URSI GASS 2023, Sapporo, Japan, 19 – 26 August 2023

Quasi-Helmholtz Projectors for High-Order Basis Functions: Definitions, Computational Strategies, Applications

Johann Bourhis*(1), Adrien Merlini(2), and Francesco P. Andriulli(1)
(1) Politecnico di Torino, Turin, Italy
(2) IMT Atlantique, Brest, France

Extended Abstract

Solutions of the low-frequency breakdown plaguing the electric field integral equation (EFIE) when discretized with the boundary element method (BEM) have been extensively studied in the past decades. These methods rely on the separation of the solenoidal and non-solenoidal contributions of the expansion and testing functions, used in BEM, in order to obtain a well conditioned system and to avoid a dramatic loss of significant digits. Among them, when low-order Rao-Wilton-Glisson basis functions are used, quasi-Helmholtz projectors are particularly effective because they do not require the computationally expensive detection of the (local and global) loops. With comparison to standard Loop/Star and Loop/Tree approaches, quasi-Helmholtz projectors also lead to a BEM linear system with a smaller condition number that grows slower when refining the mesh.

It is well known that the accuracy and convergence properties of the EFIE can be improved by employing high-order basis functions. For these functions however, quasi-Helmholtz projectors have never been investigated and this is lamentably given that the high-order EFIE is still plagued by the low-frequency breakdown.

This work will solve this problem by introducing the first ever presented generalization of the quasi-Helmholtz projectors for high-order discretizations of the EFIE. When employing arbitrary basis functions generating the Nédélec's mixed-order div-conforming spaces, it is shown that a suitable analysis of the high-order charge matrix allows to build two orthogonal projectors associated with the solenoidal and non-solenoidal subspaces that can solve the low-frequency breakdown without Loops detection. Given that Loop-finding becomes increasingly challenging for increasing order, the scheme is expected to have benefits that even exceeds those already observed in the low-order case.

Efficient algorithms are analyzed as well, included a p-multigrid, to efficiently compute the associated intermediate systems. Finally, numerical validations complements will corroborate the theory. Comparisons with the standard Loop/Star decomposed EFIE (L/S-EFIE), as well as the standard EFIE (see Figure 1) confirm the substantial impact of our new technique on conditioning and on the associated computational efficiency. The numerical study done with multiply-connected objects (see Figure 2) also shows the numerical stability of the resulting solver when the new high-order quasi-Helmholtz projectors presented here are used for modeling complex geometries.

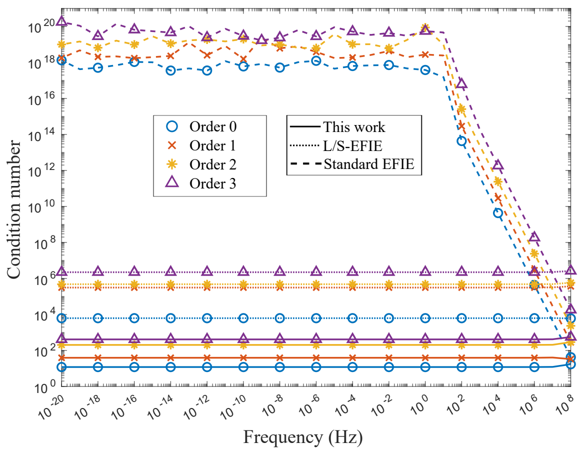

Figure 1. Condition number in function of the frequency for order zero to three (1m radius sphere).

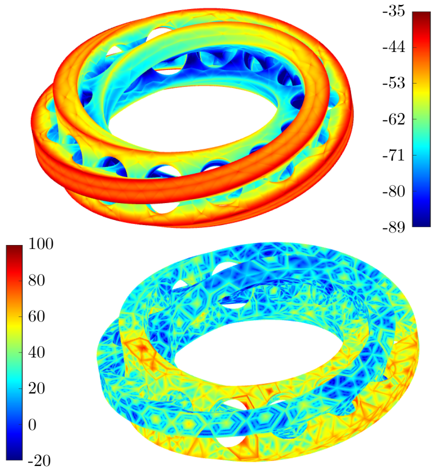

Figure 2. Norm of the surface density current in dB(A/m^2), with projectors (top) and without (bottom).