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# Towards a CO<sub>2</sub> emission standard for supersonic transport: A Mach 2 concept case study

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**Abstract.** This paper reports the work performed in the field of the H2020 MORE&LESS Project to contribute to shaping global environmental regulations for supersonic aviation. The existing CO<sub>2</sub> emission standard, which is defined for subsonic aircraft, is analyzed. However, the applicability of this standard to supersonic concepts needs to be assessed. A case study of a Mach 2 concept, based on the Concorde configuration, is considered to explore modifications needed for the CO<sub>2</sub> metric value calculation. The results provide insights into the CO<sub>2</sub> metric value for a supersonic aircraft and its comparison with subsonic aircraft limits.

## Introduction

During the last decades, the aerospace community has witnessed a renewed interest in high-speed civil passenger transport, with numerous projects underway to design civil passenger aircraft which fly faster than the speed of sound. However, it is crucial to consider the environmental impact of such concepts and prioritize environmental sustainability and social acceptance. To this end, the H2020 MORE&LESS Project (MDO and REgulations for Low boom and Environmentally Sustainable Supersonic aviation) was initiated in January 2021 with funding from the European Commission [1]. The project, which is expected to run for four years, aims to support Europe in shaping global environmental regulations for future supersonic aviation.

However, the current environmental regulations apply only to subsonic aircraft. A CO<sub>2</sub> emission standard exists and it is defined in the ICAO Annex 16 Volume III [2]. The CO<sub>2</sub> metric value is used to measure the fuel burn performance of an aircraft and it is designed to be common across different aircraft categories, regardless of their purpose or capabilities [3]. The CO<sub>2</sub> emission standard relies on three key factors linked to aircraft technology and design: cruise fuel burn performance, aircraft size and aircraft weight. It has been designed to ensure that effective enhancements measured through the system will lead to a corresponding reduction in CO<sub>2</sub> emissions during regular aircraft operations.

The work towards a potential CO<sub>2</sub> emission standard for supersonic transport (SST) is presented in this paper. One of the case studies of the More&Less project is considered for this analysis: a Mach 2 concept, derived from the Concorde configuration. More details on this case study are provided in the next sections.

The CO<sub>2</sub> metric value is currently defined for subsonic aircraft only. Nevertheless, with the growing interest in supersonic aviation, there arises a necessity for a new metric value to be established specifically for this type of aircraft. The initial step to take is to evaluate whether any elements of the existing requirements are directly applicable to civil supersonic concepts. If not, it is crucial to determine which modifications are required.

## Methodology

The CO<sub>2</sub> emission standard value is a fuel-efficiency standard, and it is based on 2 parameters:

1. Specific Air Range (SAR) during cruise flight.

2. Reference Geometric Factor (RGF), a measure of cabin size.

When an aircraft type undergoes CO<sub>2</sub> certification, a CO<sub>2</sub> metric value is calculated based on these parameters, and then compared to a limit that is dependent on the Maximum Take-Off Mass (MTOM) of the aircraft. The CO<sub>2</sub> metric value can be evaluated as:

$$CO_2 \text{ metric value} = \frac{\left(\frac{1}{SAR}\right)_{avg}}{RGF^{0.24}} \tag{1}$$

Where  $(1/SAR)_{avg}$  is the average of the reciprocal of the specific air range (SAR) [kg/km], which is evaluated at three cruise flight reference points, and the reference geometric factor (RGF) is a dimensionless measure of the cabin size. SAR values are evaluated for 3 reference cruise conditions, which are defined as a function of the MTOM:

1. High gross mass:  $0.92 \cdot MTOM$ .
2. Low gross mass:  $(0.45 \cdot MTOM) + (0.63 \cdot MTOM^{0.924})$ .
3. Mid gross mass: average of high and low gross masses.

For a generic subsonic mission, the three points should be located at beginning, end and mid of cruise, respectively, as can be seen in Fig. 1.

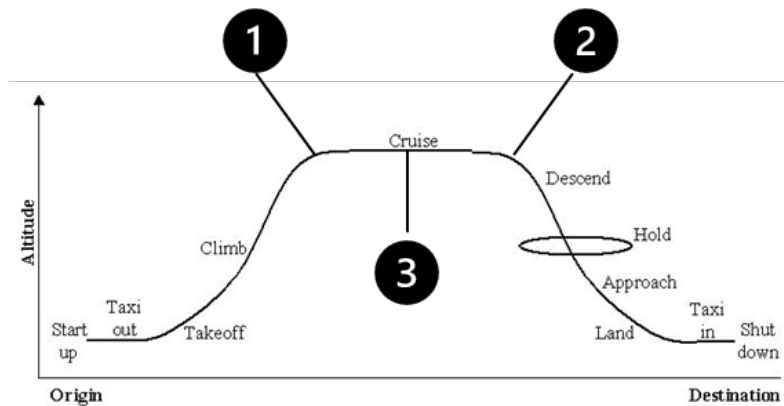


Fig. 1 – Mass points location for a generic subsonic mission.

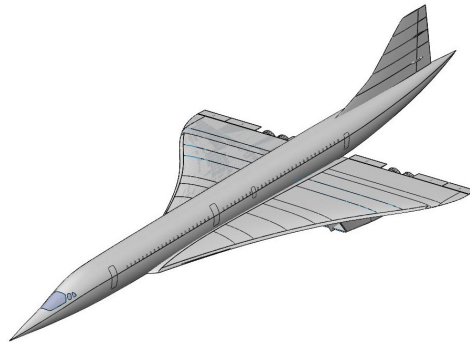
The applicability of these correlations is limited to subsonic aircraft, as they are linked to the fuel utilization percentage during ascent and cruising stages, in relation to the maximum take-off weight. Consequently, these magnitudes may vary when applied to a supersonic mission. Then, an additional evaluation is carried out to explore potential modifications in the definition of mass-points, which considers the supersonic cruise conditions. Eventually, the CO<sub>2</sub> metric value can be evaluated for these mass-points.

**Case study**

The case study considered for the analysis is a Mach 2 aircraft powered with biofuel. The Concorde aircraft has been used as reference configuration, targeting a minimization of noise and pollutant emissions as well as the lowest environmental impact at local, regional and global level. The aircraft characterization is an ongoing work and includes different aspects: vehicle design, aerodynamic characterization, propulsive characterization and mission simulation. The GTO mass of the aircraft is about 177 tons, with a maximum payload of 15280 kg and a fuel mass of 82180 kg. [4] The aircraft main geometric parameters are reported in Table 1, while an image of the aircraft is shown in Fig. 2.

*Table 1 – Mach 2 case study main parameters.*

Parameter	Value
Fuselage length [m]	62.25
Wing span [m]	25.60
Fuselage width [m]	2.90
Wing surface [m <sup>2</sup> ]	327.00
MTOM [Mg]	176.85
Passengers [-]	~120
Range [km]	6500



*Fig. 2 – Mach 2 case study.*

**Results**

First, in accordance with the standards for subsonic aircraft, Specific Air Range (SAR) values are calculated at the three specific reference mass points, which are dependent on the Maximum Take-Off Mass (MTOM). Although they are intended to represent various segments of the subsonic mission's cruise phase, they could be not situated in cruise conditions for a supersonic mission. The results obtained for the Mach 2 case study are reported in Table 2. The location of the mass points along the mission altitude profile is shown in Fig. 3. As expected, it can be seen that they do not represent the cruise conditions, but they are shifted towards the beginning of the mission. This is due to the fact that the percentage of fuel consumed to complete the climb is higher with respect to subsonic aircraft.

*Table 2 – Mass-points and SAR evaluation.*

	Mass [Mg]	Altitude [km]	SAR [km/kg]
1. High-mass point	162.70	08.84	0.047
2. Low-mass point	124.06	16.04	0.108
3. Mid-mass point	143.38	15.00	0.073

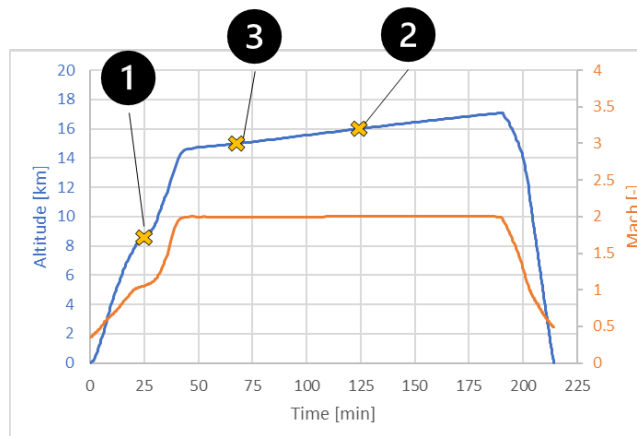


Fig. 3 – Altitude and Mach profile vs Time with mass points location.

Then, the actual cruise conditions are considered. The mass of the aircraft is evaluated at the beginning (BOC), end (EOC) and mid (MOC) of cruise and SAR is then computed accordingly. The results are reported in Table 3, while the modified position of the mass points is shown in Fig. 4.

Table 3 – Mass and SAR evaluated at real cruise conditions.

	Mass [Mg]	Altitude [km]	SAR [km/kg]
BOC	153.70	14.60	0.087
EOC	105.40	17.00	0.128
MOC	129.55	15.80	0.104

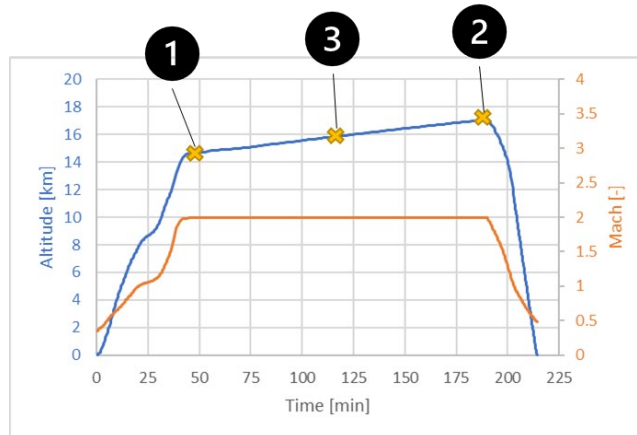


Fig. 4 – Altitude and Mach profile vs Time with mass points evaluated at real cruise conditions.

Eventually, the high-mass and low-mass points can be expressed as a function of MTOM for the actual cruise condition, as reported in Table 4. These results indicate that a representative high-mass point (at initial cruise conditions) shifts to a lower fraction of MTOM for SSTs compared to subsonic aircraft. Initial analyses with higher-Mach concepts in More&Less confirm that this trend increases with design Mach number, as more fuel is burned in the climb and acceleration segments.

Table 4 – Mass points for the different cases considered.

	High-mass point	Low-mass point
Subsonic aircrafts	$0.92 \cdot MTOM$	$(0.45 \cdot MTOM) + (0.63 \cdot MTOM^{0.924})$
Mach 2 Case Study	$0.87 \cdot MTOM$	$0.59 \cdot MTOM$

Moreover, SAR can be also evaluated during the entire mission, including climb, cruise and descent phases. An overview of the altitude and instantaneous SAR as function of ground distance is reported in Fig. 5 and Fig. 6. As expected, SAR reaches its highest level during cruise conditions, except for the final stage of the mission when the aircraft descends toward the destination airport and the thrust is kept to lower values.

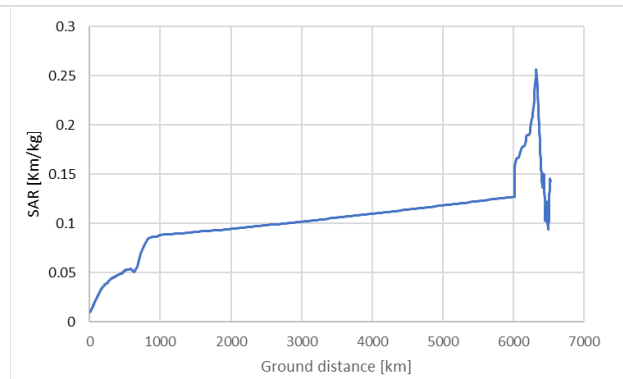
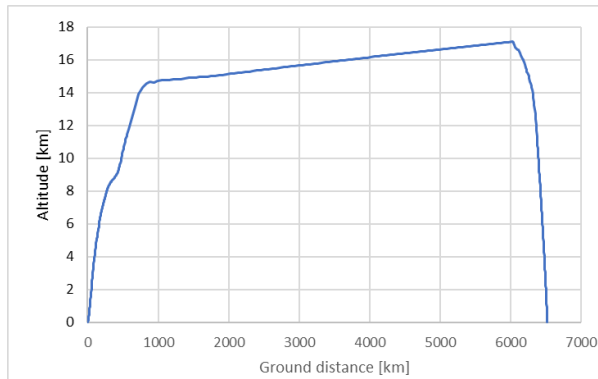


Fig. 5 – Altitude profile vs ground distance      Fig. 6 – Instantaneous SAR vs ground distance

Eventually, the CO<sub>2</sub> metric value can be evaluated considering the mass points at the representative cruise conditions:

$$CO_2 MV = \frac{\left(\frac{1}{SAR}\right)_{avg}}{RGF^{0.24}} = 3.1 \text{ kg/km}$$

The CO<sub>2</sub> metric value can be compared to the CO<sub>2</sub> limits for subsonic aircrafts, as defined in Annex 16 Vol III (Fig. 7). Some additional points are also included in the plot, representing two supersonic concepts from NASA and DLR. However, these concepts are quite different from the Mach 2 case, since they are much smaller and they are designed to fly at lower Mach numbers (1.4 and 1.6, respectively). For that reason, a direct comparison is not possible.

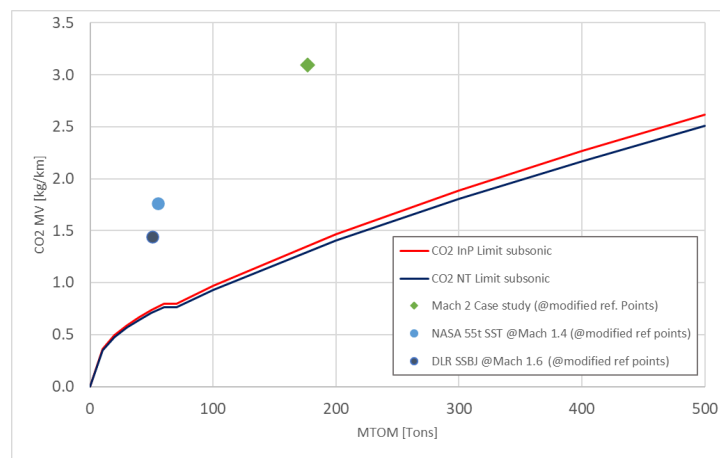


Fig. 7 – CO<sub>2</sub> metric values vs MTOM

### Conclusions and future steps

The current CO<sub>2</sub> standard is defined for subsonic aircrafts, and it cannot be directly applied to civil supersonic aircraft. The More&Less project aims to contribute to the assessment of the metric value by considering a Mach 2 case study as a baseline. Specific Air Range (SAR) and Reference Geometric Factor (RGF) are evaluated, to understand to what extent the present requirements are applicable to civil supersonic aircraft, and which modifications are needed. One crucial change is the re-definition of reference mass-points, to cover the cruise phase when evaluating SAR. However, further investigation on SAR and RGF and their impact on CO<sub>2</sub> MV is also necessary. Different configurations will be explored at various Mach numbers, such as Mach 1.5 and 1.7, to increase the number of available data. The results from these configurations can be exploited to further contribute to the assessment of CO<sub>2</sub> MV for civil supersonic aircraft.

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