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DIMOSS, a novel structural shape and stress monitoring software: theoretical background and applications review

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Abstract. Structural components of aircraft, civil and marine structures are prone to deterioration and require complex and costly maintenance activities. In recent years, much effort has been put into shifting from a preventive maintenance model, based on statistically scheduled interventions, to a predictive one, based on monitoring the actual health of the structure. The development of DIMOSS® (DIsplacement MOnitoring using Strain Sensors) software fits this context. DIMOSS® is a structural monitoring software for reconstructing fundamental quantities to assess the health of a structure, displacements and stresses. The software relies on strain sensors and it is based on the inverse Finite Element Method (iFEM). The software integrates iFEM and all the tools needed to design and operate a monitoring system. This paper introduces the theoretical background and the design approach of DIMOSS®. Moreover, it presents two successful experimental applications of the software: the monitoring of a stiffened composite wing-shaped panel and the live monitoring of an aluminium beam through the realisation of a structural digital twin.

Keywords: Structural Health Monitoring · Digital Twin · Shape sensing · iFEM

1 Introduction

Structural components in aircraft, civil engineering, and marine structures are susceptible to deterioration over time, often necessitating complex and expensive maintenance efforts. Structural Health Monitoring (SHM) techniques are advancing rapidly in response to these challenges, offering crucial insights to enhance safety and minimize maintenance costs [4]. To work effectively, a SHM framwork requires the accurate monitoring of the structure. The creation of

DIMOSS® (DIsplacement MOnitoring using Strain Sensors) software represents a significant advancement in this field. DIMOSS® (`https://dimoss.net/`) is a monitoring software designed to continuously assess the health of structures by reconstructing critical quantities such as displacements and stresses, which are key indicators of structural integrity. The software provides real-time monitoring capabilities by utilizing data from strain sensors, allowing for early detection of potential issues.

DIMOSS® relies on easy measurable structural responses, strains measurements, and it is based on the inverse Finite Element Method (iFEM) [11]. This method uses an FEM discretisation of the displacement field of a structure and, consequently, an FE discretisation of the strain field. The discretised strain field, expressed in terms of the nodal degrees of freedom (DOFs), is compared to the one discretely measured using strain sensors so that the nodal DOFs are fitted to the measured strain in a least-square sense. To summarise, the method integrates and extrapolates sparse discrete strains to obtain the strain and displacement field on the entire monitored structure. These quantities allow the computation of the stress field as well, using the constitutive equations. iFEM has been successfully applied to the monitoring of marine [7,6], civil [12,10] and aerospace [8,9] structures.

Although the iFEM has proven to be accurate and efficient, some works also proved its significant sensitivity to the sensors' configuration [1]. For this reason, in addition to enabling monitoring, DIMOSS® integrates several modules that allow the design of the optimal sensor system to achieve the most efficient monitoring system possible. In conclusion, DIMOSS® includes all the tools necessary to design and operate a monitoring system that uses iFEM effectively. The software has been successfully tested on the sensors' configuration design process and monitoring on several experimental structures. After introducing the DIMOSS® theoretical background and design philosophy, this work presents two of these successful applications: the monitoring of a stiffened composite wing-shaped plate and the live monitoring of an aluminium beam through the realisation of a structural digital twin.

The paper is structured as follows. In Section 2, some insight into the theoretical background of DIMOSS® is given. In Section 3, the characteristics and the design philosophy of the software are described. Finally, the applications of DIMOSS® are presented in Section 4 and the concluding remarks are reported in Section 5.

2 Theoretichal background

The Displacement MOnitoring using Strain Sensors (DIMOSS®) software reconstructs a structure's displacement field from discrete and sparse strain sensor configurations. The software performs this task using the inverse Finite Element Method (iFEM). This method is based on discretising the structural domain with finite elements, making it easy to use with already widespread structural models. In this section, the iFEM will be briefly described.

The iFEM has been formulated for shell [11] and beam elements [5]. We consider a generic FE discretization of the displacement field for an element [3]:

$$[u,\ v,\ w,\ \theta_x,\ \theta_y,\theta_z]^T = \mathbf{N}\mathbf{u}^e \tag{1}$$

where $[u,\ v,\ w,\ \theta_x,\ \theta_y,\theta_z]$ are the displacements and rotations along the x, y and z axis in a Cartesian reference system, $\mathbf{N}$ is the shape function matrix and $\mathbf{u}^e$ is the vector of the element's nodal degrees-of-freedom (DOFs). By deriving the shape function with respect to the spatial coordinates, it is possible to obtain the discretisation of the strain field in terms of the nodal DOFs:

$$\mathbf{e}(\mathbf{u}^e) = \mathbf{B}\mathbf{u}^e \tag{2}$$

where $\mathbf{e}$ is the vector of the strain measures for the element and $\mathbf{B}$ is the matrix of the derivatives of the shape functions. $\mathbf{B}$, $\mathbf{u}^e$ and the strain components that define the vector $\mathbf{e}(\mathbf{u}^e)$ depend on the kind and formulation of the selected inverse element.

The expression of the strain field in (2) can be used to compute the nodal DOFs that best fit the strain field measured at discrete locations of a structure. For each element of a FE mesh, it is possible to define the square error between the measured strain field, $\mathbf{e}^m$, and the one expressed in (2). For beam elements, the error functional can be formulated as follows:

$$\Phi_b^e(\mathbf{u}^e) = \sum_{k=1}^{6} \lambda_k^b w_k^b \sum_{i=1}^{N} \frac{l_e}{N} \left[e_k(x_i) - (e_k^\varepsilon)_i\right]^2 \tag{3}$$

where $e_k(x_i)$ is the k-th analytical sectional strain computed at the section identified by the x_i axial coordinate, $(e_k^\varepsilon)_i$ is the k-th measured sectional strain coming from a sensor at the same axial location, N is the number of sections within the beam element where the strains are measured and l_e is the length of the beam element. The index k identifies the six sectional strains that define the strain field for a Timoshenko beam, namely: axial stretching ($k = 1$), bending curvatures ($k = 2, 3$), transverse shear ($k = 4, 5$) and torsional strain ($k = 6$). λ_k^b are coefficients that guarantee the dimensional consistency between the different sectional strains. w_k^b is a weight used to account for sparse strain sensor configurations. It is set to 1 if the corresponding sectional strain is measured or to a small vale (10^{-5}) if it is not.

The same error functional can be defined for shell elements. In this case the functional $\mathbf{\Phi}_p^e$ is defined as:

$$\Phi_p^e(\mathbf{u}_p^e) = \sum_{k=1}^{8} \lambda_k^p w_k^p \iint_{A^e} [e_k(\mathbf{u}^e) - e_k^\varepsilon]^2 dxdy \tag{4}$$

where $e_k(\mathbf{u}^e)$ is the k-th analytical strain measure, e_k^ε is the k-th strain measure derived from sensors' measurements, and A^e is the area of the shell element. In this case, the k index identifies the eight strain measures that define the strain

field for a Mindlin plate, namely: the membrane strain ($k = 1, 2, 3$), the bending curvatures ($k = 4, 5, 6$) and the transverse shear strains ($k = 7, 8$) of the plate. In this case, the λ_k^p and w_k^p coefficient assume the same role as λ_k^b and w_k^b. Since the transverse shear strains are not measurable through strain sensors, the values of w_k^p for ($k = 7, 8$) is always set to 10^{-5}. The minimisation of the error functionals with respect to the nodal DOFs leads to the formulation of a system of linear equations.

$$\frac{\partial \Phi_b^e(\mathbf{u}_b^e)}{\partial \mathbf{u}_b^e} = \mathbf{k}_b^e \mathbf{u}_b^e - \mathbf{f}_b^e = 0 \tag{5}$$

$$\frac{\partial \Phi_p^e(\mathbf{u}_p^e)}{\partial \mathbf{u}_p^e} = \mathbf{k}_p^e \mathbf{u}_p^e - \mathbf{f}_p^e = 0 \tag{6}$$

The standard FE assembly procedure for all the elements of a mesh, taking into account the necessary transformations from the local DOFs of the elements to the global ones, generates the system of linear equations that, upon enforcement of the displacement boundary conditions, can be inverted to compute the vector of the global DOFs, $\mathbf{U}$, that best fit in a least-square sense the measured strains:

$$\mathbf{U} = \mathbf{K}^{-1}\mathbf{F} \tag{7}$$

$\mathbf{K}$ is a matrix that depends on the mesh and the strain sensor configuration, whereas $\mathbf{F}$ is a vector that depends on the mesh, the strain sensor configuration and the values of the discrete strain measurements. When considering the monitoring of displacements during the service life of a structure, it is important to highlight that when the sensor configuration is fixed, $\mathbf{K}$ does not change. That means that, for a structure instrumented with a defined sensor layout, the inversion of $\mathbf{K}$ can be done only once, whereas the measured values coming from these sensors, which will change with time, only influence the $\mathbf{F}$ vector. This can be computed and multiplied by the already inverted $\mathbf{K}$ to get the displacement at each time step at which the strains are measured. This makes the process computationally efficient. In fact, the most computationally demanding procedure, the inversion of a matrix, is performed only once before starting the live measurements. In contrast, the displacement computation at each time step relies on easy matrix-vector multiplication. This allows the method to monitor a structure in real-time, as will be proved in the 4.2 section.

The monitoring capabilities of iFEM can be easily extended from displacement to strain and stress fields. From the reconstructed DOFs, the strain field can be easily computed through strain-displacement relations. Moreover, if the material properties are known, the stress field can also be derived from the computed strain field through the constitutive equations, thus allowing a complete monitoring of the structure.

3 DIMOSS®: characteristics and design philosophy

The idea behind DIMOSS® is the design of integrated software for the monitoring of the displacement and stress field of a structure. Therefore, although iFEM

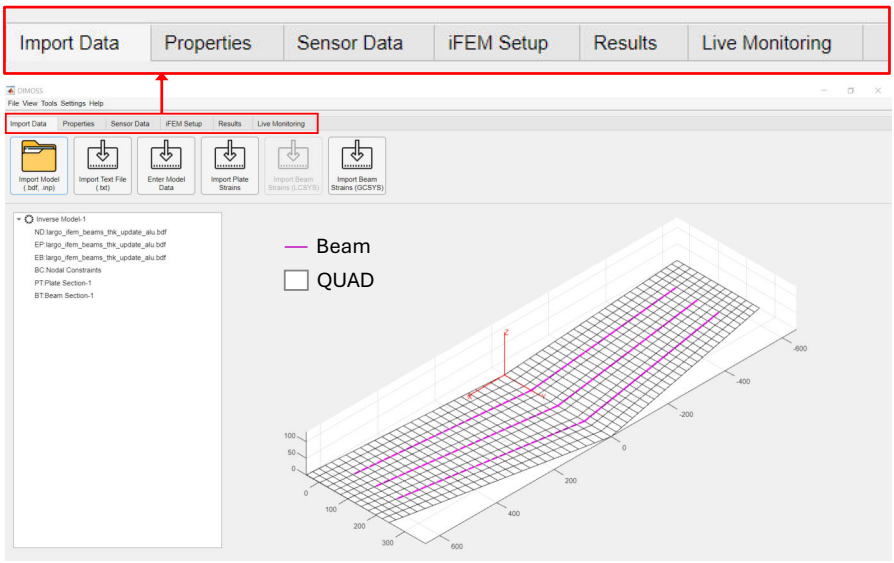

Fig. 1. DIMOSS® graphical user interface.

is the software's core, it is integrated with all the ingredients needed to design and operate a structural monitoring system. For this reason, many modules are included in DIMOSS®, each related to a different step of the monitoring process. The graphical user interface of DIMOSS®, where these modules are shown, is presented in Fig. 1. The characteristics that guided the design philosophy and the relative modules implementing them are described in the following.

- **Compatibility with FE commercial software**. DIMOSS® was created with the idea that it could be easily used by the same people with experience in standard structural analysis. Moreover, it relies on Finite Element models of the structure to be monitored. For these reasons, the software includes an *Import Data* module, designed to import FE models from two widespread FE commercial software, Abaqus® and PATRAN®. This solution allows expert users to create the models using existing and established tools without forcing them to learn different and complex procedures.
- **Independent software**. Although compatibility with other software is important, the idea is to create software that does not need any other third-party program to work. Therefore, all the functions that can be performed by importing an existing model can also be performed directly in DIMOSS®. Within the module *Properties* and also *Import Data*, the user can create a finite element model from scratch by directly inputting the model properties, i.e. nodal coordinates, element thickness and connectivity, etc.
- **Sensors optimisation**. One crucial aspect related to the applicability of iFEM is the availability of strain sensors. The method is sensitive to the

sensor array layout to guarantee accuracy in monitoring displacements and stresses. For this reason, DIMOSS® includes a sensor optimization module. The strain sensor placement is optimised using a genetic algorithm. The algorithm can produce configurations including different kinds of sensors: mono-axial strain gauges, strain rosettes and fibre optics sensors. Moreover, in the case of poor physical sensor availability, the software includes a strain pre-extrapolation tool to expand the sensors with virtually extrapolated data based on polynomial fitting. Thanks to these features, included in the *Sensor Data* module, the user can easily design the hardware part of the monitoring system, represented by the strain sensor layout. The module also allows the user to define a sensor configuration without optimisation if the configuration is already known.

- **Accurate and efficient monitoring**. The software relies on the efficiency of iFEM. Many studies have proved this method accurate and efficient, even considering experimental contests and uncertainties deriving from this kind of applications. In DIMOSS®, the iFEM is implemented considering the possibility of modelling structures with shell and beam elements separately or combined in a hybrid shell-beam model. The core module of the software, where the iFEM is applied to reconstruct displacements and stresses from strain data, is implemented within the *iFEM Setup* module.
- **Post-processing and exporting capabilities**. Once the monitoring results are elaborated, it is fundamental to have the possibility to post-process, analyse and elaborate them. Two previously highlighted principles are simultaneously considered: independence and compatibility with FE software packages. In fact, the software includes a *Results* module that allows the user to visualize and manipulate result data inside DIMOSS® or export the results in a format importable by Abaqus® or PATRAN®. This allows the user to visualize and elaborate the data using the same model previously built with these software packages.
- **Live monitoring**. The development of a live monitoring tool is paramount for Structural Health Monitoring software. The possibility of checking the status of a structure online, in every moment of its service life, can accurately guide predictive maintenance algorithms and lead to the achievement of the digital twin paradigm. The module *Live Monitoring* realizes this function. Once again, DIMOSS® exploit the efficiency of iFEM. As anticipated in Section 2, thanks to iFEM formulation, the displacement and stress field reconstruction can be obtained from simple matrix multiplication, thus allowing for computations extremely fast that allow the in-service monitoring of a structure at high-frequency rates.

4 Applications

The capabilities and functions of DIMOSS® have been tested on several complex numerical and experimental applications. The most recent ones are briefly described in this section. In particular, the shape sensing of a stiffened composite wing-shaped panel is presented in order to show the software's design and

monitoring capabilities. On the other hand, a simpler structure, a cantilevered aluminium beam, is analysed to show the digital twin capabilities of DIMOSS®.

4.1 Monitoring of a composite wing-shaped panel

The wing-shaped stiffened panel adopted for this application is shown in Fig. 2. Three T-section stringers stiffen the wing-shaped panel on one face. The entire structure is made of a TWILL T-300 carbon-fibre fabric prepreg with the following layup, $[45/0/0/45/0/0/0/45]_s$. The panel is loaded with a transverse concentrated force, $F_1 = 200N$, and has simply supported boundary conditions applied at the two wing tips. This experimental testing configuration is realized as shown in Figs. 3. For this experiment, the whole monitoring system is

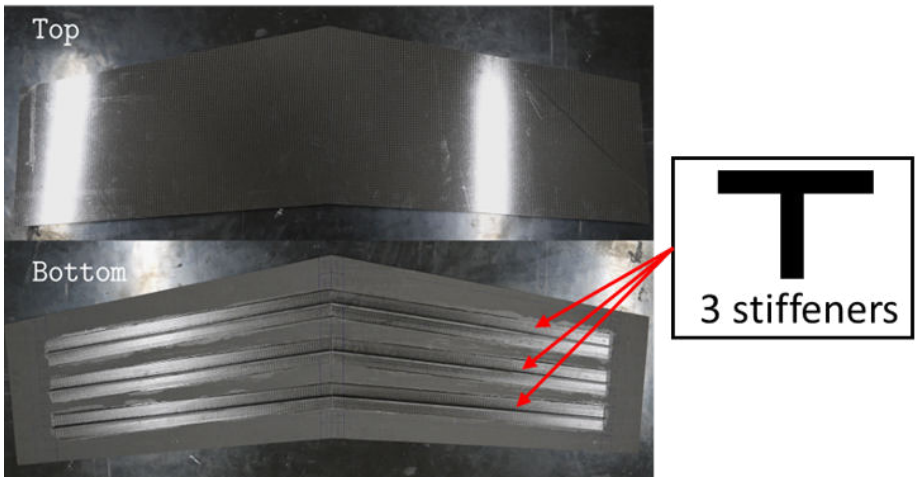

Fig. 2. Composite wing-shaped panel.

designed using DIMOSS®. The first step of the process requires the definition of an FE model of the structure. The model is created using PATRAN® and then, exploiting the software's features, imported into DIMOSS®. The model is made of 672 quad elements for the skin and 144 beam elements to simulate the stiffeners (Figs. 1 and 4). From this model, a sensor optimisation campaign with DIMOSS® is performed. To perform the optimisation, simulated data of the experimental test are used to perform numerical shape sensing. The optimisation tool of DIMOSS® allows the search for the optimal sensor configuration for a given set of available sensors. In this case, the available physical sensors are two 10-meter-long distributed strain sensing fibres and 16 strain rosettes. With this set of sensors, the genetic algorithm searches for the configuration that minimises the errors in the displacement field reconstructions. The optimisation module returned the optimised sensor configuration in Fig. 4. The designed configuration of sensors is then installed on the structure. To assess the accuracy

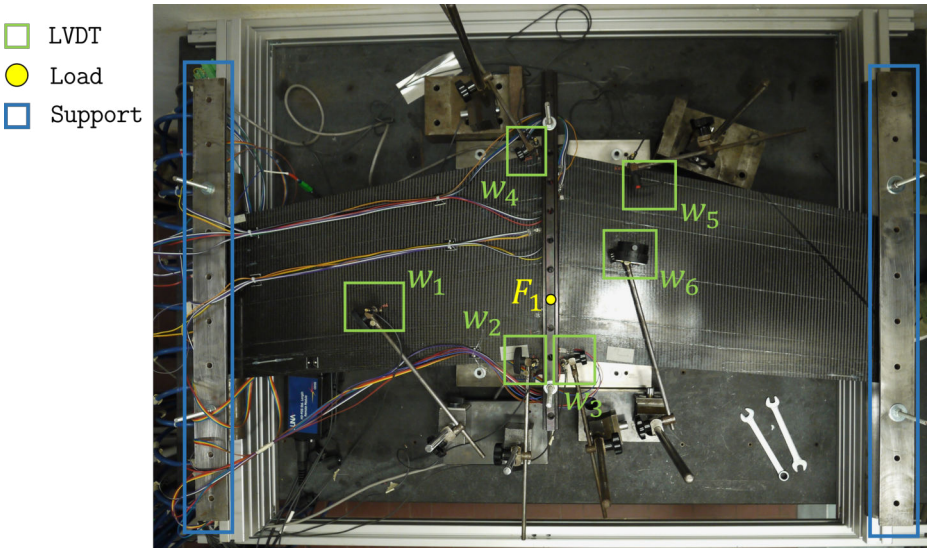

Fig. 3. Experimental testing configuration for the composite wing-shaped panel [2].

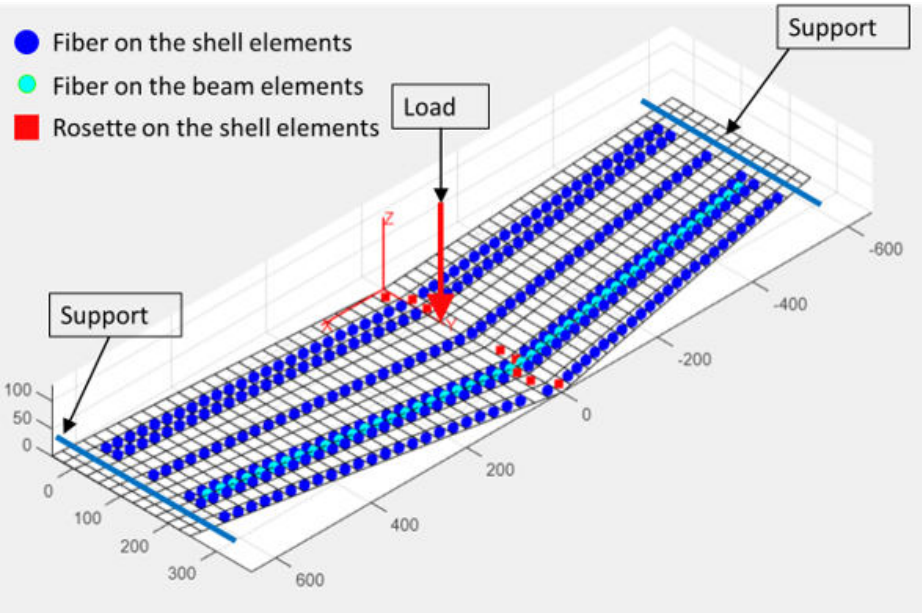

Fig. 4. Optimised sensor configuration.

of the experimental shape sensing, the panel is also instrumented with six randomly distributed LVDTs to measure the experimental transverse displacements

(Fig. 3). From the experimental test, the strain data are downloaded and used as inputs for DIMOSS®. At the same time, the data from the LVDTs are collected. The reconstructed transverse displacements, compared with the ones from the LVDTs, show impressive accuracy. The average absolute error over the six deflections is only 4.1% and the maximum percentage error 5.3%. More details on this experimental application are reported in [2].

4.2 Live monitoring of an aluminium beam

Since the next application represents the first application of DIMOSS®, and more in general of iFEM, for live monitoring, the considered structure is relatively simple. It is a cantilevered aluminium beam whose geometric properties are shown in Fig 5. The beam is 3 mm thick, and it is made of a 7075T6

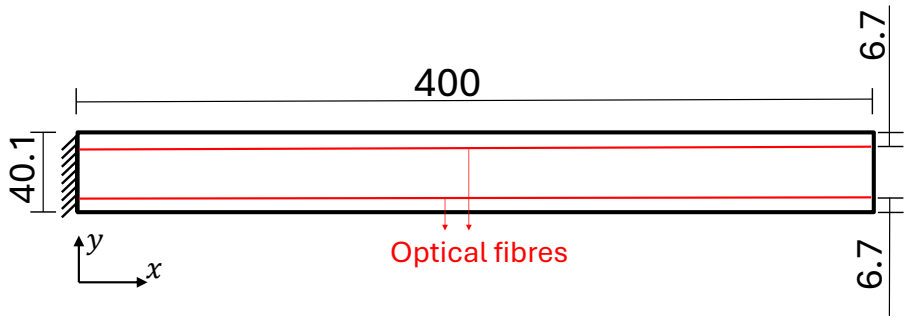

Fig. 5. Aluminum beam geometry - The figure shows the geometry of the aluminium beam and its boundary conditions. All dimensions are expressed in mm.

Aluminium alloy (Ergal), whose experimentally measured Young Modulus is $E = 67545.6\ MPa$ and Shear Modulus $G = 25393.1\ MPa$. The beam is instrumented with a 2 m fibre-optic strain sensor. In this case, since the complexity of the test did not require any optimisation, the senor layout was chosen based on physical considerations and experience. Two lines of measure in a back-to-back configuration are installed with the fibre. They measure the strain along the x direction, as shown in Fig. 5. The live monitoring requires an effective interfacing between the sensors and the monitoring software. In this case, the communication between the sensors and the computer where DIMOSS® is running is realised with a Wi-Fi connection. In detail, the acquisition system for the fibre is connected through a local Wi-Fi network to a computer where the monitoring through DIMOSS® is performed. The testing configuration is schematised in Fig. 6. This kind of configuration is designed to test the live and remote monitoring capabilities of DIMOSS® simultaneously. During the test, the beam is loaded by hand with concentrated transverse loads with different application

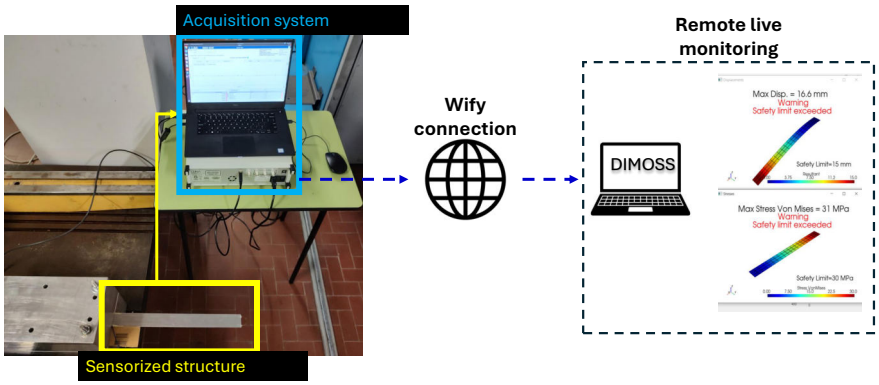

Fig. 6. Live monitoring testing configuration.

points. Since the load is not an input for the shape seeing, no sensing system for this quantity is included. The live monitoring of the structure can be viewed at this link (https://www.youtube.com/watch?v=vuUK1tuwv3Y), or on this website (https://DIMOSS®.net/). The video shows the live monitoring of the beam with a sensor acquisition rate of 21 Hz. It can be noticed that the reconstructed displacements and stresses are flawless and show no delay at this acquisition frequency. Figure 7 shows the detail of the monitoring at a specific time step. The figure shows that DIMOSS® also includes a warning system for exceeding displacement and stress safe limits during the monitoring. This tool has been included to show one of the possible crucial applications of live tracking, a warning system for excessive load limit reach during service life.

5 Conclusions

This work introduces a novel SHM software, DIMOSS® (DIsplacement MOnitoring using Strain Sensors). A structure is monitored with DIMOSS® by computing the displacements, strain and stress field from discrete strain sensors. The software is based on the inverse Finite Element Method. Anyway, it is not a simple implementation of the iFEM. In fact, it integrates different tools that allow the realization of a monitoring system from the design phase to the operative one. Among them, the most important is the optimisation tool to design each application's best strain sensor layout. This tool also includes a virtual sensor expansion feature. The software is fully compatible with existing software packages. It can import the FE structural model and export the monitoring results in formats that are readable by Abaqus® and PATRAN®. DIMOSS®, thanks to the iFEM efficiency, is computationally efficient and allows the real-time monitoring of structures. All these features are described in this paper. Moreover, they are verified for two experimental applications. The analysis of a compos-

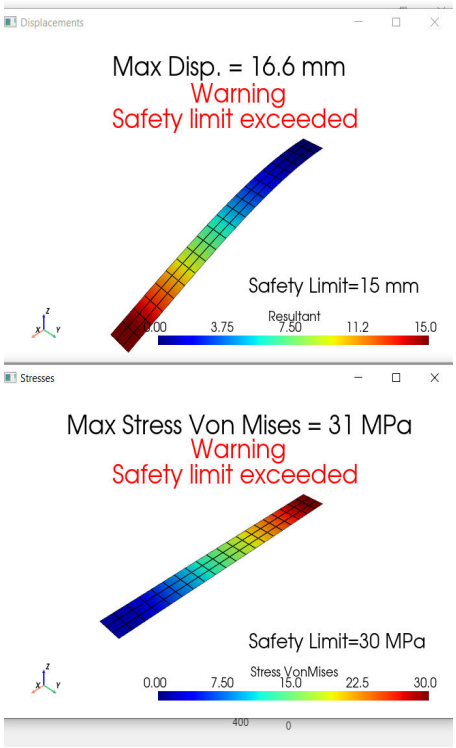

Fig. 7. Detail of the live monitoring at a specific time step.

ite wing-shaped panel is performed, from the design of the sensor layout to the actual monitoring of the displacement field of the structure. The results of this applications show significant accuracy in the reconstruction of the monitored displacement. The second application concerns the real time monitoring of an aluminium beam. This application proves the software's capabilities in realizing a structural digital twin. The live monitoring of the displacements is successfully obtained at an acquisition rate of 21 Hz from the sensors.

The development of DIMOSS® still requires further improvements. The live monitoring capabilities should be verified on more complex structures, which require more refined models and increase the computational burden. Moreover, the optimisation tool for the sensors should be improved with more effective strain pre-extrapolation techniques, able to reconstruct more complex strain field distributions than the already implemented polynomial fitting method.

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