering outliers

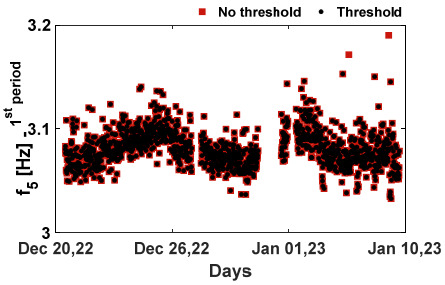
Table 4. Values of MAC, MAC_{err} and $Freq_{err}$ (Running 4)

	MAC_{err}	MAC_{err}	$Freq_{err}$	$\text{Weig}_{\text{freq}}$
f_5	0.6929	0.3071	0.4348	0.5350

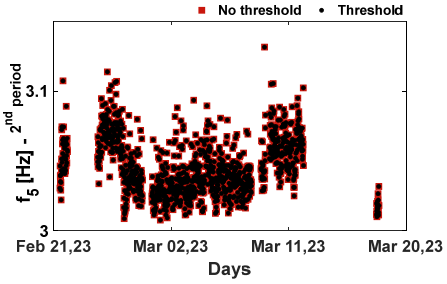
To obtain satisfactory results, a filtering of all the values considered outliers was applied and thus the trends of mode 5 represented in Fig. 6 (c-d) were obtained.

4.3 Analysis with second method

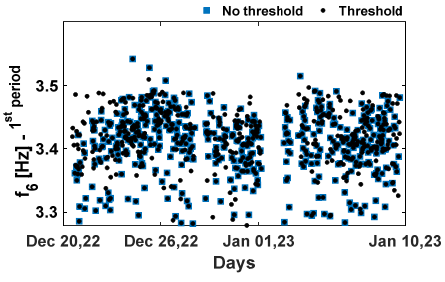
In the case of the second method, it is sufficient to carry out less analysis as the algorithm is able to analyse all modes simultaneously. Indeed, once the mode tracking has been obtained, a manual definition of the thresholds for the computation of the MAC_{err} and the $Freq_{err}$ is necessary. The definition of these thresholds is used to remove errors in the final analysis, indeed the algorithm with the insertion of these thresholds is able to trace historical series characterized by much more data and less outliers and therefore more reliable. In Fig. **7** the results are shown.
The time series indicated with the square represents the result of the first analysis, while the one with the circles represents the results of the analysis after establishing of the thresholds. In Table 5 the value of thresholds for the definition of MAC_{err} and the $Freq_{err}$ are reported.


No threshold
Threshold
3.2
3.1
3
f5 [Hz] - 1st period
Dec 20,22
Dec 26,22
Jan 01,23
Jan 10,23
Days

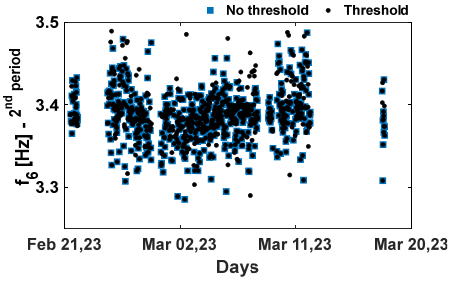
(a)


No threshold
Threshold
3.1
3
f5 [Hz] - 2nd period
Feb 21,23
Mar 02,23
Mar 11,23
Mar 20,23
Days

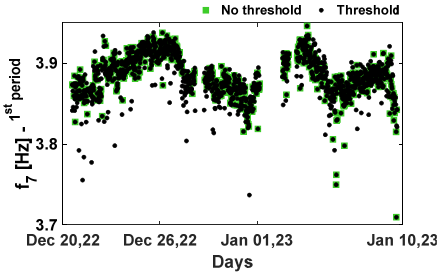
(b)


No threshold
Threshold
3.5
3.4
3.3
f6 [Hz] - 1st period
Dec 20,22
Dec 26,22
Jan 01,23
Jan 10,23
Days

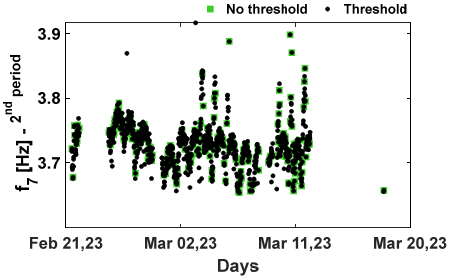
(c)


No threshold
Threshold
3.5
3.4
3.3
f6 [Hz] - 2nd period
Feb 21,23
Mar 02,23
Mar 11,23
Mar 20,23
Days

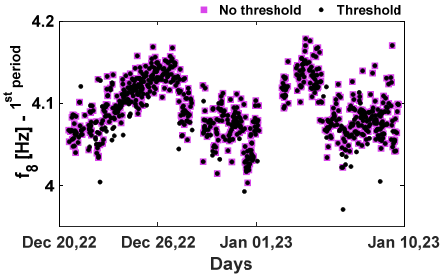
(d)


No threshold
Threshold
3.9
3.8
3.7
f7 [Hz] - 1st period
Dec 20,22
Dec 26,22
Jan 01,23
Jan 10,23
Days

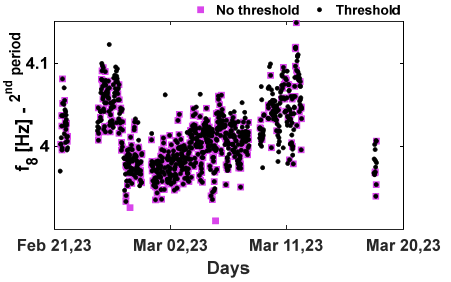
(e)


No threshold
Threshold
3.9
3.8
3.7
f7 [Hz] - 2nd period
Feb 21,23
Mar 02,23
Mar 11,23
Mar 20,23
Days

(f)


No threshold
Threshold
4.2
4.1
4
f8 [Hz] - 1st period
Dec 20,22
Dec 26,22
Jan 01,23
Jan 10,23
Days

(g)


No threshold
Threshold
4.1
4
f8 [Hz] - 2nd period
Feb 21,23
Mar 02,23
Mar 11,23
Mar 20,23
Days

(h)

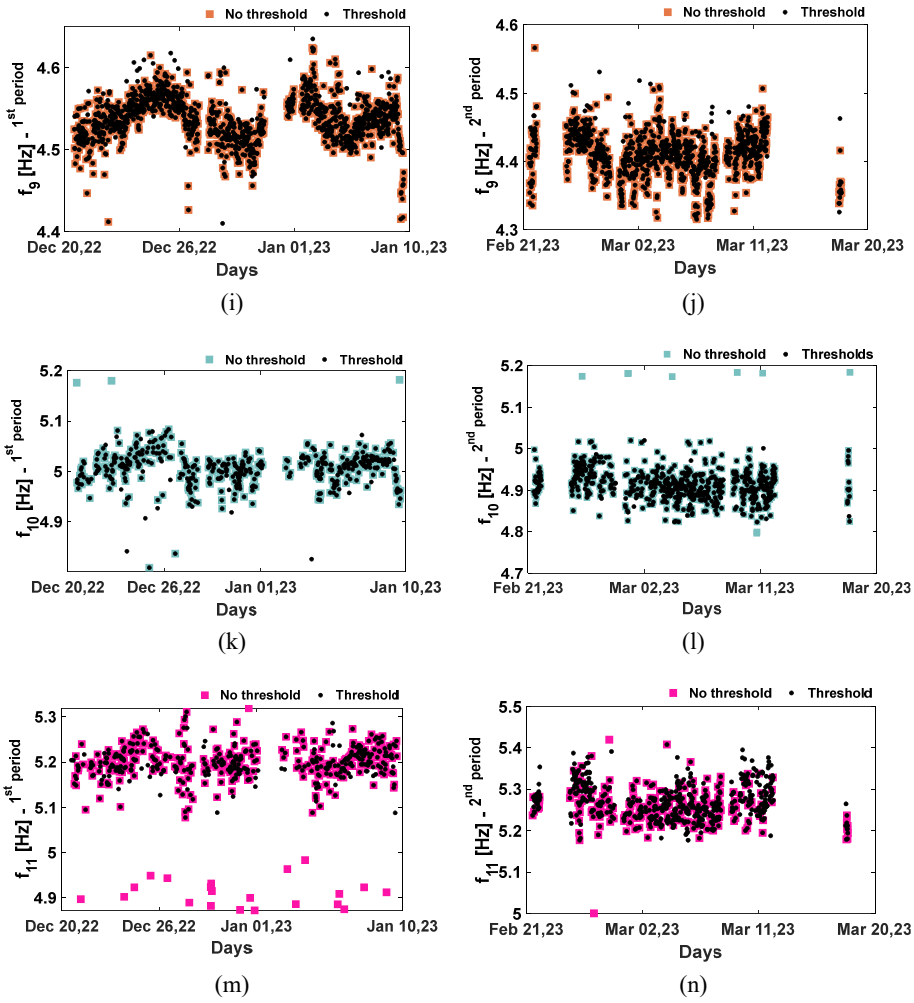


Fig. 7. Comparison of time series of natural frequencies of the structure before and after the manual definition of thresholds: (a) Frequency 5, 1st campaign; (b) Frequency 5, 2nd campaign; (c) Frequency 6, 1st campaign; (d) Frequency 6, 2nd campaign; (e) Frequency 7, 1st campaign; (f) Frequency 7, 2nd campaign; (g) Frequency 8, 1st campaign; (h) Frequency 8, 2nd campaign; (i) Frequency 9, 1st campaign; (j) Frequency 9, 2nd campaign; (k) Frequency 10, 1st campaign; (l) Frequency 10, 2nd campaign; (m) Frequency 11, 1st campaign; (n) Frequency 11, 2nd campaign.

Table 5. Values of MAC, MAC_{err} and $Freq_{err}$ for the second algorithm

	MAC	MAC_{err}	$Freq_{err}$
f_5	0.9985	0.0015	0.02875
f_6	0.9923	0.0077	0.04115
f_7	0.9835	0.0165	0.04181

f_8	0.9774	0.0226	0.03214
f_9	0.9818	0.0182	0.03707
f_{10}	0.9854	0.0146	0.02912
f_{11}	0.9917	0.0083	0.02582

5 Discussion on the results

In the previous sections, the analysis with both method used in the paper have been presented. It seems clear that the second method presented is much more convenient and efficient than the first. Firstly, much less analysis is required and there is not the need to select which modes to track and the reference mode is not based on a particular identification, but takes into account the characteristics of all modes by calculating the mean value. The second method is able not only to obtain all modes simultaneously, but also takes into account the *"intrinsic"* variability of a mode. This means that if a mode by its nature has large oscillations, no value will be excluded and therefore the tracking result will not be altered. Furthermore, in the case of the second method the thresholds used in the case of the MAC are much closer to the value used during the frequency identifications, a minimum of 0.95 is considered. Considering this value it can be stated that the thresholds defined by the first method are not so reliable, which is why classification problems are easily encountered. The further advantage of the second method is that the analysis is always carried out both in terms of MAC and frequency, which is not ensured in the first case (refer to running 1). Focusing on the second method, it is noted that the less stable modes, i.e. the higher ones, are more sensitive to the removal of outliers. In the case of the modes 10 and 11 there is a certain difference between the two historical series, those before and after the outlier rejection process. While for the more stable modes the two historical series are almost overlapping and the difference lies in the fact that after the outlier rejection the number of NaN decreases and consequently the series are more populated. In general, the second method guarantees less data loss.

6 Conclusions

In this paper, an automatic procedure for MT has been shown. The procedure has been presented through two different methods that are based on distinct parameters for the definition of threshold to compute; in the first methods MAC_{err},and $Freq_{err}$. The first method computes them relying on minimization of σ and reduction of number of NaN, while the second method is based on the minimization of the number of the modes of the distribution and reduction of number of NaN. These methods are both applied to a monumental building placed in Casale Monferrato in Piedmont that was first

instrumented with long-term monitoring between December 2022 and March 2023 and since July 2023 it has been instrumented with a permanent monitoring system with low-cost sensors. For this study, data of first monitoring system was used. From the comparison of the two methods it emerges that the first method is not very efficient and convenient, both because several analyses are necessary to find a satisfactory result and because despite the various analyses, the method is not able to find all the modes of the structure. Instead, the second method is able to trace all modes of the structure with a single analysis, the only manual intervention lies in defining the thresholds for the MAC_{err},and $Freq_{err}$, once these values have been set the result can be obtained. Furthermore, in the second case the risk of losing data is much lower, in fact once the thresholds have been set manually the number of NaNs is reduced and the time series are repopulated.

When dealing with MT problem, care should be taken to the type of structure, the number of sensors it is equipped with and the characteristics of its vibration modes. Indeed, if it is structure such as reinforced concrete or steel bridges, equipped with many sensors, the problem could become simpler. On the contrary, in the case of monumental masonry buildings which are perhaps equipped with few sensors, the problem becomes very complicated. In addition, if the structural modes are not well defined, i.e. the frequency ranges are not well separated, the problem becomes even more difficult. Therefore, there is the need to always consider the particularities of the case treated and from there, if necessary, readjust the procedure. However, the second algorithm presented in this paper could be a valid solution applicable to more types of buildings.

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