Finite Element Model Updating Applied to a Lower Limb Prosthesis Through the Optimisation of Its Mechanical Properties

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Abstract. In the context of elite sports for lower limb amputees, the use of advanced materials and pioneering designs has enhanced athletes' performance by improving the energy storage and return capability of prosthesis feet. The knowledge of the behaviour of these components is crucial to meet athletes' requirements and needs. Given their inherent anisotropic nature, the modelling of these components entails fine-tuning several parameters, *i.e.,* Young moduli, Poisson ratios and shear moduli along three directions. This research aims to develop an automated algorithm, implemented in Matlab, for the automated fitting of the material properties. An experimental modal analysis in free-free conditions has been conducted on the blade prosthetic to extract natural frequencies and mode shapes. Subsequently, in the optimisation code, modal simulations are performed on a finite element model, using Nastran. The optimisation procedure is based on the comparison to the experimental data previously evaluated. The optimisation outcomes, in terms of material properties, enable the development of a numerical model capable of predicting the experimental dynamic behaviour up to 400 Hz.

Keywords: Running specific blade prosthetic; Automated code; Experimental modal analysis; Nastran simulations; Orthotropic material.

1 Introduction

The development of Energy Storing And Return (ESAR) prosthetic feet has revolutionised the way individuals with limb loss experience mobility, comfort, and athletic performance by mimicking natural lower limb functions [1, 2]. These components, typically made of lightweight carbon fiber composites, offer superior strength, reactivity, and durability, contributing to symmetric gait performances [2].

Insights into blade prostheses' mechanical behaviour are crucial for their accurate design, development, and customisation to effectively meet individual requirements. In the studies involving modal dynamic analyses on blade prostheses, it was observed that foot description should not be solely based on the weight of the amputees but also on their activity level aiming to attain the component's first bending natural frequency and thus enhancing performance [3, 4]. While these works highlighted the significance of conducting dynamic tests on blade prosthetics, they did not focus on the definition of the anisotropic material properties of the component. However, it is crucial for modelling the free dynamic behaviour of the component and fine-tuning it to meet athletes' requirements, as highlighted by Noroozi et al. [5]. Typically, mechanical analyses on blades simplify the component as isotropic, which is a significant oversimplification, especially for describing dynamic properties, given their inherent anisotropic nature. The motivation which leads to this simplification lies in the complexity of considering the actual characteristics of the component, which involves tuning numerous unknown independent parameters. Several works, in many research fields, have been presented on the tuning of multiple unknowns through an optimisation approach from experimental evidence [6–9].

This paper presents an optimisation algorithm which allows tuning the modal behaviour of an orthotropic FE prosthetic model, by varying the material properties. This target requires the challenging task of integrating the modal analysis in MSC Nastran with Matlab optimisation tools. The optimisation process is necessary due to the large number of material parameters to be tuned. Thanks to this approach, it will be possible to replicate, through FEA, the static and dynamic behaviour of the real component, which is not isotropic, in different operative conditions. In this paper, a description of the Experimental Modal Analysis (EMA) performed on the blade prosthetic is first presented. Then, the characteristics of the Finite Element (FE) model created in Lupos [10] are discussed. Finally, the automated optimisation code is described by specifying the workflow, the definition of the objective function, the initial values selected and the obtained results.

2 Experimental Modal Analysis

A roving hammer test has been conducted on the blade prosthetic Cheetah Xcel, Össur, designed for athletes between 89 and 100 kg (stiffness category 6).

The comprehensive description of this experimental analysis, including the test setup and the obtained experimental results, has been previously detailed in [11].

The prosthetic leg has been tested under free-free conditions by suspending the component with an elastic band, as shown in Figure 1 (a). The node configuration selected for testing (Figure 1 (b)) comprises 57 nodes. In five of those nodes, one mono-axial and four tri-axial accelerometers are placed, depicted by coloured dots in Figure 1 (b). The testing involved hammering the 57 nodes in all feasible directions permitted by the prosthetic shape, resulting in a total of 96 possible strokes. This process resulted in 1248 Frequency Response Functions (FRFs), each one estimated from an average over three repetitions.

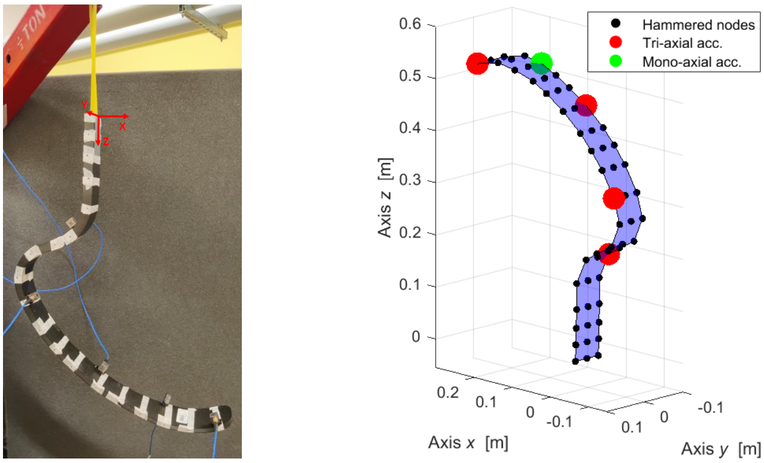

Fig. 1 Experimental setup (a) and location of the hammered and measured nodes (b).

The prosthesis' modal properties *i.e.,* the natural frequencies (f_r) and mode shapes ($\mathbf{\Phi}_r$) have been evaluated from the Experimental Modal Analysis (EMA) in the frequency range from $0Hz$ to $750Hz$ using PolyMAX algorithm [12].

3 Finite Elements Model

The FE prosthetic model has been developed in LUmped Parameter Open Source (LUPOS) FEM code developed in Matlab, Lupos 2023 [10], and the real modal analyses are computed with MSC Nastran 2017.

The model is discretised with 3D hexahedral elements (CHEXA) with a specified density calculated from the measured weight and the volume derived from the 3D CAD model created in Solidworks. It consists of 6 elements along the thickness and 20 along the width, resulting in a total of 32640 hexahedral elements. The model accurately replicates the gradual changes in the thickness and width of the prosthesis. It comprises 40131 nodes and has 240786 degrees of freedom.

4 Optimisation Code

The automated tuning of the FE model is developed in Matlab to evaluate the material properties for which the prosthetic dynamic behaviour, experimentally measured, is well simulated, in terms of natural frequencies and mode shapes. As previously demonstrated by Barattini et al. [11], the free dynamic behaviour of the first mode shapes is not well simulated when isotropic material is applied to the blade prosthetic, hence, for the tuning, anisotropic orthotropic material properties are considered. As a consequence, the tuning is performed considering the following 9 independent material parameters:

- E_1, E_2 and E_3: the longitudinal and transversal Young moduli, respectively;
- ν_{12}, ν_{13} and ν_{23}: the in-plane and out-of-plane Poisson ratios, respectively;
- G_{12}, G_{13} and G_{23}: the in-plane and out-of-plane shear moduli.

The subscripts 1, 2 and 3 correspond to the main directions of the local reference frame of the hexahedral elements: 2 is directed along the width, while 3 is along the thickness (refer to Figure 1 (b)). According to the Maxwell reciprocity theorem, the stiffness matrix is symmetric [13], hence *i.e* $k_{i,j} = k_{j,i}$. Given the definition of the stiffness matrix of an orthotropic material, it follows that $-\frac{\nu_{j,i}}{E_j} = -\frac{\nu_{i,j}}{E_i}$, consequently the Poisson ratios ν_{21}, ν_{31} and ν_{32} are dependent parameters of the three Young moduli and the other three independent Poisson ratios.

4.1 Workflow

The optimisation strategy is summarised in Figure 2. A hybrid optimisation technique is developed: Genetic Algorithm (GA) and Pattern Search (PS) methods are applied in series to evaluate the minimum of a multi-objective function by varying the independent parameters. Upper and lower bounds are used to constrain the optimisation variables. GA [14] is initially used for randomly exploring a wide parameter space and restricting the research space. Afterwards, PS [15] is employed to thoroughly identify the optimum starting from the result of the GA optimisation.

At the beginning of the code, the initial conditions are extrapolated from the Nastran input file, where all the model characteristics are specified, *i.e* geometry, FE elements characteristics, boundary conditions. In this case, the model is not constrained to reproduce the free-free conditions of the experimental test. The experimental modal results are also necessary since they are the target of the optimisation logic.

Subsequently, within the optimisation loop, where material properties are iteratively adjusted to minimise the objective function, the material values are updated in the input Nastran file to conduct modal simulations. At each iteration, before computing the objective function, the simulated frequencies and mode shapes are reordered according to experimental mode shapes order. This phase of the algorithm relies on the computed Modal Assurance Criterion (MAC) index [16] which allows selecting the corresponding EMA and FEA modes. Indeed, the MAC is adopted to compare the experimental and numerical mode shapes by measuring the correlation level between two mode shapes $\mathbf{\Phi}_j$ and $\mathbf{\Phi}_q$, respectively.

Once the function tolerance requirements δ or the maximum number of iterations i_{max} are achieved, the code stops and outputs the optimal results. Parameters δ and i_{max} chosen for Genetic Algorithm (GA) are 10^{-4} and 55, respectively, while for Pattern Search (PS), they are 10^{-5} and 45, respectively.

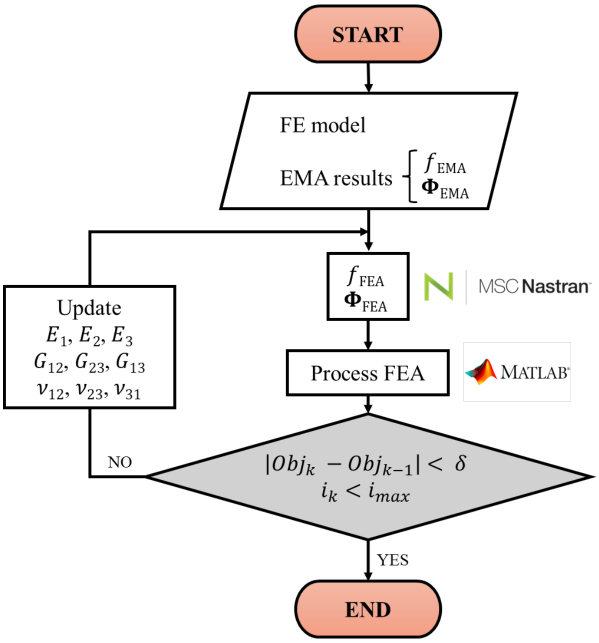

Fig. 2 Scheme of the optimisation process.

4.2 Objective Function Definition

The multi-objective function to be minimised is made of two contributions corresponding to the error based on the difference between experimental and numerical results, in terms of both natural frequencies (f_n) and eigenvectors (Φ). This function is used for both the optimisation algorithms employed in the code.

The objective function is defined according to Eq. 1, where the two error functions are combined using a weight parameter α, as done by Cirimele et al. [17] to explore the Pareto front. For this study, the α value selected is 0.5, to confer the same weight to natural frequencies and mode shapes.

$$Obj_k = \alpha \epsilon_{f_n,k} + (1-\alpha)\epsilon_{\Phi,k} \tag{1}$$

where:

$$\epsilon_{f_n,k} = \frac{\sum_{r=1}^{n} \frac{|f_{\mathrm{EMA},r} - f^*{}_{\mathrm{FEA},r}|}{f_{\mathrm{EMA},r}}}{n} \tag{2}$$

$$\epsilon_{\Phi,k} = \frac{n - \sum_{r=1}^{n} \mathrm{MAC}^*{}_r}{n} \tag{3}$$

n is the number of modes considered (equal to 4). $\mathrm{MAC}^*{}_r$ represents the diagonal elements of the reordered MAC matrix and $f^*{}_{\mathrm{FEA},r}$ indicates the reordered FEA natural frequencies (as mentioned in Paragraph 4.1). In this research work, the focus lies on the first four modes, hence up to $400Hz$ (the results are provided in Table 2, in Paragraph 4.3). Indeed, at higher modes, there is slight coupling between torsional and

Table 1 Material properties boundaries (**Min.** and **Max.**), initial (**Initial**) values selected and resulting values obtained after the *Genetic Algorithm* optimisation (**G.A. (1^{st} stage)**) and after the *Pattern Search* optimisation, hence at the end of the optimisation (**S.P. (2^{nd} stage)**).

Property	Min.	Max.	Initial	G.A. (1^{st} stage)	S.P. (2^{nd} stage)
E_1 [GPa]	10	180	10.06	28.79	28.79
E_2 [GPa]	5	80	14.96	15.99	20.68
E_3 [GPa]	1	50	30.00	17.24	17.24
G_{12} [GPa]	1.5	50	10.00	47.12	4.68
G_{23} [GPa]	1.5	20	2.50	8.44	20.00
G_{13} [GPa]	1.5	20	2.00	3.18	3.18
ν_{12} [-]	0.1	0.8	0.19	0.789	0.789
ν_{13} [-]	0.1	0.7	0.20	0.698	0.698
ν_{23} [-]	0.1	0.7	0.30	0.695	0.695

bending behaviour. Hence, the further modes are neglected to place greater emphasis on the first four and ensure their simulation is as accurate as possible.

4.3 Results

Since the internal stacking sequence of the prosthetic is unknown, a wide range of variance of the material properties has been selected. Only the upper and lower bound of these parameters are specified (Table 1), deduced from the values provided by manufacturers and researchers on the mechanical properties of carbon fibre composite materials [18, 19], without imposing additional constraints.

The material values coming from the simulation are reported in Table 1.

The material results obtained reasonably well align with the expected outcomes. As also declared by Noroozi et al. [4], it is expected that the majority of the layers are unidirectional with the fibres directed along the prosthetic shape, given its primary function of bending on the XZ plane (reference system of Figure 1 (b)). Consequently, the elastic modulus along the prosthetic shape should be highest, as confirmed by the automated optimisation process performed. Following these considerations, the transversal moduli are less than half of the longitudinal one, thus confirming the aforementioned assumptions.

In Table 2 the resulting natural frequencies of EMA (f_{EMA}) and optimal FEA (f_{FEA}) and the corresponding MAC values are reported.

The minimum objective function achieved is 0.141 with an error on the eigenvalues ϵ_{ω^2} (Eq. 2) of 0.132 and on the mode shapes ϵ_Φ (Eq. 3) of 0.149. This implies a mean difference of $25Hz$ in the natural frequencies, down from the initial $65Hz$ difference observed. The optimal results exhibit a really good match for the first two natural frequencies, with just a mean difference of $0.4Hz$. On the other hand, for the fourth mode, there is a higher discrepancy but with a mode shape correspondence of nearly 95.3% according to the MAC value.

A significant success in the results is the achievement of a diagonal MAC matrix. Initially, the modes' order in the simulation differed from that observed in experimental tests, a discrepancy noted across various material value selections.

Table 2 Natural frequencies of the EMA and the optimal FEA and the corresponding MAC values.

Mode n.	f_{EMA} [Hz]	f_{FEA} [Hz]	Mode shape	MAC [%]
1	86.3	86.1	1^{st} bending XZ	88.1
2	226.5	227.1	Torsion RZ / 1^{st} bending XY	80.0
3	275.2	257.3	2^{nd} bending XZ	76.8
4	371.8	289.6	Torsion RZ / 2^{nd} bending XY	95.3

5 Conclusions

In this research activity, an automated algorithm for tuning the orthotropic material characteristics of a FEM model is presented. The optimisation is carried out considering an objective function based on the error between the simulated and the experimental modal results. The proposed methodology is built on the case study of a lower limb prosthetic (Cheetah Xcel, Össur).

The material properties obtained validate the assumptions of predominantly unidirectional carbon layers with the fibres disposed along the prosthetic shape. The presented method proves successful for the selected case study, achieving good agreement in natural frequencies and mode shapes compared to experimental results. A mean difference of just $0.4Hz$ is observed for the first two natural frequencies, while the best mode shapes match between EMA and FEA is verified for the fourth mode with a MAC index of 95.3%.

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