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HIGHLIGHTS

- We consider the definition of a value function in multicriteria decision aiding
- We introduce the Deck-of-cards-based Ordinal Regression (DOR) method
- DOR conjugates the deck-of-cards method with ordinal regression
- We propose to guide Multiobjective Optimization Problem (MOP) with DOR
- We apply the proposed methodology for planning a sustainable ecovillage

The Deck-of-cards-based Ordinal Regression method and its application for the development of an ecovillage

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Abstract

This paper presents the deck-of-cards-based Ordinal Regression (DOR), a new multicriteria decision-aiding procedure that conjugates the deck-of-cards method with an ordinal regression approach to define a multicriteria value function representing the preferences of the decision maker (DM). The deck-of-cards method allows the DM to express the ranking order of a set of reference alternatives along with the intensity of preferences between reference alternatives. An ordinal regression procedure is then used to define a multicriteria value function that represents the ranking of the reference alternatives as well as the preference intensity. This approach can be applied to define value functions with different formulations, such as weighted sum, additive value, or Choquet integral. The value function thus obtained can be used to comprehensively evaluate alternatives of a multi-criteria decision problem. The value function provided by DOR can also be applied to a multi-objective optimisation problem. In this study, we applied DOR to handle urban and regional planning decisions in which facilities are required to be selected, located, and planned. In particular, we consider the interactions between criteria and synergies between facilities in an enriched version of the so-called space-time model. We applied this methodology to a real-world problem to plan the development of a sustainable ecovillage in the province of Turin (Italy), thus supporting the president of the cooperative owning the ecovillage in his decisions regarding which structures to select, where to locate them, and when to plan their realisation.

Keywords: Urban and Regional Planning; Ordinal Regression; Deck-of-Cards Method; Interactive Multi-objective Optimisation

1. Introduction

Decisions usually require a comparison of alternatives based on different perspectives, which are technically referred to as criteria. For example, when choosing an office to rent (Hammond et al., 1998), one may consider different aspects of candidate locations, such as commuting time from home, access to clients, office services, space, and costs. Generally, when comparing two alternatives, one is better in some respects and the other is superior in others. For example, when considering locations A and B, A may have better access to customers, offer better office services, and have more space, while B may be closer to home and less expensive. To handle similar situations, in the research on Multiple-Criteria Decision Analysis (MCDA), a large corpus of methodologies, procedures, and techniques have been proposed (for an updated and comprehensive collection of state-of-the-art surveys, see (Belton and Stewart, 2002; Greco et al., 2016) and for their historical importance (Köksalan et al., 2016)). Many MCDA approaches are aimed at aggregating evaluations with respect to the considered criteria through a value function that provides a comprehensive evaluation of the available alternatives. The value function must be defined using an appropriate preference elicitation procedure (Keeney and Raiffa, 1976). In this study, we propose a preference elicitation procedure for constructing a value function that conjugates two main approaches from the MCDA domain: the deck-of-cards method (Figueira and Roy, 2002; Abastante et al., 2020) and ordinal regression (Jacquet-Lagrèze and Siskos, 1982, 2001). The deck-of-cards method permits the DM to express their preferences in a simple and understandable form, while ordinal regression permits the effective induction of the parameters of the adopted decision model. With respect to the basic model of ordinal regression, the advantage of the proposed methodology is the consideration not only of ordinal information of the type "alternative a is preferred to alternative b", but also of more cardinal information of the type "a is more preferred to b, than c is preferred to d", that–owing to the deck-of-cards method–can be handled using a "user-friendly" procedure. We call this new methodology a deck-of-cards-based ordinal regression (DOR).

The advantages of user-friendly elicitation procedures, such as DOR, are highly beneficial in any MCDA context, but they can become extremely relevant in complex multi-objective optimisation problems wherein the DM has to be placed in a position of expressing preferences with respect to alternatives that should not only be selected, but also constructed and defined, that is, created (Keeney, 1994).

The handling of multi-objective optimisation problems is not straightforward (Ehrgott and Gandibleux, 2000) and several methods have been proposed, as described in many surveys, books, and collections that address such problems (e.g., Steuer, 1986; Marler and Arora, 2004; Gunantara, 2018). The basic concept of multi-objective optimisation is that, in general, it is not possible to achieve the best possible level of satisfaction for all the objectives; therefore, it is necessary to seek a compromise solution that takes into consideration the preferences of the DM. In this context, a key focus is on Pareto-optimal solutions, which are solutions for which there is no alternative solution that is not worse with respect to all the objectives considered and strictly better than at least one of them. The set of Pareto-optimal solutions is called the Pareto front and contains all the solutions that can potentially be considered to select the best solution. However, the Pareto front may contain a disproportionate number of solutions, often reaching infinity. In addition, the solutions in the Pareto front are generally overwhelmingly heterogeneous. Consequently, the selection of the best solution after the DM has individually examined all the solutions in the Pareto front is an unreasonable approach to multi-objective optimisation problems, even in cases wherein the entire Pareto front or part of it can be analytically described (Zhou et al., 2018). In any case, although several methods have been proposed to determine the entire Pareto front (see, e.g., regarding exact methods (Mavrotas et al., 2015) and, regarding heuristic and metaheuristic methods, (Ehrgott and Gandibleux, 2008)) in order to select the most desirable solution the DM's preferences must be taken into account appropriately. In addition to that, when the problem size increases, the difficulty of finding the non-dominated set of solutions increases as in the case of Multi-objective Integer Programs (Özarık et al., 2020) or even more in the case of mixed integer linear programmming problems (Doğan et al., 2022).

Based on the aforementioned perspective, an interactive multiple-objective optimization (IMOO) methodology is often adopted (Wallenius, 1975; Zionts and Wallenius, 1976; Zionts, 1981; Zionts and Wallenius, 1983; Miettinen and Mäkelä, 2000). While acknowledging that, in general, the DM has no a priori global stable preference when approaching the problem, IMOO methods support the DM in learning about the decision problem and in constructing and updating their preferences during a decision procedure in which the phases of preference elicitation (decision phase) and solution generation (computation phase) alternate (Benayoun et al., 1971; Miettinen et al., 2008). Here, we propose the use of the aforementioned DOR procedure in the elicitation phases. Consequently, the proposed IMOO procedure proceeds as follows. First, we compute the reference solutions for a given optimisation problem. We then present these solutions to the DM and ask them to rank and compare them pairwise in terms of the intensity of preferences using the deck-of-cards method (Figueira and Roy, 2002; Abastante et al., 2020). Using the DOR method, a value function representing the preferences of the DM is then defined. The obtained value function is optimised to determine candidate solutions to the multi-objective optimisation problem. New candidate solutions can be proposed to the DM, who is again asked to comment on those solutions and rank and compare them pairwise. This process continues until the DM is satisfied with one of the proposed solutions. The entire iterative process can be supported using appropriate graphical charts to illustrate the solutions obtained to support the DM throughout the process. As we use a value function that aggregates criteria to evaluate the solutions of the multi-objective optimisation problem, in the following, we use the terms criterion and objective as equivalents.

The proposed approach has several advantages:

- The DM can participate in the decision-making process by expressing their preferences easily thanks to the use of the deck-of-cards method.
- The deck-of-cards method is applied for eliciting the preferences of the DM and incorporating them in the solutions of an optimization model instead of being used for expressing more abstract judgments on the importance and interaction of criteria. In this manner, the cognitive burden of the DM is reduced, thus allowing the DM to directly comment on some "feasible" plans and making the process easier and more similar to what occurs in reality.
- On the basis of the preferences elicited from the DM, the ordinal regression model permits the definition of a value function with a degree of complexity that can range, for instance, from the basic weighted sum to the more sophisticated Choquet integral.
- The DM can iteratively build the solutions along with the analyst while returning to their preferences at every step of the process.
- The whole process is transparent and straightforward for the DM and provides arguments to explain the selected solutions to other stakeholders to arrive at a participated decision.

We applied the above methodology to urban and regional planning, which we approached in terms of multi-objective optimisation (Miettinen et al., 2008) to make decisions regarding the choice of facilities to implement, their location, and their time of implementation under certain constraints (Pujadas et al., 2017). Such decisions are very complex as many perspectives must be taken into consideration and many actors are involved. From this perspective, transparent and participatory procedures are beneficial for supporting decision-making in this domain. We applied the above methodology to a sustainable territorial decision-making process, whereby the following three questions should be answered in the context of the so-called *space–time model* proposed by (Barbati et al., 2020):

1. What facilities are required to be selected when planning for a territory?
2. Where should we locate these facilities?
3. When should those facilities be activated?

In complex real-world decision problems, these three questions should be considered simultaneously. Indeed, it is sporadic, particularly in large multi-million-euro planning procedures, that a developer can do everything in one shot (Ingaramo et al., 2022). Furthermore, administrators and developers are increasingly pushing for a careful study of the scheduling of interventions in the plan owing to several restrictions or constraints, such as budget constraints, that need to be considered. Several optimisation models consider only certain aspects of the urban and regional planning, while answering only one of the three aforementioned questions, e.g. questions 1), 2), and 3) were respectively answered in (Tervonen et al., 2017), (Farahani et al., 2019), and (Le Bivic and Melot, 2020), while a combination of questions 2) and 3) was answered in (Sarnataro et al., 2021). Instead, while adopting the space-time model, we developed an approach that supports the strategic decision of answering all three questions simultaneously.

We tested a methodology for establish an ecovillage in the Piedmont region of Italy. According to the Global ecovillage Network, (The Global Ecovillage Network, 2023), an ecovillage is "an intentional, traditional, or urban community that is consciously designed through locally owned participatory processes in all four dimensions of sustainability (social, cultural, ecological, and economic) to regenerate social and natural environments". The principles of this type of community tend to be the voluntary adhesion of participants and sharing of the founding principles, the creation of living nuclei designed to minimise environmental impact, the use of renewable energy, and food self-sufficiency based on organic forms of agriculture. In this sense, the reality of ecovillages intends to give life to new forms of cohabitation, such as responding to the current disintegration of the family, cultural, and social fabric, constituting a laboratory for research and experimentation towards alternative lifestyles to the most widespread socioeconomic models. The use of the space–time model and interactive procedure is particularly indicated for such a problem for the following reasons:

- The DM can realize that the ecovillage should be treated as a whole system in which the decisions related to the facilities to be installed, their location, and when they should be executed are interrelated in a common overall perspective strategy for which the space–time model appears to be the most natural methodological scheme.
- The DM can verify that the budget and technical requirements impose constraints regarding when each facility can and should be built.
- The DM can recognize that in the setup of an ecovillage, a variety of criteria have to be considered because of its characteristic of being a self-sufficient village and not a mere profitable investment. These criteria can also be different from more classical criteria in terms of decisions related to conventional touristic structures.
- The criteria can present a certain interaction between them that has to be taken into consideration appropriately and, in this perspective, we generalize the space–time model to the consideration of the interaction between the criteria (more precisely and more technically, representing the preferences of the DM with a value function formulated in terms of a Choquet integral). Moreover, the weights and the interaction of the considered criteria and their definition and interaction are not always clearly intelligible, even for the DMs. Therefore, the use of the DOR methodology permits an easily understandable indirect preference elicitation procedure because, in this manner, the DM was asked to compare some feasible plans comprehensively through a user-friendly and straightforward procedure, i.e., the deck-of-cards method, which is characterized, in our opinion, by a minimum level of cognitive burden and by several other advantages (Corrente et al., 2021). Instead, a preference elicitation procedure requiring DM's preferences information expressed in relatively abstract terms such as the importance and interaction of the considered criteria would be much more complex and cognitively demanding, with the risk of obtaining insufficiently reliable results.
- Finally, the introduction of an interactive multi-objective methodology helps in making a participatory decision, also owing to DOR elicitation procedure. It takes into consideration the perspective of the DM in guaranteeing openness and transparency to the public, in the general perspective of a decision model co-constructed by the analyst with the DM (Roy, 1993).

This is a 'non-ordinary' case study that intercepts an increasingly widespread demand for new ways of living, dwelling, working and relating to the planet. It is likely that experts will increasingly be asked to help make decisions considering unconventional criteria and alternatives; thus, this specific case study constitutes a type of stress test for the methodology, precisely because of the nature of the reasoning and decisions to be made.

The remainder of this paper is organised as follows. After the introduction, Section 2 outlines the DOR elicitation procedure, while Section 3 introduces the DOR-guided interactive multi-objective optimisation procedure and explains the method of applying it to the space–time model to handle regional and urban planning problems. Section 4 describes the real-world problems analysed. Section 5 illustrates the interaction process conducted with the DM and the results obtained, and the last section presents the conclusions of this study and possible research developments.

2. Deck-of-cards-based ordinal regression method

In this section, we present the DOR method. This is based on a combination of the deck-of-cards method (Figueira and Roy, 2002) in the formulation proposed in (Abastante et al., 2020) (SRF-II) with an ordinal regression method (Jacquet-Lagrèze and Siskos, 1982). In Section 3, this elicitation procedure is used to handle an optimisation problem formulated in terms of the space–time model (Barbati et al., 2020). However, in general, it has an autonomous interest in MCDA problems. It can be used to induce the preference parameters of the Choquet integral (Choquet, 1953; Grabisch, 1997) and other multicriteria aggregation procedures such as the most straightforward weighted sum or piecewise additive value function considered in the UTA method (Jacquet-Lagrèze and Siskos, 1982).

We assume that the set of alternatives $\mathcal{A}$ to be considered in the decision problem at hand are evaluated with respect to a set of criteria $\mathcal{G} = \{g_1, \ldots, g_m\}$ for which, without the loss of generality, $g_j : \mathcal{A} \rightarrow \mathbb{R}^+$, and for all $a, b \in \mathcal{A}$, a is at least as good as b with respect to the criterion g_j if $g_j(a) \geqslant g_j(b), j = 1, \ldots, m$. In this context, for each alternative $a \in \mathcal{A}$, the weighted sum assigns an overall evaluation

$$U(a) = \sum_{j=1}^{m} w_j g_j(a)$$

where $w_j \geqslant 0, j = 1, \ldots, m, \sum_{j=1}^{m} w_j = 1$, and for all $a, b \in \mathcal{A}$, a is comprehensively at least as good as b if $U(a) \geqslant U(b)$.

A slightly more sophisticated formulation for the overall evaluation of alternatives from $\mathcal{A}$ is provided by the piecewise additive value function proposed in the UTA method (Jacquet-Lagrèze and Siskos, 1982). Let us assume that the criteria $g_j \in G$ assign to the alternatives $a \in \mathcal{A}$ values $g_j(a)$ in the interval $[y_j^0, y_j^{\gamma_j}]$ divided into sub-intervals

$$[y_j^0, y_j^1], \ldots, [y_j^r, y_j^{r+1}], \ldots, [y_j^{\gamma_j - 1}, y_j^{\gamma_j}].$$

The overall value function $U : \mathcal{A} \rightarrow [0, 1]$ assigns each alternative $a \in \mathcal{A}$ the following overall evaluation:

$$U(a) = \sum_{j=1}^{m} u_j(g_j(a)) \tag{1}$$

with

$$u_j(g_j(a)) = u_j(y_j^r) + \frac{g_j(a) - y_j^r}{y_j^{r+1} - y_j^r}[u_j(y_j^{r+1}) - u_j(y_j^r)]$$

for $g_j(a) \in [y_j^r, y_j^{r+1}]$, where $j = 1, \ldots, m$. Therefore, once the values $u_j(y^r), r = 0, \ldots, \gamma_{j-1}, j = 1, \ldots, m$ are fixed, the values $u_j(g_j(a))$, where $a \in \mathcal{A}$, are assigned using linear interpolation. The monotonicity of the overall evaluation $U(a)$ with respect to the evaluations $g_j(a), j = 1, \ldots, m$, requires that $u_j(y_j^{r+1}) \geq u_j(y_j^r)$ for all $j = 1, \ldots, m$. Moreover, the normalisation of the overall evaluations $U(a), a \in \mathcal{A}$, for which

$0 \leqslant U(a) \leqslant 1$, is ensured by imposing $u_j(y_j^0) = 0$ for all $j = 1, \ldots, m$, and $\sum_{j=1}^{m} u_j(y^{\gamma_j}) = 1$.

It is observed that the normalisation constraint

$$\sum_{g_j \in \mathcal{G}} u_j(y^{\gamma_j}) = 1$$

can be substituted with any constraint.

$$\sum_{g_j \in \mathcal{G}} u_j(y^{\gamma_j}) = \overline{U}, \overline{U} \in \mathbb{R}^+.$$

For example, in the didactic example in Section 2.3, for the sake of a greater expressivity, we consider $\overline{U} = 100$.

In the next section, we introduce the formulation of the overall value function $U(\cdot)$ expressed in terms of the Choquet integral (Choquet, 1953) to represent the interaction between the criteria, which deserves a specific space, as it represents a more complex model than the previous formulations in terms of the weighted sum and piecewise value function.

2.1. Modelling interaction between the criteria through the Choquet integral

To take into consideration the interaction between the criteria, a comprehensive value function $U(\cdot)$ can be expressed in terms of the Choquet integral (Choquet, 1953; Grabisch, 1996). With this aim, we introduce the concept of capacity as a function $\mu : 2^{\mathcal{G}} \rightarrow [0, 1]$ that satisfies the following properties:

- Normalization: $\mu(\emptyset) = 0$ and $\mu(\mathcal{G}) = 1$
- Monotonicity: for all $A \subseteq B \subseteq \mathcal{G}, \mu(A) \leq \mu(B)$

For all $A \subseteq \mathcal{G}, \mu(A)$ can be interpreted as a value such that, taking into consideration an alternative a for which $g_j(a) = k > 0$ for all $g_j \in A$ and $g_j(a) = 0$ for all $g_j \notin A$, we have $U(a) = k \cdot \mu(A)$. Given an alternative a and capacity μ, the Choquet integral assigns a comprehensive evaluation to each alternative a formulated as

$$U(a) = \sum_{j=1}^{m} \mu(\{g_h \in \mathcal{G} : g_h(a) \geq g_{(j)}(a)\}) \cdot [g_{(j)}(a) - g_{(j-1)}(a)] \tag{2}$$

with $g_{(1)}(a), \ldots, g_{(m)}(a)$ being a reordering of $g_1(a), \ldots, g_m(a)$ such that

$$g_{(0)}(a) \leq g_{(1)}(a) \leq \ldots \leq g_{(m)}(a),$$

with $g_{(0)}(a) = 0$. It is observed that the formulation (2) of the Choquet integral can be rewritten as

$$U(a) = \mu(\{g_{(m)}\}) \cdot g_{(m)}(a) + \sum_{j=1}^{m-1} \left[\mu(\{g_h \in \mathcal{G} : g_h(a) \geq g_{(j)}(a)\}) - \mu(\{g_h \in \mathcal{G} : g_h(a) \geq g_{(j+1)}(a)\})\right] \cdot g_{(j)}(a) \tag{3}$$

It should be noted that a capacity is additive if for all $A, B \subseteq \mathcal{G}$ such that $A \cap B = \emptyset, \mu(A \cup B) = \mu(A) + \mu(B)$. In this case, we can set $\mu(\{g_j\}) = w_j$ for all $g_j \in \mathcal{G}$, and owing to the normalisation and monotonicity properties of μ, we obtain $w_j \geq 0$ for all $g_j \in \mathcal{G}$ and $w_1 + \ldots + w_m = 1$. Moreover, we also obtain $U(a) = \sum_{j \in J} w_j g_j(a)$; that is, if the capacity μ is additive, the Choquet integral formulation (3) collapses to the weighted sum formulation (1).

If additivity does not hold, the criteria g_j from G interact with each other. For simplicity, we consider a specific form of interaction that permits to obtain manageable models, while still allowing us to represent general situations. More precisely, we consider a two-additive capacity (Grabisch, 1997), that is, a capacity μ such that there exist w_j, $j = 1, \ldots, m$, and $w_{jj'}, \{j, j'\} \subseteq \mathcal{G}$, such that for all $A \subseteq \mathcal{G}$,

$$\mu(A) = \sum_{g_j \in A} w_j + \sum_{\{g_j, g_{jj'}\} \subseteq A} w_{jj'} \tag{4}$$

With respect to the two-additive capacities, the normalisation and monotonicity properties can be reformulated as

- Normalization: $\sum_{g_j \in A} w_j + \sum_{\{g_j, g_{jj'}\} \subseteq A} w_{jj'} = 1$,
- Monotonicity: $w_j \geq 0$ for all $g_j \in \mathcal{G}$ and

$$w_j + \sum_{g_{j'} \in T} w_{jj'} \geq 0, \text{ for all } g_j \in \mathcal{G} \text{ and for all } T \subseteq \mathcal{G} \setminus \{g_j\}, T \neq \emptyset. \tag{5}$$

If μ is a two-additive capacity, then the Choquet integral, which in this case we call the two-additive Choquet integral, can be expressed as follows:

$$U(a) = \sum_{g_j \in \mathcal{G}} w_j g_j(a) + \sum_{\{g_j, g_{jj'}\} \subseteq \mathcal{G}} w_{jj'} \min\{g_j(a), g_{j'}(a)\}. \tag{6}$$

(6) can be obtained by observing that if the capacity μ is two-additive, then

$$\mu(\{g_h \in \mathcal{G} : g_h(a) \geq g_{(j)}(a)\}) - \mu(\{g_h \in \mathcal{G} : g_h(a) \geq g_{(j+1)}(a)\}) = w_{(j)} + \sum_{h>j} w_{(j)(h)}$$

such that, from (3), we obtain

$$U(a) = w_{(m)} g_{(m)}(a) + \sum_{j=1}^{m-1} [w_{(j)} + \sum_{h>j} w_{(j)(h)}] g_{(j)}(a)$$

where, after observing that for all $h > j, j = 1, \ldots, m-1, g_{(j)}(a) = \min\{g_{(h)}(a), g_{(j)}(a)\}$, we obtain (6).

2.2. *Deck-of-cards-based ordinal regression*

To define the comprehensive value function $U(\cdot)$, we must elicit its parameters, that is,

- The weights w_j, where $j = 1, \ldots, m$, for the weighted sum
- The values $u_j(y^r)$, where $r = 0, \ldots, \gamma_j$, where $j = 1, \ldots, m$, for the piecewise linear value function,
- The weights $w_j, j = 1, \ldots, m$ and $w_{j,j'}, j = 1, \ldots, m-1, j' = j+1, \ldots, m$, for the two-additive Choquet integral.

With this aim, we propose DOR, which is a new ordinal regression procedure that takes into consideration the intensity of preferences expressed through the deck-of-cards method (Figueira and Roy, 2002; Abastante et al., 2020). The procedure consists of the following steps:

- A set of reference alternatives $\mathcal{A}^* \subseteq \mathcal{A}$ is presented to the DM.

- The DM rank orders the alternatives from $\mathcal{A}^*$ from worst to best with possible ex-aequo, in r, where $r \leq p$, with equivalence classes $C_1, \ldots, C_r$, such that C_1 contains the alternatives that are considered the worst, C_r contains the alternatives considered the best, and, in general, if the alternative a is contained in the equivalence class C_s, and if the alternative b is contained in the equivalence class $C_{s'}$ with $s' > s$, then b is preferred to a. In particular, a DM is given a set of cards, with each one representing an alternative from $\mathcal{A}^*$, and the DM orders these cards in agreement with the expressed preferences.

- The DM puts a certain number of blank cards e_s, $s = 1, \ldots, p-1$, between the cards representing the alternatives in the equivalence class C_s and the cards representing the alternatives in the equivalence class C_{s+1}, where $s = 1, \ldots, r-1$, such that the greater the number of blank cards, the greater the difference in the preferences between the alternatives $b \in C_{s+1}$ and $a \in C_s$; the DM also has the option to put e_0 blank cards between a "zero level" and the equivalence class C_1.

- An evaluation $\nu(a) = v_s, s = 1, \ldots, p$, is assigned to each alternative from C_s while applying the following rule.
$$v_s = v_{s-1} + e_{s-1} + 1$$
so that
$$v_s = \sum_{z=0}^{s-1}(e_z + 1) = \sum_{z=0}^{s-1} e_z + s.$$

- The parameters of the comprehensive value function $U(\cdot)$ are elicited by minimizing the sum of the positive and negative deviations $\sigma^+(a)$ and $\sigma^-(a)$, $a \in \mathcal{A}^*$, between the evaluations $U(a)$ assigned by the value function and the evaluations $\nu(a)$ assigned via the deck-of-cards method, appropriately scaled through a multiplicative positive constant k. With this aim, one has to solve the following linear programming (LP) problem with variables that are the parameters of the value function $U(\cdot)$, the deviations $\sigma^+(a)$ and $\sigma^-(a)$, $a \in \mathcal{A}^*$, and the scaling constant k:
$$\begin{array}{l} \min \sum_{a \in \mathcal{A}^*} \sigma^+(a) + \sigma^-(a) \\ \text{subject to} \\ \left.\begin{array}{l} E_{Deck-of-cards\ basis} \\ E_{value\ function} \end{array}\right\} \end{array} \tag{7}$$
with
$$\left.\begin{array}{l} U(a) - \sigma^+(a) + \sigma^-(a) = k \cdot \nu(a) \ \text{ for all } a \in \mathcal{A}^*, \\ k \geq 0, \\ \sigma^+(a) \geqslant 0, \sigma^-(a) \geqslant 0 \text{ for all } a \in \mathcal{A}^* \end{array}\right\} E_{Deck-of-cards\ basis} \tag{8}$$
an $E_{value\ function}$ being a set of constraints related to the specific formulation of the value function $U(\cdot)$. Furthermore, the above LP problem can be applied to any form of the value function $U(\cdot)$, such as the aforementioned weighted sum, additive piecewise linear value function, and Choquet integral. For the remaining three cases of the weighted sum, additive piecewise linear value function, and Choquet integral, the set of constraints $E_{value\ function}$ is formulated as follows:

$$\left.\begin{array}{l} U(a) = \sum_{g_j \in \mathcal{G}} w_j g_j(a), \\ \sum_{j=1}^{m} w_j = 1, \\ w_j \geqslant 0, \text{ for all } j = 1, \ldots, m \end{array}\right\} E_{value\ function\ (weighted\ sum)} \tag{9}$$

$$\left.\begin{array}{l} U(a) = \sum_{g_j \in \mathcal{G}} u_j(g_j(a)) \\ u_j(g_j(a)) = u_j(y_j^r) + \frac{g_j(a) - y_j^r}{y_j^{r+1} - y_j^r}[u_j(y_j^{r+1}) - u_j(y_j^r)] \\ \qquad \text{for } g_j(a) \in [y_j^r, y_j^{r+1}] \\ u_j(y_j^{r+1}) \geq u_j(y_j^r) \text{ for all } j = 1, \ldots, m, r = 0, \ldots, \gamma_j - 1, \\ u_j(y_j^0) = 0 \quad \text{for all } j = 1, \ldots, m, \\ \sum_{g_j \in \mathcal{G}} u_j(y^{\gamma_j}) = 1 \end{array}\right\} E_{value\ function\ (piecewise\ linear)} \tag{10}$$

$$\left.\begin{array}{l} U(\mathbf{x}) = \sum_{g_j \in \mathcal{G}} w_j g_j(a) + \sum_{\{g_j, g_{jj'}\} \subseteq, \mathcal{G}} w_{jj'} \min\{g_j(a), g_{j'}(a)\}, \\ \sum_{g_j \in \mathcal{G}} w_j + \sum_{\{g_j, g_{jj'}\} \subseteq \mathcal{G}} w_{jj'} = 1, \\ w_j \geqslant 0, \text{ for all } j = 1, \ldots, m, \\ w_j + \sum_{g_{j'} \in T} w_{jj'} \geq 0, \text{ for all } g_j \in \mathcal{G} \text{ and for all } T \subseteq \mathcal{G} \setminus \{g_j\}, T \neq \emptyset \end{array}\right\} E_{value\ function\ (Choquet\ integral)} \tag{11}$$

We now discuss the ordinal regression optimisation problem (7). Ideally, one would define a value function $U(\cdot)$ that can perfectly represent the value $\nu(a)$ assigned to the reference alternatives a from $\mathcal{A}^*$ through the deck-of-cards method, appropriately scaled using a scaling constant $k > 0$, which formally means that one is looking for a value function satisfying the following condition:

$$U(a) = k\nu(a), a \in \mathcal{A}^*. \tag{12}$$

As, in general, this could not be possible, the optimization problem (7) searches for the value function that, among the possible value functions belonging to a given class (weighted sum, additive piecewise linear value function, and Choquet integral), the best approximates the desired condition (12). To this end, for each alternative $a \in \mathcal{A}^*$, a positive and a negative deviation $\sigma^+(a)$ and $\sigma^-(a)$, where $\sigma^+(a) \geqslant 0$ and $\sigma^-(a) \geqslant 0$, are introduced such that condition (12) is reformulated as

$$U(a) - \sigma^+(a) + \sigma^-(a) = k\nu(a), a \in \mathcal{A}^* \tag{13}$$

Through the optimisation problem (7), the value function $U(\cdot)$ is searched for, and the total sum of the deviations $\sum_{a \in \mathcal{A}^*} \sigma^+(a) + \sigma^-(a)$ is minimised because this is one possible formulation of the concept of the value function that best approximates the condition (12) (we discuss other possible formulations of this concept in this same section). The ordinal regression optimisation problem (7) minimises the sum of the deviations subject to two sets of constraints:

- $E_{Deck-of-cards\ basis}$, containing conditions (13) expressing the general requirement of adherence of the value function to the DM's preference information as represented by the value $\nu(a)$ assigned to the alternatives a from $\mathcal{A}^*$ via the deck-of-cards method plus the non-negativity of deviations $\sigma^+(a)$ and $\sigma^-(a)$,

- $E_{value\ function}$ ($E_{valuefunction(weighted\ sum)}$, $E_{valuefunction(piecewise\ linear)}$, and $E_{valuefunction(Choquet\ integral)}$ for the three cases of the weighted sum, piecewise value function, and Choquet integral, respectively) containing conditions defining the value function $U(\cdot)$ in terms of the parameters to be determined through the solution of (7).

If the optimisation problem (7) provides a solution for which $\sum_{a\in\mathcal{A}^*}\sigma^+(a)+\sigma^-(a)=0$, then in the class of the considered value functions, there is one that can perfectly represent the DM's preference information. The concept of the best-approximating value function (12) can also be formulated in terms of a value function that minimises the maximal deviations $\sigma^+(a)$ and $\sigma^-(a), a \in \mathcal{A}^*$. This can be obtained by reformulating the ordinal regression optimisation problem (7) as follows:

$$
\begin{aligned}
&\min \gamma\\
&\text{subject to}\\
&\left.\begin{array}{l}
\gamma \geqslant \sigma^+(a),\ a \in \mathcal{A}^*\\
\gamma \geqslant \sigma^-(a),\ a \in \mathcal{A}^*\\
E_{Deck-of-cards\ basis}\\
E_{value\ function}
\end{array}\right\}
\end{aligned}
\tag{14}
$$

Other possible formulations of the ordinal regression optimisation problem can be obtained by combining the two above formulations (7) and (14), for example, as follows:

- By minimizing the maximum deviation in the set of the value functions in the considered class, having a sum of deviations $\sum_{a\in\mathcal{A}^*}\sigma^+(a)+\sigma^-(a)$ not greater than $S^*+\varepsilon^S$, with S^* being the minimal possible sum of deviations provided by the optimization problem (7), and ε^S being a predefined tolerance threshold, that is,

$$
\begin{aligned}
&\min \gamma\\
&\text{subject to}\\
&\left.\begin{array}{l}
\sum_{a\in\mathcal{A}^*}\sigma^+(a)+\sigma^-(a) \leqslant S^*+\varepsilon^S\\
\gamma \geqslant \sigma^+(a),\ a \in \mathcal{A}^*\\
\gamma \geqslant \sigma^-(a),\ a \in \mathcal{A}^*\\
E_{Deck-of-cards\ basis}\\
E_{value\ function}
\end{array}\right\}
\end{aligned}
\tag{15}
$$

- By minimizing the sum of deviations in the set of value functions in the considered class having deviations $\sigma^+(a)$ and $\sigma^-(a)$, $a \in \mathcal{A}^*$, not greater than $\gamma^*+\varepsilon^\gamma$, with γ^* being the minmax of the deviations provided by optimization problem (14), and ε^γ being a predefined tolerance threshold, that is,

$$\left.\begin{array}{l}\min \sum_{a\in\mathcal{A}^*} \sigma^+(a)+\sigma^-(a)\\ \text{subject to}\\ \sigma^+(a)\leqslant \gamma^*+\varepsilon^\gamma,\ a\in\mathcal{A}^*\\ \sigma^-(a)\leqslant \gamma^*+\varepsilon^\gamma,\ a\in\mathcal{A}^*\\ E_{Deck-of-cards\ basis}\\ E_{value\ function}\end{array}\right\} \quad (16)$$

Some concluding remarks are useful at the end of this section:

- The selection of the analytical form of the value function depends on the specific nature of the decision problem. In general, to select from among the three cases considered above, the weighted sum, Choquet integral, or additive piecewise linear value function, we can say the following:
  - If there is an interest in working with a decision model that is as simple as possible, the weighted sum should be selected.
  - If interactions between the criteria have to be taken into consideration, as is the case for the case study we are considering in the real-world application presented in Sections 4 and 5, the Choquet integral appears to be the most adequate formulation of the value function.
  - If there is an interest in considering how the contribution to the value function of each criterion changes from one level to the other, the additive piecewise linear value function should be selected.
  - In this first proposal of the DOR method, we do not extend our approach to the multiplicative function (Keeney and Raiffa, 1976) that would imply the adoption of nonlinear methods. Another interesting form for the value function $U(\cdot)$ is the enriched additive value function proposed in (Greco et al., 2014), wherein the aforementioned additive piecewise linear value function is augmented by components modelling positive and negative interactions between pairs of criteria. Moreover, in this case, we do not extend our approach to computational problems here (related to the formulation of a specific problem).
- We considered the elicitation of the DM's preference information using the deck-of-cards method. However, similar preference information can be collected using different scaling methods, such as AHP (Saaty, 1977), BWM (Rezaei, 2015) and MACBETH (Bana e Costa and Vansnick, 1994). In any one of these cases, as in the considered deck-of-cards method, a set of reference alternatives $\mathcal{A}^*$ can be presented to the DM that can provide the pairwise judgments required by each of these methods, such that, by applying the same methods, a comprehensive value $\nu(a)$ can be assigned to each alternative $a \in \mathcal{A}^*$. Once the above values $\nu(a)$ are obtained, the value function $U(\cdot)$ can be obtained by solving the ordinal regression optimisation problem discussed in this section.

2.3. Didactic example

In this section, with a simple didactic example, we illustrate the procedure for inducing a value function by means of the DOR method. Let us suppose that we have six projects P_1, P_2, P_3, P_4, P_5 and P_6 evaluated on a $[0-100]$ scale with respect to the three criteria of economic aspects g_1, social aspects g_2, and environmental aspects g_3, as shown in Table 1.

Using the deck-of-cards method and taking into consideration a "zero project" P_0 as a reference of a null value level, the DM orders the projects from the worst $P_{\{1\}}$ to the best $P_{\{6\}}$, with the number of blank

Table 1: Evaluations of projects with respect to considered criteria

Projects	Economic aspects: g_1	Social aspects: g_2	Environmental aspects: g_3
$\mathtt{P}_1$	80	50	75
$\mathtt{P}_2$	60	60	60
$\mathtt{P}_3$	60	80	50
$\mathtt{P}_4$	70	60	70
$\mathtt{P}_5$	50	70	60
$\mathtt{P}_6$	90	50	40

cards e_s between the project $P_{\{s-1\}}$ and the following $P_{\{s\}}$, where $s = 1, \ldots, 6$, written between brackets [ ], as follows:

$$\mathtt{P}_0 \; [40] \; \mathtt{P}_5 \; [1] \; \mathtt{P}_2 \; [1] \; \mathtt{P}_3 \; [6] \; \mathtt{P}_6 \; [1] \; \mathtt{P}_4 \; [4] \; \mathtt{P}_1$$

On applying the deck-of-cards method, we assign the following value to each project:

- $\nu(P_0 = [0, 0, 0]) = 0$,
- $\nu(P_{\{1\}} = P_5 = [50, 70, 60]) = \nu(P_0) + e_1 + 1 = 41$,
- $\nu(P_{\{2\}} = P_2 = [60, 60, 60]) = \nu(P_5) + e_2 + 1 = 43$,
- $\nu(P_{\{3\}} = P_3 = [60, 80, 50]) = \nu(P_2) + e_3 + 1 = 45$,
- $\nu(P_{\{4\}} = P_6 = [90, 50, 40]) = \nu(P_3) + e_4 + 1 = 52$,
- $\nu(P_{\{5\}} = P_4 = [70, 60, 70]) = \nu(P_6) + e_5 + 1 = 54$,
- $\nu(P_{\{6\}} = P_1 = [80, 50, 75]) = \nu(P_4) + e_6 + 1 = 59$.

Considering the value function $U(\cdot)$ expressed in terms of a weighted sum, the ordinal regression methodology proposed in Section 2.2 can then be applied to solve the following LP problem for the variables $w_1, w_2, w_3, \sigma^+(\mathbf{P_i}), \sigma^-(\mathbf{P_i}), i = 1, \ldots, 6$, and k:

$$\begin{array}{l} \min \sum_{i=1}^{6} \sigma^+(\mathbf{P}_i) + \sigma^-(\mathbf{P}_i) \\ \text{subject to} \\ \left. \begin{array}{l} U(\mathbf{P}_i) = w_1 g_1(\mathbf{P}_i) + w_2 g_2(\mathbf{P}_i) + w_3 g_3(\mathbf{P}_i), \quad i = 1, \ldots, 6 \\ U(\mathbf{P}_i) - \sigma^+(\mathbf{P}_i) + \sigma^-(\mathbf{P}_i) = k \cdot \nu(\mathbf{P}_i), \quad i = 1, \ldots, 6, \\ w_1 + w_2 + w_3 = 1, \\ w_1 \geqslant 0, w_2 \geqslant 0, w_3 \geqslant 0, \\ k \geq 0, \\ \sigma^+(\mathbf{P}_i) \geqslant 0, \sigma^-(\mathbf{P}_i) \geqslant 0, \quad i = 1, \ldots, 6. \end{array} \right\} \end{array} \quad (17)$$

The solution of the LP problem (17) yields the results listed in Table 2 with a scaling constant $k = 1.282$ and the following weights for the considered criteria: $w_1 = 0.517, w_2 = 0.079, w_3 = 0.404$. The total sum of the errors $\sum_{i=1}^{6} \sigma^+(\mathbf{P}_i) + \sigma^-(\mathbf{P}_i)$ is 8.09.

Table 2: Scores assigned to projects by the value function $U(\cdot)$ obtained solving the LP problem (17)

Projects	$U(\mathbf{P}_i)$	$\nu(\mathbf{P}_i)$	$k \cdot \nu(\mathbf{P}_i)$	$\sigma^+(\mathbf{P}_i)$	$\sigma^-(\mathbf{P}_i)$
$\mathtt{P}_1$	75.62	59	75.62	0	0
$\mathtt{P}_2$	60	43	55.12	4.88	0
$\mathtt{P}_3$	57.53	45	57.68	0	0.15
$\mathtt{P}_4$	69.21	54	69.21	0	0
$\mathtt{P}_5$	55.61	41	52.55	3.06	0
$\mathtt{P}_6$	66.65	52	66.65	0	0

When considering a value function expressed in terms of an additive piecewise linear value function, we divide the interval $[0, 100]$ of possible values assigned by the criteria g_1, g_2, g_3 into the intervals

$$[0, 50], [50, 75], [75, 100].$$

The following LP problem in the variables $u_j(0), u_j(50), u_j(75)$, and $u_j(100)$, where $j = 1, 2, 3$, $\sigma^+(\mathbf{P_i})$ and $\sigma^-(\mathbf{P_i})$, where $i = 1, \ldots, 6$, and k is required to be solved:

$$\left.\begin{array}{l}
\min \sum_{i=1}^{6} \sigma^+(\mathbf{P}_i) + \sigma^-(\mathbf{P}_i) \\
\text{subject to} \\
\quad U(\mathbf{P}_i) - \sigma^+(\mathbf{P}_i) + \sigma^-(\mathbf{P}_i) = k \cdot \nu(\mathbf{P}_i), \;\; i = 1, \ldots, 6, \\
\quad U(\mathbf{P}_i) = \sum\limits_{g_j \in G} u_j(g_j(\mathbf{P}_i)), \\
\quad u_j(g_j(\mathbf{P}_i)) = u_j(y_j^r) + \frac{g_j(\mathbf{P}_i) - y_j^r}{y_j^{r+1} - y_j^r}[u_j(y_j^{r+1}) - u_j(y_j^r)] \text{ for } g_j(\mathbf{P}_i) \in [y_j^r, y_j^{r+1}], \\
\quad u_j(50) \geq u_j(0), j = 1, 2, 3, \\
\quad u_j(75) \geq u_j(50), j = 1, 2, 3, \\
\quad u_j(100) \geq u_j(75), j = 1, 2, 3, \\
\quad u_j(0) = 0, j = 1, 2, 3, \\
\quad u_1(100) + u_2(100) + u_3(100) = 100, \\
\quad k \geq 0, \\
\quad \sigma^+(\mathbf{P}_i) \geqslant 0, \sigma^-(\mathbf{P}_i) \geqslant 0 \;\; i = 1, \ldots, 6.
\end{array}\right\} \quad (18)$$

The solution to the LP problem (18) provides the marginal value function determined by the values $u_j(0), u_j(50), u_j(75)$, and $u_j(100)$, where $j = 1, 2, 3$, as shown in Table 3, with the scaling constant $k = 1.11$. The projects $\mathbf{P}_i$, where $i = 1, \ldots, 6$, receive the evaluations listed in Table 3. The total sum of errors $\sum_{i=1}^{6} \sigma^+(\mathbf{P}_i) + \sigma^-(\mathbf{P}_i)$ is equal to zero. We observe that in the LP problem (26), through the constraint

$$u_1(100) + u_2(100) + u_3(100) = 100$$

we set $\overline{U} = 100$.

Finally, taking into consideration a value function expressed in terms of the Choquet integral, the following LP problem (19) must be solved for the variables $w_1, w_2, w_3, w_{12}, w_{23}, w_{13}$, k, $\sigma^+(\mathbf{P_i})$, and $\sigma^-(\mathbf{P_i})$, where $i = 1, \ldots, 6$:

Table 3: Reference values defining the piecewise additive value function U obtained on solving the LP problem (18)

	$u_j(0)$	$u_j(50)$	$u_j(75)$	$u_j(100)$
Economic aspects	0	31.48	47.22	64.81
Social aspects	0	0	10.19	20.37
Environmental aspects	0	0	14.81	14.81

Table 4: Scores assigned to projects by the value function $U(\cdot)$ obtained on solving the LP problem (18)

Projects	$U(\mathbf{P}_i)$	$\nu(\mathbf{P}_i)$	$k\cdot\nu(\mathbf{P}_i)$	$\sigma^+(\mathbf{P}_i)$	$\sigma^-(\mathbf{P}_i)$
$\mathtt{P}_1$	65.56	59	65.56	0	0
$\mathtt{P}_2$	47.78	43	47.78	0	0
$\mathtt{P}_3$	50.00	45	50.00	0	0
$\mathtt{P}_4$	60.00	54	60.00	0	0
$\mathtt{P}_5$	45.56	41	45.56	0	0
$\mathtt{P}_6$	57.78	52	57.78	0	0

$$\begin{array}{l}\min \sum_{i=1}^{6} \sigma^+(\mathbf{P}_i)+\sigma^-(\mathbf{P}_i)\\ \text{subject to}\\ \left.\begin{array}{l} U(\mathbf{P}_i)-\sigma^+(\mathbf{P}_i)+\sigma^-(\mathbf{P}_i)=k\cdot\nu(\mathbf{P}_i)\ \ i=1,\ldots,6,\\ U(\mathbf{P}_i)=w_1g_1(\mathbf{P}_i)+w_2g_2(\mathbf{P}_i)+w_3g_3(\mathbf{P}_i)+\\ +w_{12}min\{g_1(\mathbf{P}_i),g_2(\mathbf{P}_i)\}+w_{13}min\{g_1(\mathbf{P}_i),g_3(\mathbf{P}_i)\}+w_{23}min\{g_2(\mathbf{P}_i),g_3(\mathbf{P}_i)\},\\ w_1+w_2+w_3+w_{12}+w_{23}+w_{13}=1,\\ w_j+\sum_{g_{j'}\in T} w_{jj'}\geq 0, \text{ for all } g_j\in\{g_1,g_2,g_3\} \text{ and for all } T\subseteq\{g_1,g_2,g_3\}\setminus\{g_j\}, T\neq\emptyset,\\ k\geq 0,\\ \sigma^+(\mathbf{P}_i)\geqslant 0, \sigma^-(\mathbf{P}_i)\geqslant 0,\ \ i=1,\ldots,6.\end{array}\right\}\end{array} \quad (19)$$

The solution to the LP problem (19) yields $w_1=0.52, w_2=0.08, w_3=0.09, w_{12}=0, w_{13}=0.32$, and $w_{23}=0$ with the scaling constant $k=1.28$, with the projects $\mathbf{P}_i, i=1,\ldots,6$, receiving the evaluations listed in Table 5 and the total sum of errors $\sum_{i=1}^{6}\sigma^+(\mathbf{P}_i)+\sigma^-(\mathbf{P}_i)$ being equal to 4.99.

Table 5: Scores assigned to projects by the value function $U(\cdot)$ obtained on solving the LP problem (19)

Projects	$U(\mathbf{P}_i)$	$\nu(\mathbf{P}_i)$	$k\cdot\nu(\mathbf{P}_i)$	$\sigma^+(\mathbf{P}_i)$	$\sigma^-(\mathbf{P}_i)$
$\mathtt{P}_1$	75.51	59	75.58	0	0.07
$\mathtt{P}_2$	60	43	55.08	4.92	0
$\mathtt{P}_3$	57.64	45	57.64	0	0
$\mathtt{P}_4$	69.17	54	69.17	0	0
$\mathtt{P}_5$	52.52	41	52.52	0	0
$\mathtt{P}_6$	66.61	52	66.61	0	0

On considering only the weighted sum, we obtain the following:

- On minimizing the maximum deviation, through the solution of the ordinal regression optimization problem (14), we obtain $w_1 = 0.63, w_2 = 0.04$, and $w_3 = 0.33$ with $k = 1.34$ and a maximum deviation $\gamma^* = 2.56$;
- On minimizing the sum of the deviations under the constraint that deviations should be not greater than the minmax deviation γ^* plus a tolerance $\varepsilon^\gamma = 0.5$, through the solution of the ordinal regression optimization problem (15), we obtain $w_1 = 0.57, w_2 = 0.03$, and $w_3 = 0.4$ with $k = 1.32$ and the sum of deviations 9.11.
- On minimizing the maximal deviation under the constraint that deviations should be not greater than the minimal sum of the deviation provided by the solution of the ordinal regression optimization problem (7) $S^* = 8.09$ plus a tolerance $\varepsilon^S = 1$, through the solution of the ordinal regression optimization problem (16), we obtain $w_1 = 0.57, w_2 = 0.03$, and $w_3 = 0.4$ with $k = 1.32$ and the maximum deviation 3.06.

The value function elicited through DOR method can be used to evaluate any project. Consider, for example, the three new projects P_7, P_8, and P_9, whose evaluations with respect to the considered criteria as well as overall evaluations with respect to all the elicited value functions expressed as weighted sum, additive piecewise linear value function, and Choquet integral are shown in Table 6.

Table 6: Evaluations of projects with respect to considered criteria (g_1, Economic aspects; g_2, Social aspects; g_3, Environmental aspects; U^{WS_1},weighted sum by minimimization of the sum of deviations; U^{PL}, additive piecewise linear value function; $U^{Choquet\ integral}$, Choquet integral; U^{WS2}, weighted sum by minimization of the maximal deviation; U^{WS3}, weighted sum by minimization of the maximal deviation with a constraint on the sum of the deviations; U^{WS4}, weighted sum by minimization of the sum of the deviations with a constraint on the maximal deviation)

Projects	g_1	g_2	g_3	U^{WS_1}	U^{PL}	$U^{Choquet\ integral}$	U^{WS2}	U^{WS3}	U^{WS4}
P_7	60	70	90	72.91	60.74	67.41	70.21	72.30	72.31
P_8	85	90	65	77.31	79.4	77.38	78.68	77.12	77.11
P_9	75	75	80	77.02	72.22	75.45	76.63	77.00	77.01

3. DOR-guided interactive multi-objective optimization and space–time model

3.1. DOR-guided interactive multi-objective optimization

The DOR approach introduced in Section 2.2 can be integrated into an interactive multi-objective optimisation procedure following the approach of (Jacquet-Lagrèze et al., 1987), with respect to which we propose the replacement of the classical ordinal regression procedure based on the mere ranking of the reference alternatives (Jacquet-Lagrèze and Siskos, 1982) with our DOR method that takes into consideration the intensity of the preference in addition to the ranking of reference alternatives. The interactive multi-objective optimisation procedure that we consider is articulated in the following steps:

- Generation of a small subset of representative feasible efficient solutions to be presented to the DM;
- Elicitation of DM's preference information through the deck-of-cards methods;
- Assessment of a value function $U(\cdot)$ through the DOR method;
- Optimization of the value function $U(\cdot)$ on the original set of feasible solutions defining a new subset of representative solutions to be presented to the DM;

- If the DM is satisfied by the proposed solutions, the procedure stops, else the cycle restarts.

Let us observe that the above interactive procedure, although simple, has several positive aspects.

- Through the deck-of-cards method, the DM's preference information is elicited in an easy and understandable manner.
- During the iteration of the procedure, the value function can change according to the new preference information provided by the DM on the solutions that, at each iteration, are proposed to them.
- There is a possibility of considering different formulations of the value function (weighted sum, piecewise linear value function, and Choquet integral) according to the type of decision problem at hand.
- It is possible to change the formulation of the value function during the procedure: for example, one can start with a simple weighted sum, and later switch to the Choquet integral to take into consideration the interaction between the considered objectives.

3.2. Space–time model

In the real-world problem proposed in Section 4, we apply the DOR-guided interactive multi-objective optimisation procedure described in the previous subsection, formulating a territorial planning problem in terms of the space–time model introduced by (Barbati et al., 2020), which we recall as follows. Let us consider a set of facilities $I = \{1, \ldots, I, \ldots, n\}$. For each facility $i \in I$, we define a set of potential locations $L(i) = \{1(i), \ldots, l(i), \ldots, n(i)\}$. A facility can be assigned a location in different time epochs $T = \{0, \ldots, t, \ldots, p\}$. Each facility is evaluated with respect to a set of criteria $G = \{g_j, j \in J\}$ and $J = \{1, \ldots, m\}$. The evaluation of the facility $i \in I$ activated at location $l \in L(i)$ with respect to criterion $g_j \in J$ is denoted by $y_{ijl} \in \mathbb{R}^+$. For simplicity, without the loss of generality, we suppose that all the criteria $g_j \in G$ should be maximised, that is, the greater y_{ijl}, the better the evaluation of facility $i \in I$ on criterion $g_j \in J$ in location $l \in L(i)$.

For each time epoch $t \in T$, a discount factor $v(t)$, with $0 \leq v(t) \leq 1$ and v being a nonincreasing function of t, is defined to discount the evaluation of the performances y_{ijl}, where $i \in I, j \in J$, and $l \in L(i)$ in future periods. The values $v(t)$, where $t \in T$, represent the DM's intertemporal preferences. A constant discount rate is proposed according to (Samuelson, 1937). Although several other methods of taking into consideration the time preferences of future utilities can be defined (see Frederick et al., 2002), the discount rates can be assumed to be relatively constant over time while considering the DM's subjective estimates of duration, as highlighted by (Zauberman et al., 2009). Moreover, given the interactive nature of our method, the initial discount rate proposal can be discussed with the DM, and its impact on the analysis can be investigated.

For simplicity, the performances on the different criteria are first aggregated by abstracting from any consideration of the interaction between criteria to realize homogeneous performances on the considered criteria g_j, taking into consideration the weights $w_j \geq 0$, where $j = 1, \ldots, m$, which permits the definition of an overall value of each plan by summing up the weighted discounted single criterion performances $w_j \cdot y_{ijl} \cdot v(t)$. A plan is understood as the solution to the decision-making problem, and thus, in the case of urban and regional transformations, as the definition of the facility allocation choices. Each facility $i \in I$ incurs a cost $c_{il} \in \mathbb{R}^+$. We denote the available budget for each period $t \in T$ as B_t.

The following decision variables are considered to define the adopted plan $\mathbf{x}$:

$$x_{ilt} = \begin{cases} 1, & \text{if facility } i \in I \text{ is installed in location } l \in L(i) \text{ in period } t \in T - \{0\}; \\ 0, & \text{otherwise.} \end{cases}$$

For example, with a set of facilities $I = \{1, 2\}$, set of locations $L(1) = \{1, 2\}$ and $L(2) = \{1, 2, 3\}$, and set of time epochs $T = \{0, 1, 2\}$, we have to consider the following vector of the decision variables:

$$\mathbf{x} = [x_{110}, x_{111}, x_{120}, x_{121}, x_{210}, x_{211}, x_{220}, x_{221}, x_{230}, x_{231}].$$

If we have

$$x_{110} = x_{111} = x_{120} = 0, x_{121} = x_{230} = 1, x_{210} = x_{211} = x_{220} = x_{221} = x_{231} = 0,$$

then the adopted plan consists of installing facility 1 to its second potential location in period 1, and facility 2 in its third potential location in period 0.

If no interaction between the criteria is considered, the overall objective function of the space–time optimisation model aggregating all the contributions of all the criteria in all the locations and at all times with respect to a plan $\mathbf{x}$ can be formulated as follows:

$$U(\mathbf{x}) = \sum_{i \in I} \sum_{j \in J} \sum_{l \in L(i)} \sum_{t \in T - \{0\}} \sum_{\tau=0}^{t-1} v(t) w_j x_{il\tau} y_{ijl}. \tag{20}$$

Let us observe that, for each criterion $g_j \in G$ and plan $\mathbf{x} = [x_{ilt}]$, it is possible to define the overall contribution of criterion $g_j(\mathbf{x})$ as

$$g_j(\mathbf{x}) = \sum_{i \in I} \sum_{l \in L(i)} \sum_{t \in T - \{0\}} \sum_{\tau=0}^{t-1} v(t) x_{il\tau} y_{ijl}, \tag{21}$$

such that we can write

$$U(\mathbf{x}) = \sum_{j \in J} w_j g_j(\mathbf{x}). \tag{22}$$

It is observed that not all 0–1 vectors $\mathbf{x} = [x_{ilt}]$ are feasible. A variety of constraints can be defined according to the particular application at hand:

1. **Budget constraints** according to which, in each period $t \in T$, the expenses cannot be greater than the available budget B_t, which is increased by the possible unspent budgets from previous periods:

$$\sum_{i \in I} \sum_{l \in L(i)} c_{il} x_{ilt} \leq B_t + \sum_{\tau \in T: \tau < t} B_\tau - \sum_{\tau \in T: \tau < t} \sum_{i \in I} \sum_{l \in L(i)} c_{il} x_{il\tau}, \quad \forall t \in T, \tag{23}$$

   that is, in an equivalent formulation,

$$\sum_{\tau \in T: \tau \leq t} \sum_{i \in I} \sum_{l \in L(i)} c_{il} x_{il\tau} \leq B_t + \sum_{\tau \in T: \tau < t} B_\tau, \quad \forall t \in T, \tag{24}$$

   which can be interpreted by considering that, in each period t, the total expenses cannot be greater than the sum of all the available budgets until t.

2. **Single opening constraints**, i.e., each facility can be activated once at most

$$\sum_{l(i) \in L(i), t \in T} x_{ilt} \leq 1, \quad \forall i \in I. \tag{25}$$

3. **Exclusion constraints**: Potential locations for different facilities may be the same. In this case, it

may be impossible to activate both facilities. Let us define the set of exclusions $E = \{1, \ldots, e_k, \ldots, e_{\overline{K}}\}$. Each $e_k \in E$ is identified by a quadruple (i, i', l, l'), with facilities $i, i' \in I$, and potential locations $l \in L(i)$ and $l' \in L(i')$. If the facility i is planned in location l, then facility i' cannot be located at l' at any period $t \in T$. This can be described by the following constraints:

$$\sum_{t\in T} x_{ilt} + \sum_{t\in T} x_{i'l't} \leq 1, \quad \forall (i, i', l, l') = e_k \in E. \tag{26}$$

4. **Scheduling constraints**: Some facilities may need to be scheduled earlier or later than other facilities. For instance, if a facility i is required to be scheduled after a facility i', then the following constraints have to be considered:

$$x_{ilt} \leq \sum_{\tau=0}^{t-1} x_{i'l\tau}, \quad \forall t \in T, \forall l \in L. \tag{27}$$

Other types of constraints are related to the consideration of synergistic effects between selected facilities in the objective function of the space–time model. More precisely, we consider the case in which the contribution to the different criteria $g_j \in J$ is boosted when some facilities are implemented conjointly in some "favourable" locations. Thus, we define a set of synergies $S = \{s_1, \ldots, s_r, \ldots, s_{\overline{r}}\}$, with $s_r = (i, i', l, l')$, $i, i' \in I, l \in L(i), l' \in L(i')$. The synergy s_r is realised when facility i is located in l, and facility i' is located in l'. In this case, for period t in which the synergy is realised, there is an additional contribution $y^r_{jt} = \sigma_r \cdot (y_{ijl} + y_{i'jl'})$, with $\sigma_r \geq 0$. To consider these synergies in our model, we define for each synergy $s_r = \{i, i', l, l'\} \in S$ and for each $t \in T$, the auxiliary variables γ^r_t as

$$\gamma^r_t = \begin{cases} 1, & \text{if facilities } i \text{ and } i' \text{ result implemented in } l \text{ and } l' \text{ at period } t \in T \text{ or earlier;} \\ 0, & \text{otherwise.} \end{cases}$$

Thus, $\gamma^r_t = 1$ if the synergy $s_r \in S$ is realised in $t \in T$, and $\gamma^r_t = 0$ otherwise, which is ensured by the following constraints:

$$\sum_{\tau\in T:\tau\leq t} x_{il\tau} + \sum_{\tau\in T:\tau\leq t} x_{i'l'\tau} - 1 \leq \gamma^r_t, \quad \forall s_r \in S, \forall t \in T; \tag{28}$$

$$\sum_{\tau\in T:\tau\leq t} x_{il\tau} \geq \gamma^r_t; \forall s_r \in S, \quad \forall t \in T; \tag{29}$$

$$\sum_{\tau\in T:\tau\leq t} x_{i'l'\tau} \geq \gamma^r_t; \forall s_r \in S, \quad \forall t \in T. \tag{30}$$

Considering the contributions of the synergies between the facilities, we can reformulate the objective function of the space–time model as follows:

$$U(\mathbf{x}) = \sum_{i\in I}\sum_{j\in J}\sum_{l\in L(i)}\sum_{t\in T-\{0\}}\sum_{\tau=0}^{t-1} v(t) w_j x_{il\tau} y_{ijl} + \sum_{s_r\in S}\sum_{t\in T-\{0\}} v(t) w_j \gamma^r_t y^r_{jt}. \tag{31}$$

We observe that the objective function in the formulation (31) can be expressed in terms of the overall contribution of the criteria $g_j \in G$ with respect to plan $\mathbf{x} = [x_{ilt}]$ appropriately redefined as

$$g_j(\mathbf{x}) = \sum_{i\in I}\sum_{l\in L(i)}\sum_{t\in T-\{0\}} v(t)(\sum_{\tau=0}^{t-1} x_{il\tau} y_{ijl} + \sum_{s_r\in S} \gamma^r_t y^r_{jt}), \tag{32}$$

such that we can write

$$U(\mathbf{x}) = \sum_{j \in J} w_j g_j(\mathbf{x}). \tag{33}$$

It should be noted that the above contributions could be split in relation to one or more elements, such as the facility, period, or criterion. For instance, one can consider the overall performance in period $t \in T - \{0\}$ of all the facilities $i \in I$, all criteria $j \in J$, and all locations $l \in L$, that is, $y_t^T(\mathbf{x}) = \sum_{i \in I} \sum_{j \in J} \sum_{l \in L} \sum_{\tau=0}^{t-1} w_j x_{il\tau} y_{ijl}$. This could be helpful in understanding how the contributions of all the activated facilities to the criteria evolved over time.

A further enrichment of the objective function of the space–time model we consider in the following is related to the consideration of the interaction between the criteria, which can be obtained by generalising the formulation (33) of $U(\mathbf{x})$ in terms of the Choquet integral introduced in Section 2.1, that is,

$$U(\mathbf{x}) = \sum_{j=1}^{m} \mu(\{g_h \in \mathcal{G} : g_h(\mathbf{x}) \geq g_{(j)}(\mathbf{x})\}) \cdot [g_{(j)}(\mathbf{x}) - g_{(j-1)}(\mathbf{x})], \tag{34}$$

where μ denotes the capacity of G. As detailed in Section 2.1, if the capacity μ is two-additive, the formulation (34) of the Choquet integral can be expressed as

$$U(\mathbf{x}) = \sum_{g_j \in \mathcal{G}} w_j g_j(\mathbf{x}) + \sum_{\{g_j, g_{j'}\} \subseteq \mathcal{G}} w_{jj'} \min\{g_j(\mathbf{x}), g_{j'}(\mathbf{x})\} \tag{35}$$

with weights w_j, where $j = 1, \ldots, m$, and $w_{j,j'}$, where $\{j, j'\} \subset \mathcal{G}$ satisfying the constraints presented in Section 2.1, that can be induced from the DM's preference information through the DOR method presented in Section 2.2.

3.3. Summary of steps

In the following section, we present a summary of the steps for the proposed methodology:

1. **Structuring the problem:** The analyst and the DM define the main elements of the problems in terms of objectives/criteria to take into consideration, the facilities, their location, and their evaluations. They also specify the planning horizon and other characteristics that the plans should comprise.
2. **Identification of potential plans:** The analyst selects some plans to submit to the DM. This step can be conducted with the definition of some plans obtained, for example, including relevant constraints related to the desired characteristics of the plan in the space–time model of Subsection 3 and optimising the single criteria.
3. **Ranking of the proposed plans and elicitation of the DM preferences:** The DM ranks the proposed plans and compares them with the deck-of-cards method, thus obtaining an evaluation $\nu(\mathbf{x})$ for each plan $\mathbf{x}$. With the applications of the regression model of Subsection 2.2, taking into consideration, for example, a value function formulated in terms of the Choquet integral, a set of weights w_j for each criterion g_j and a set of interaction coefficients $w_{jj'}, \{g_j, g_{j'}\} \subseteq \mathcal{G}$ is derived, and a new value function for the space–time model is defined. The DM also comments on the plans obtained, and their indications can be introduced as constraints in the multi-objective optimization problems expressed in terms of the space–time model.
4. **Definition of a new set of plans:** Owing to the application of the space–time model of Subsection 3 and the value function obtained in the previous step, new plans are generated. If the DM is satisfied with one of the proposed plans, the procedure is stopped. Else, we return to step 3, ask the DM to express their preferences for the newly generated plans, and the procedure is iterated until the DM is satisfied with one of the proposed plans.

4. Real-world application

The real-world application comprises the development of an ecovillage in Italy. Ecovillages may be considered as rural enterprises that combine sustainable and environment-friendly technologies, organic agriculture, and other farming activities and tourism services. Ecovillages represent a type of lifestyle. Based on this philosophy, they are usually designed and built within the framework of four foci: ecologic, social, cultural, and spiritual concepts. The case under analysis is a project for the revitalisation of a rural settlement built at the end of the 18th century in dry stone at an altitude of 1000 m, located in the mountains approximately an hour from Turin (the capital of the region), and abandoned in the 1950s. It comprises two small boroughs, the Upper and Lower Boroughs, with 11.4 hectares of woodland in the surrounding area (see Figure 1). After years of searching and negotiation, a cooperative bought this rural settlement to create an ecovillage called "The House of the Sun". Their motto is "Another world is possible, we are building it... here!". The objective of this project is to be able to restore the relationship of the settlement with nature and the environment more harmoniously, through food, furnishings, clothing, and a whole series of practices, in addition to those already working, which may be organic farming, even a little more unusual and holistic as the martial arts, yoga, or meditation, rather than shiatsu treatment or tai chi chuan, but also more simply traditional folk dances to recover the Occitan tradition of these cross-border valleys. This is part of a dynamic exchange with the territory to reactivate the economic fabric of the valley–the experience of artisans who have knowledge of how to build with stone and wood– and involve those who want to help the cooperative in revitalising the valley.

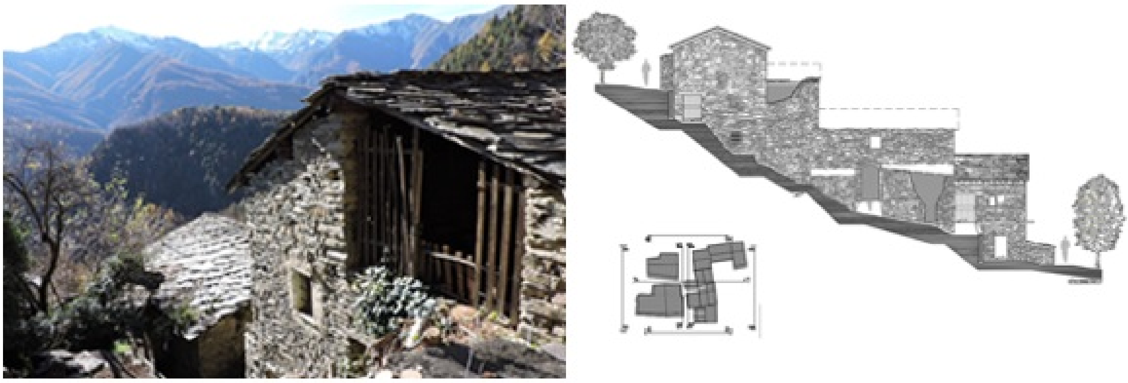

Figure 1: One of the buildings of the "House of the Sun" and a transformation hypothesis (source: libertarea.org)

Defining the facilities, their locations, and the timing of an ecovillage is undoubtedly challenging because it is a unique case of regional transformation with non-ordinary logic, wherein, for example, money has a very different value compared to urban transformation contexts in which the goal of the developer is to maximise income. There are several unique aspects of an ecovillage that must be considered:

- The informal economy plays a fundamental role as one has to also consider exchanges that take place via the social network, without the exchange of money (e.g., barter). This is an important aspect to consider in the location of facilities, which follows non-commercial logic for residents.
- There is no certain right or wrong concept while developing an ecovillage. What is generally recognized is that a careful and specific design is important for healthy development in the long run. Therefore, ecovillages use technologies such as passive solar energy designs, natural isolation materials, and biomass gas converters.

- The social aspect is fundamental to an ecovillage. In each ecovillage, a conscious effort is made towards developing the community environment and creating a sense of belonging.
- The ecovillage involves the presence of three types of users: i) residents, i.e., people living there all year round; ii) temporary residents who work in the village for a period ranging from 2 weeks to 6 months by taking advantage of opportunities referred to using a specific name, i.e., WWOOFER (worldwide opportunities on organic farms); iii) guests (in hotels) and keen tourists with a strong environmental connection (eco-tourism).

The last point implies that the allocation of services takes into consideration which facilities could be used temporarily or permanently by different types of beneficiaries. For instance, it is possible that the first two types of users could have similar residential spaces and temporarily share common areas. In general, all spaces must be created to stimulate interactions, protect privacy, and encourage the possibility of developing a sense of community. The decision to use this case study was based on the opportunity to interact with the president of the cooperative owning "The House of the Sun" (hereinafter defined as the DM, and to whom we shall refer with masculine pronouns, being a man). The strong conviction to create an alternative way of living and working conflicted with severe budget constraints. Therefore, the application of the DOR-based interactive optimisation procedure described above for handling the ecovillage planning problems formulated in terms of a space–time model appeared to fit perfectly.

5. Results and Implementation of the methodology

5.1. Structuring the problem

In collaboration with the DM, we structured the problem, considering the following elements:

- The set of facilities $I = \{1, \ldots, 10\}$ is distinguished by those for the residents and those for the tourists (including the WWOOFERs). The facilities to be included concern these two types of users, although the level of interaction between the two could be very strong, particularly in the first years of the ecovillage. Both residents and tourists will need a kitchen, dining room, and rooms; then there are the tailoring/laundry, woodworking, and recreational rooms (destined for yoga, meditation, martial arts, and dance). Table 7 lists the facilities with their respective symbols and labels in detail. These facilities can be briefly described as follows: regarding the spaces for WWOOFERS, the residence consists of the private spaces designated for sleeping for those who will reside in the ecovillage and for tourists with long stays; the kitchen is the room reserved and equipped for preparing and cooking food; the refectory is the room designated for the eating of meals in buildings in which the community lives. The spaces for "guests" (i.e. tourists staying here for a short time) concern the bedrooms ("rooms"), the kitchen for food preparation ("kitchen") and the room for eating meals ("dining room"). There are also a series of common spaces intended for all types of users: two laboratories, one for tailoring and the other for woodworking, and a recreation room adaptable to different types of activities, such as yoga and dance. Finally, there are the technical spaces, which contain "machinery" necessary for the functioning of the ecovillage, such as the heating system
- The sets of locations $L(i) = \{l_1(i), l_2(i)\}$ define for each facility $i \in I$ the potential location for each facility in the Upper or Lower boroughs (see Figure 2). The two locations are a short distance apart; the upper location is a little larger, but both are in a serious state of disrepair and require extensive renovation. According to the technical and positional characteristics of the different rooms in the buildings in the Upper Borough and the Lower Borough, the facilities can be located only in specific spaces (primarily according to the surfaces required). All locations are the result of significant

Facility	Label	Symbol
Residence for the WWOOFER	(`RES-WWO`)	$_R$
Kitchen for the WWOOFER	(`KIT-WWO`)	$_R$
Refectory for the WWOOFER	(`REF-WWO`)	$_R$
Guest Rooms	(`ROM-GUE`)	$_G$
Guest Kitchen	(`KIT-GUE`)	$_G$
Guest Dining room	(`DIN-GUE`)	$_G$
Laboratory 1: tailoring	(`TAI-LAB`)	
Laboratory 2: woodworking	(`WOO-LAB`)	
Recreational room (yoga / meditation martial arts dance)	(`ROM-REC`)	
Main technical room	(`ROM-TEC`)	

Table 7: List of facilities

renovation of existing buildings, considering only a new construction being a pavilion for recreational activities. In Table 8, the different spaces are identified with a letter (corresponding to the building) and a number (to distinguish the different rooms located at the different levels of the buildings).

- The cost c_{il} associated to each location $l \in L(i)$ and to each facility $i \in I$ (see Table 8). The cost represents an estimation of the implementation costs. In addition to the construction costs indicated in the table, the following items of expenditure have been estimated, and appropriately distributed over the four years considered: design costs; general expenses; primary and secondary urbanization charges; initial costs (purchase of furniture and machinery); annual running costs.

- The set of periods $T = \{t_0, t_1, t_2, t_3\}$, with $t_0 = 0, t_1 = 1, t_2 = 2, t_3 = 3$, i.e. we are investigating the possibility that the planning period will last for three years.

- The set of criteria $G = \{g_1, g_2, g_3, g_4\}$ that have been derived by the analysis for the aims of installing ecovillages were extensively discussed with the DM. More in detail:
  - Environmental aspects (g_1): it is the "mother principle" that determines everything else; it is considered the fundamental value that motivates this peculiar choice of life;
  - Social aspects (g_2): it is related to the will to repopulate inland territories (an objective recognized as particularly important at the European level and, paradoxically, less at the Italian level), while encouraging urban congestion;
  - Economic aspects (g_3): it considers two main aspects. On the one hand, a principle of self-sustainability with a low environmental impact is a fundamental and structural objective to be pursued; on the other hand, the issue of running a profitable activity related to eco-tourism;
  - Cultural aspects (g_4): it takes into account how activities in the area are intertwined with social and cultural themes (e.g. guided socio-hiking, rediscovery of local history, aggregation of schooling, etc.).

Theoretically, these four criteria must always be optimised together because the ecological-cultural holistic basic assumption implies the consideration of strong interactions between these four criteria. Considering its capacity to model the interaction between criteria, the Choquet integral model appears to be the most appropriate formulation of the value function $U(\cdot)$ for the decision problem presently. In Table 9, we can see that for each facility $i \in I$ and for each location $l \in L$, the evaluations y_{ijl} for each criterion $g_j \in G$; these estimates were provided by the expert and consistent with the DM and for the sake of simplicity, are expressed with values between 0 and 100.

Facilities	Location 1	c_1	Location 2	c_2
(RES-WWO)	B1, B7, B8, B9, B10, A3, A4, A5, A6, A7	212,175 €	H1, H2, H3, H4, I1, I2, I3, I4, L1, L2	233,390 €
(KIT-WWO)	B4	26,560 €	M1	29,215 €
(REF-WWO)	B3	15,955 €	M2	17,550 €
(ROM-GUE)	F4, F6, A7, D4, D6, C4, C5, C6	185,515 €	B1, B7, B8, B9, B10, A3, A4, A5, A6, A7	212,175 €
(KIT-GUE)	C2	18,235 €	D3	30,090 €
(DIN-GUE)	C1	31,910 €	D1, E6,E5	73,800 €
(TAI-LAB)	B6	14,865 €	C2	35,100 €
(WOO-LAB)	C7	31,910 €	F6	8,720 €
(ROM-REC)	C8	21,405 €	Pavillon	23,545 €
(ROM-TEC)	F5	13,975 €	H5	20,060 €

Table 8: Locations of each facility and the associated costs

Facilities	g_1		g_2		g_3		g_4	
	l_1	l_2	l_1	l_2	l_1	l_2	l_1	l_2
(RES-WWO)	80	80	82	70	40	35	80	80
(KIT-WWO)	80	80	82	70	40	35	80	80
(REF-WWO)	80	80	82	70	40	35	80	80
(ROM-GUE)	60	60	70	0	72	80	70	70
(KIT-GUE)	55	60	70	70	72	80	65	70
(DIN-GUE)	55	60	62	70	72	80	65	70
(TAI-LAB)	70	62	43	38	50	50	70	72
(WOO-LAB)	70	65	45	40	55	65	70	72
(ROM-REC)	72	60	55	42	55	70	62	78
(ROM-TEC)	75	75	35	35	42	48	72	72

Table 9: Criteria evaluations for each facility and for each location

- In terms of characteristics that the plans must have, the DM and the analysts agreed that:
  - A facility could be activated only once and only in one location.
  - The pairs of facilities (RES-WWO) and (ROM-GUE), (KIT-GUE) and (WOO-LAB) and (ROM-TEC) should not be opened in the same location.
  - The facilities (KIT-GUE) and (DIN-GUE) if opened at the same time would cause an increase in the evaluation of the facilities with respect to the considered criteria of $\sigma_r = 20\%$.

Each of the above requirements was considered in the definition of the plans by means of specific constraints included in the formulation of the space-time model. The plans proposed for the DM were obtained by maximising a specific value function $U(\cdot)$ as detailed in the following.

5.2. Identification of potential plans

To propose some plans to the DM, the analysts adopted the space-time model introduced in Section 3. Additionally, we simulated two different scenarios according to two different budget configurations:

- 100,000 Euro in every period $t \in T$, called budget configuration B_1;
- 50,000 Euro in every period $t \in T$, called budget configuration B_2.

In this initial stage, we aggregated the evaluations of the considered criteria using a value function $U(\cdot)$ expressed in terms of a weighted sum considering four different weight vectors $\mathbf{w} = [w_1, w_2, w_3, w_4]$, collecting weights w_j for criteria $g_j, j = 1, 2, 3, 4$, as reported in Table 10. These initial weights were chosen to represent equal weights or to give significantly more importance to one of the criteria than to the others. For this initial stage, we did not consider potential interactions among the criteria and, consequently, we did not adopt a more complex and sophisticated Choquet integral model because we only wanted to propose some initial plans to the DM to start the discussion. In other words, in the first step, we fixed the interaction coefficients $w_{j,j'}, \{g_j, g_{j'}\} \subseteq \mathcal{G}$ equal to zero.

	w_1	w_2	w_3	w_4
$\mathbf{w^1}$	0.25	0.25	0.25	0.25
$\mathbf{w^2}$	0.997	0.001	0.001	0.001
$\mathbf{w^3}$	0.001	0.997	0.001	0.001
$\mathbf{w^4}$	0.001	0.001	0.997	0.001
$\mathbf{w^5}$	0.001	0.001	0.001	0.997

Table 10: Selected set of weights for the initial stage

To the formulation of the space-time model, we added the single-opening activation constraints (25) for each facility $i \in I$ and the exclusion constraints (26) among the pairs of facilites (`RES-WWO`) and (`ROM-GUE`), (`KIT-GUE`) and (`WOO-LAB`) and (`ROM-TEC`) according to the DM's preferences. We also defined the discount factor $v(t) = 1.10^{-t}$. In addition, we ran all the scenarios defined above with the synergy constraint between facilities (`KIT-GUE`) and (`DIN-GUE`). If these facilities were opened simultaneously, they would make an additional contribution of 20% to the four criteria considered. During our initial discussion with the DM, he expressed that this synergy would be important, but he also kindly discussed plans without any synergy. Therefore, to attain a set of initial plans that are as different as possible, we simulated all scenarios with this synergy constraint, identified as SG_1, and without the synergy constraint, identified as SG_2. In this way, maximizing the value function $U(\mathbf{x}) = \sum_{g_j \in G} w_j g_j(\mathbf{x})$ in the different scenarios $(B_r, \mathbf{w^s}, S_k)$ obtained by the combination of the budget $B_r, r = 1, 2$, the weight vectors $\mathbf{w^s}, s = 1, \ldots, 5$, and the presence of synergy constraint $SG = \{SG_1, SG_2\}$, we obtained 20 initial plans. Some plans were identical. In addition, to reduce the cognitive burden of the DM, we decided to select only the most representative ones and those that presented more differences. In the end, eight different plans $\mathbf{x}_1, \ldots, \mathbf{x}_8$ were presented to the DM as reported in Table 11 with the first four plans obtained with budget configuration B_1 and the other four plans obtained with budget configuration B_2. The symbol $\times$ means that a particular facility has not been selected; otherwise, the location $l \in L$ and the period $t \in T$ in which the facility is implemented were presented. The selected plans were obtained as follows:

- $\mathbf{x}_1$, for budget B_1, weights $\mathbf{w^1}$, presence of synergy SG_1;
- $\mathbf{x}_2$, for budget B_1, weights $\mathbf{w^5}$, absence of synergy SG_2;
- $\mathbf{x}_3$, for budget B_1, weights $\mathbf{w^4}$, absence of synergy SG_2;

- $\mathbf{x}_4$, for budget B_1, weights $\mathbf{w^3}$, absence of synergy SG_2;
- $\mathbf{x}_5$, for budget B_2, weights $\mathbf{w^3}$, presence of synergy SG_1;
- $\mathbf{x}_6$, for budget B_2, weights $\mathbf{w^1}$, presence of synergy SG_1;
- $\mathbf{x}_7$, for budget B_2, weights $\mathbf{w^5}$, absence of synergy SG_2;
- $\mathbf{x}_8$, for budget B_2, weights $\mathbf{w^4}$, absence of synergy SG_2;

Each plan can be obtained maximizing the value function $U(\mathbf{x})$ in different scenarios related to different parameter combinations, such as plan $\mathbf{x}_6$, which is the optimal plan also for budget B_2, weights $\mathbf{w^1}$, in the absence of synergy SG_1.

	(RES-WWO)	(KIT-WWO)	(REF-WWO)	(ROM-GUE)	(KIT-GUE)	(DIN-GUE)	(TAI-LAB)	(WOO-LAB)	(ROM-REC)	(ROM-TEC)
$\mathbf{x}_1$	$\times$	l_1t_1	l_1t_1	l_2t_3	l_1t_0	l_1t_0	l_1t_0	l_2t_0	l_2t_1	l_2t_1
$\mathbf{x}_2$	$\times$	l_1t_1	l_1t_0	l_2t_3	l_1t_0	l_1t_1	l_1t_0	l_2t_0	l_1t_0	l_2t_1
$\mathbf{x}_3$	l_1t_3	l_1t_1	l_1t_0	$\times$	l_2t_1	l_1t_1	l_1t_0	l_2t_0	l_2t_0	l_1t_0
$\mathbf{x}_4$	l_1t_3	l_1t_1	l_1t_0	$\times$	l_2t_1	l_1t_1	l_1t_0	l_2t_0	l_1t_0	l_1t_0
$\mathbf{x}_5$	$\times$	l_1t_1	l_1t_0	$\times$	l_1t_2	l_1t_2	l_1t_1	l_2t_0	l_1t_3	l_1t_0
$\mathbf{x}_6$	$\times$	l_1t_3	l_1t_0	$\times$	l_1t_1	l_1t_2	l_1t_0	l_2t_0	l_2t_1	l_2t_2
$\mathbf{x}_7$	$\times$	l_1t_1	l_1t_0	$\times$	l_1t_1	l_1t_2	l_1t_2	l_2t_0	l_2t_3	l_1t_0
$\mathbf{x}_8$	$\times$	l_1t_1	l_1t_0	$\times$	l_2t_2	l_1t_3	l_1t_0	l_2t_0	l_1t_2	l_1t_1

Table 11: Plans presented to the DM during the first iteration

5.3. Ranking of the proposed plans and elicitation of the preferences

The DM, faced with the plans in Table 11, pointed out that there were some priorities and requirements to bear in mind:

- The tailor's laboratory (TAI-LAB), which also contains the laundry, should be built immediately so that the residents can be accommodated. This service cannot be outsourced because it is based on the crucial principles of ecovillage, such as water recycling.
- In the identified plans, a mixed use of kitchens and refectories for guests and residents was implemented at the starting period t_0; the DM considered this to be very reasonable. From a strategic point of view, the DM pointed out that it made sense to have alternatives where guest kitchens were implemented initially because there might be catering without residents initially, but not vice versa.
- Preference had to be given to plans where the recreational room (ROM-REC) was in the Upper Borough, where all other facilities were located, because it was more convenient for guests. In overnight accommodations, the spaces could be used interchangeably between residents and external guests. Moreover, in the first phase of the settlement, there was a high degree of adaptability because guests and residents were not very dissimilar. Again, the above requirements were considered by adding corresponding constraints to the optimization problems to be solved to define the plans for the DM.

Moreover, commenting on the first four plans related to the budget B_1, the DM observed that plan $\mathbf{x}_1$ was preferred over plan $\mathbf{x}_2$ because the kitchen (KIT-GUE) and guest dining room (DIN-GUE) were located in a building that was most suitable for hospitality in the medium to long term; plan $\mathbf{x}_4$ was preferred over plan $\mathbf{x}_3$ because the recreational room (ROM-REC) was located in the Upper Borough, which is more convenient for short-stay guests. The DM also underlined that plan $\mathbf{x}_1$ was preferred to plan $\mathbf{x}_3$ because higher income could be provided as the catering could be obtained immediately. Then, by applying the deck-of-cards method, we asked the DM to rank the plans related to budget B_1, also providing a measure of the strength of the preferences in terms of the number of blank cards between each plan and the following one in the preference ranking. The DM provided the following ranking, identified with R_{50} with the number of blank cards shown between parenthesis [ ], with $\mathtt{x}_0^1$ representing a fictitious plan identifying a zero level for budget B_1:

$$\mathtt{x}_0^1\ [5]\ \mathtt{x}_3\ [0]\ \mathtt{x}_4\ [2]\ \mathtt{x}_2\ [3]\ \mathtt{x}_1$$

Commenting on the plans for budget configuration B_2, the DM stated that they were less preferred because there were no residential facilities in any of them. Plan $\mathbf{x}_6$ was preferred because it selected a kitchen for guests (`KIT-GUE`) and a refectory (`DIN-GUE`). For the guests, the most connotative room was for recreational activities (`ROM-REC`), which were rare and uncommon for the region (such as yoga and martial arts), and together with the dining activity, were also the most profitable. The worst plan was $\mathbf{x}_8$ because it did not schedule the opening of the technical room (`ROM-TEC`) at the starting period. Plan $\mathbf{x}_7$ was worse than plan $\mathbf{x}_5$ because there was no tailoring laboratory (`TAI-LAB`). We then asked the DM to rank the plans and insert blank cards representing the strength of preferences concerning plans related to budget B_2. The DM provided the following preference information with $\mathtt{x}_0^2$ representing a fictitious plan and identifying a zero level for budget B_2:

$$\mathtt{x}_0^2\ [2]\ \mathtt{x}_8\ [3]\ \mathtt{x}_7\ [2]\ \mathtt{x}_5\ [5]\ \mathtt{x}_6$$

To create a single ranking between the plans related to budget configuration B_1 (considered in general favorite) and the plans related to budget configuration B_2, we asked the DM to define the number of cards between the worst plan related to B_1, that is $\mathbf{x_3}$, and the best plan related to B_2, that is x_6. The DM established a distance of seven cards, justifying this significant distance, considering that plans related to budget configuration B_2 did not present any housing facilities, which would mean creating more restaurants with related activities than a real ecovillage. In addition, if the first four plans required twice the budget of the others, then they provided more than double the revenue. The final ranking was accordingly identified with the following preference information R_{Tot} where cards measure the strength of the preferences between one plan and the following ones, and $\mathtt{x}_0 = \mathtt{x}_0^2$ is interpreted as a general zero level:

$$\mathtt{x}_0\ [2]\ \mathtt{x}_8\ [3]\ \mathtt{x}_7\ [2]\ \mathtt{x}_5\ [5]\ \mathtt{x}_6\ [7]\ \mathtt{x}_3\ [0]\ \mathtt{x}_4\ [2]\ \mathtt{x}_2\ [3]\ \mathtt{x}_1$$

Using the preference information supplied by the DM in terms of the ranking and preference pairwise comparisons of plans, we induced the parameters of a more complex value function, considering the interaction between criteria and the synergy between projects. Specifically, we proceeded as follows. We considered a value function $U(\mathbf{x})$ expressed in terms of a Choquet integral aggregating evaluation on the previously considered four criteria g_1, g_2, g_3 and g_4 plus the further criterion syn taking a value of 1 if in the considered plan there is synergy between facilities and zero vice versa. The criterion syn was added because the DM felt a specific relevance to the interaction between facilities (`KIT-GUE`) and (`DIN-GUE`), going beyond the increase σ_r given to the evaluation of the considered facilities on the considered criteria. We considered the interaction between the pairs of the four criteria g_1, g_2, g_3 and g_4, whereas we did not consider any interaction between synergy syn and one of the criteria g_1, g_2, g_3 and g_4. Consequently, the adopted value function had the following formulation

$$U(\mathbf{x}) = \sum_{j=1}^{4} w_j g_j(\mathbf{x}) + \sum_{j,j'=1,2,3,4,j<j'} w_{jj'} min(g_j(\mathbf{x}), g_{j'}(\mathbf{x})) + w_{syn} syn(\mathbf{x})$$

with $\sum_{j=1}^{4} w_j + \sum_{j,j'=1,2,3,4,j\neq j'} w_{jj'} + w_{syn} = 1$, $w_{syn} \geqslant 0$, $w_j, j = 1,2,3,4$, and $w_{j,j'}, j, j' = 1,2,3,4, j < j'$, satisfying all constraints of the Choquet non-additive weights. We applied the DOR methodology to the preference information provided by the DM in terms of the SRFII deck-of-cards method to:

1. the ranking of plans related to budget B_2, identified as R_{50};
2. the ranking of plans related to budget B_1 identified as R_{100};
3. the whole ranking of plans related to budget B_1 and B_2, identified as R_{Tot}.

Then, by formulating the problem in terms of LP (19) in Section 2, we computed three vectors of non-additive weights, as reported in Table 12, for the Choquet integral formulation of the value function $U(\mathbf{x})$, which corresponds to the ranking obtained using the deck-of-cards method.

	w_1	w_2	w_3	w_4	w_{12}	w_{13}	w_{14}	w_{23}	w_{24}	w_{34}	w_{sin}
$\mathbf{w}^{\mathbf{R_{50}}}$	0.05	0	0.502	0	0	0	0	0	0.175	0	0.273
$\mathbf{w}^{\mathbf{R_{100}}}$	0	0	0	0	0	0	0.468	0	0	0	0.532
$\mathbf{w}^{\mathbf{RTot}}$	0.306	0	0.455	0	0	0	0	0	0	0	0.239

Table 12: Nonadditive weights for the value function expressed in terms of a Choquet integral

In Tables 13, 14 and 15 we reported the values assigned to each plan with the deck-of-cards method, the value function $U(\cdot)$, the corrected value function and the deviations $\sigma^+(\mathbf{x})$ and $\sigma^-(\mathbf{x})$ for each of the configuration introduced, respectively.

Table 13: Scores assigned to plans by the value function $U(\cdot)$ obtained solving the LP problem (19) for ranking R_{50}

Plans	$U(\mathbf{x}_i)$	$\nu(\mathbf{x}_i)$	$k \cdot \nu(\mathbf{x}_i)$	$\sigma^+(\mathbf{x}_i)$	$\sigma^-(\mathbf{x}_i)$
$\mathtt{x}_5$	0.31	10	0.31	0	0
$\mathtt{x}_6$	0.5	16	0.5	0	0
$\mathtt{x}_7$	0.22	7	0.22	0	0
$\mathtt{x}_8$	0.09	3	0.09	0	0

Table 14: Scores assigned to plans by the value function $U(\cdot)$ obtained solving the LP problem (19) for ranking R_{100}

Plans	$U(\mathbf{x}_i)$	$\nu(\mathbf{x}_i)$	$k \cdot \nu(\mathbf{x}_i)$	$\sigma^+(\mathbf{x}_i)$	$\sigma^-(\mathbf{x}_i)$
$\mathtt{x}_1$	0.53	14	0.53	0	0
$\mathtt{x}_2$	0.70	10	0.38	0	0.32
$\mathtt{x}_3$	0.25	6	0.23	0	0.02
$\mathtt{x}_4$	0.27	7	0.27	0	0

5.4. Definition of a new set of plans

Based on the discussion with the DM, we generated a new set of plans to optimise the value function $U(\cdot)$ formulated in terms of a Choquet integral related to the weight vectors $\mathbf{w}^{\mathbf{R_{50}}}, \mathbf{w}^{\mathbf{R_{100}}}$ and $\mathbf{w}^{\mathbf{R_{Tot}}}$ induced in the previous step. We considered two budget configurations B_1 and B_2, as previously defined. We also imposed the constraint that at least one kitchen should be selected and that facility (`TAI-LAB`) should be selected earlier than facilities (`RES-WWO`) and (`WOO-LAB`), according to the preferences expressed by the DM during the second discussion. We also included a plan for each of the budget configurations with

Table 15: Scores assigned to plans by the value function $U(\cdot)$ obtained solving the LP problem (19) for ranking R_{Tot}

Plans	$U(\mathbf{x}_i)$	$\nu(\mathbf{x}_i)$	$k \cdot \nu(\mathbf{x}_i)$	$\sigma^+(\mathbf{x}_i)$	$\sigma^-(\mathbf{x}_i)$
x_1	0.89	32	0.89	0	0
x_2	0.96	28	0.78	0	0.18
x_3	0.72	24	0.67	0	0.06
x_4	0.7	25	0.7	0	0
x_5	0.28	10	0.28	0	0
x_6	0.11	16	0.45	0.34	0
x_7	0.06	7	0.19	0.14	0
x_8	0.06	3	0.08	0.03	0

the complete order and with an additional constraint on the presence of at least one of the residences to investigate if the DM would prefer plans that would allow him since the beginning to host guests in the ecovillage. The synergy constraint related to the activation of facilities (KIT-GUE) and (DIN-GUE) was always included, according to the DM preferences expressed in the previous step. In total, we generated eight plans by combining the two budget scenarios, three sets of weights $\mathbf{w}^{\mathbf{R50}}$, $\mathbf{w}^{\mathbf{R100}}$ and $\mathbf{w}^{\mathbf{R_{Tot}}}$ and the presence of at least one of the residences with a set of weights $\mathbf{w}^{\mathbf{R_{Tot}}}$. The selected plans were obtained as follows:

- $\mathbf{x}_1^{'}$, for budget B_1, weight vector $\mathbf{w}^{\mathbf{R_{50}}}$;
- $\mathbf{x}_2^{'}$, for budget B_1, weight vector $\mathbf{w}^{\mathbf{R_{100}}}$;
- $\mathbf{x}_3^{'}$, for budget B_1, weight vector $\mathbf{w}^{\mathbf{R_{Tot}}}$;
- $\mathbf{x}_4^{'}$, for budget B_1, weight vector $\mathbf{w}^{\mathbf{R_{Tot}}}$, with the residence constraint;
- $\mathbf{x}_5^{'}$, for budget B_2, weight vector $\mathbf{w}^{\mathbf{R_{100}}}$;
- $\mathbf{x}_6^{'}$, for budget B_2, weight vector $\mathbf{w}^{\mathbf{R_{100}}}$;
- $\mathbf{x}_7^{'}$, for budget B_2, weight vector $\mathbf{w}^{\mathbf{R_{Tot}}}$;
- $\mathbf{x}_8^{'}$, for budget B_2, weight vector $\mathbf{w}^{\mathbf{R_{Tot}}}$, with the residence constraint.

These new plans are presented to the DM in Table 16.

	(RES-WWO)	(KIT-WWO)	(REF-WWO)	(ROM-GUE)	(KIT-GUE)	(DIN-GUE)	(TAI-LAB)	(WOO-LAB)	(ROM-REC)	(ROM-TEC)
$\mathbf{x}_1^{'}$	×	l_1t_1	l_1t_1	l_1t_3	l_1t_0	l_1t_0	l_1t_0	l_1t_3	l_2t_1	l_1t_0
$\mathbf{x}_2^{'}$	×	l_1t_0	l_1t_0	l_1t_3	l_2t_1	l_1t_1	l_1t_1	l_1t_3	l_1t_0	l_1t_0
$\mathbf{x}_3^{'}$	×	l_1t_1	l_1t_1	l_2t_3	l_1t_0	l_1t_0	l_1t_0	l_1t_3	l_2t_1	l_1t_0
$\mathbf{x}_4^{'}$	×	l_1t_1	l_1t_1	l_2t_3	l_1t_0	l_1t_0	l_1t_0	l_1t_3	l_2t_1	l_1t_0
$\mathbf{x}_5^{'}$	×	l_1t_2	l_1t_0	×	l_1t_0	l_1t_1	l_1t_1	l_1t_3	l_2t_2	l_2t_3
$\mathbf{x}_6^{'}$	×	l_1t_2	l_1t_0	×	l_1t_0	l_1t_1	l_1t_1	l_1t_3	l_2t_2	l_2t_3
$\mathbf{x}_7^{'}$	×	l_1t_3	l_1t_1	×	l_1t_0	l_1t_2	l_1t_0	l_1t_3	l_1t_1	l_1t_1
$\mathbf{x}_8^{'}$	×	×	×	l_1t_3	l_1t_0	×	×	×	×	×

Table 16: Plans presented to the DM during the second iteration

The DM expresses his preference for plan $\mathbf{x}_1^{'}$. He pointed out that the only inconsistency was that the recreational room (ROM-TEC) in the new pavilion was too distant.

In this sense, the DM stated that the recreational room (ROM-REC) should have been close to the guest refectory (DIN-GUE) (which, in turn, had to be close to the guest kitchen (KIT-GUE)) and that the space was not less than 30 m^2. Otherwise, everything was congruent, and the principle of environmental protection was respected. With regard to the plan obtained with budget configuration B_2, the DM underlined that even considering the actual economic difficulties in starting the transformation process of the area, it constituted a "horizontal cut" that implied no overnight hospitality solution: having only the facility (DIN-GUE) was not interesting enough. Generally, the DM expressed a preference for having at least two facilities in each transformed building. Therefore, we formulated these constraints and adopted the same weight vector $\mathbf{w}^{R_{Tot}}$ for budget configuration B_1, which produced the preferred plan for the DM in the previous step, i.e. $\mathbf{x}_1'$. The following three new plans were generated:

- plan $\mathbf{x}_1^{''}$, obtained imposing that facilities (WOO-LAB) and (ROM-REC) should not be both located in Location 1;
- plan $\mathbf{x}_2^{''}$, obtained imposing that in each building in which a facility is activated, at least two facilities were activated;
- plan $\mathbf{x}_3^{''}$, obtained, imposing that at least two facilities should be activated in each building.

	(RES-WWO)	(KIT-WWO)	(REF-WWO)	(ROM-GUE)	(KIT-GUE)	(DIN-GUE)	(TAI-LAB)	(WOO-LAB)	(ROM-REC)	(ROM-TEC)
$\mathbf{x}_1^{''}$	×	l_1t_1	l_1t_1	l_2t_3	l_1t_0	l_1t_0	l_1t_0	×	l_2t_1	l_1t_0
$\mathbf{x}_2^{''}$	×	l_1t_1	l_1t_1	l_2t_3	l_1t_0	l_1t_0	l_1t_0	l_1t_3	l_1t_1	l_1t_0
$\mathbf{x}_3^{''}$	×	l_1t_3	l_1t_1	×	l_1t_0	l_1t_2	l_1t_2	l_1t_3	l_2t_1	l_2t_0

Table 17: Strategies presented to the DM during the third iteration

Observing plan $\mathbf{x}_1^{''}$, the DM noted compact timing for the renovations, whereas the locations were acceptable. He also pointed out that there were only two critical points: the recreational room (ROM-REC) remained disconnected from the transformed village and there was no woodworking room (WOO-LAB). Plan $\mathbf{x}_2^{''}$ was the most interesting for the DM for its compactness, with all the facilities placed in the borough above, simplifying the management of the space for guests and residents, and it had all the facilities. There was a problem that the woodworking room (WOO-LAB) was too close to the recreational room (ROM-REC), so this location should be changed. Plan $\mathbf{x}_3^{''}$ was the least preferred, especially concerning the timing of the implementation of various facilities, with some facilities having to be activated together (e.g. the food serving space away from the kitchens). Therefore, the DM selected plan $\mathbf{x}_2^{''}$ as the most representative opinion. We also note that we interacted with the DM thanks to the use of the technical representation of ecovillage in which the selected facilities and their timing were represented. For example, Figure 2 presents a representation of the selected facilities for the most representative plan for the DM. Figure 2 illustrates the "architectural plan" of the various floors of the buildings that constitute the Upper Borough. In architecture, the "plan" is the top view of a building sectioned with a horizontal plane. Specifically, the Figure is divided into columns and rows. The three periods in which the work was conducted and the various facilities in the buildings are indicated in the columns. The numbers indicated at the bottom right of each image represent the level heights, i.e. the relative heights of the floors, which may be preceded by a + or - sign in reference to the appropriately chosen 0.00 height. Thus, if one looks at the six images in a column, one is "looking" at the architectural plans of each floor of the buildings in the Upper Borough, where the colours indicate the works carried out and the facilities inserted at the specific time. Different colours have been used to facilitate the DM's understanding of the temporal sequence of the realisation of the facilities: facilities activated at t_1 are light blue, those at t_2 are pink and those at t_3 are light blue. If one reads Figure 2 through the lines, one can see how the new facilities could be realised and how the ecovillage project could be gradually developed. The arrangement of the floor plans made communication and evaluation of the different plans particularly effective.

6. Conclusions

We presented deck-of-cards-based ordinal regression (DOR), a new multicriteria decision aiding procedure. To ensure the ease and understandability of the interaction with the DM, the richness of the obtained preference information and the flexibility of the decision model to construct, DOR conjugates the deck-of-cards method with the ordinal regression approach to define a multicriteria value function representing the Decision Maker's (DM's) preferences. Thanks to the deck-of-cards method, the preference information collected in the DOR methodology also considers the intensity of preferences (measured in terms of the number of blank cards between reference alternatives). Therefore, it is finer than the mere ranking of reference alternatives considered by standard ordinal regression methods such as UTA. However, thanks to the deck-of-cards method, the preference information required can be considered easy and understandable for the DM. We also showed that, owning to its specific ordinal regression optimisation model, DOR can consider value functions that can have different forms, such as weighted sum, additive value function, or Choquet integral. This is another advantage of the proposed methodology because it permits the selection of a more appropriate value function formulation in consideration of the decision problem at hand; for example, using a weighted sum in case there is a necessity to be as simple as possible, or adopting the Choquet integral in case it is convenient to consider interactions between criteria. Moreover, this flexibility can be further augmented by the possibility of modifying the formulation of the value function during the decision process. For example, the decision aiding procedure can start with the weighted sum, when the DM initially needs a simpler decision model to familiarise itself with the decision problem at hand and, after, one can pass to the Choquet integral, when the DM has gained some awareness of the crucial points of the decision problem and more specific aspects need to be taken into consideration, such as the interaction between criteria. Because these are useful properties of a decision-aiding methodology, we are convinced that DOR can constitute a relevant evolution in the domain of ordinal regression models.

We also showed that the value function obtained from the application of DOR can be applied to a multi-objective optimisation problem. In particular, the solutions maximizing the value function aggregating the considered objective functions can be searched for and proposed to the DM, which can further rank and pairwise compare them with the deck-of-cards method. With this new preference information, a new value function can be defined and optimised, obtaining other solutions to be proposed to the DM. This process can be iterated until the DM is satisfied with the proposed solutions. Let us point out that the size of the problem at hand will impact our procedure during the computation phase; for example, if we deal with a very large instance of a combinatorial optimization problem, perhaps we may need to apply some specific algorithms to solve the problem and find some solutions to propose to the DM. However, in the decision phase, we do not need a very large number of solutions; it is up to the analyst, based on the problem, to decide how many solutions to propose to the DM.

We also discuss the application of this DOR-guided multi-objective optimization procedure to urban and regional planning problems in which facilities need to be selected, located and planned. With this aim, we considered the formulation of these territorial planning problems in terms of the so-called space-time model (Barbati et al., 2020), which in turn, was generalised by considering the interactions between criteria (through the use of a value function $U(\mathbf{x})$ formulated in terms of a Choquet integral) and synergies between facilities.

Figure 2: Selected facilities and their timing for the most representative plan $\mathbf{x}_2^{''}$

Finally, we applied the above-described methodology to a real-world problem to plan the development of a sustainable ecovillage in the province of Turin (Italy), supporting the president of the cooperative owning the ecovillage in his decisions regarding which structures to select, where to locate them and when to plan their realisation. In this specific context, the challenge is to create an environmentally responsible settlement that can reconcile two conflicting perspectives: the desire to pursue an informal economy that is entirely unrelated to commercial logic and, at the same time, the need to achieve economic self-sufficiency in the settlement. In addition, there are three types of users: residents, WWOOFERs and guests, imposing location choices with very different timeframes (short, medium and long term), relating to both the construction of various buildings and the subsequent management of the functions to be performed in them. This type of application is specifically relevant because it can be viewed as a case study for decision-making related to choices involving aspects such as sustainability and social responsibility which are fundamental for planet Earth's future generations. Regarding the actual realisation of the "House of the Sun", it must be said that the construction work on ecovillage has unfortunately not started. However, the application of our model has not been in vain because the president of the association that owns the buildings to be transformed into the "House of the Sun", i.e. the DM who interacted with us, is using these results to discuss both with various banks to obtain financing and with the architects to define the final design. According to his statement, what has been particularly helpful is the awareness he has gained regarding the most urgent facilities to be realised, the possible synergies between the facilities and the values guiding his choices. With respect to future research, the following points are seemingly the most promising:

- The urban and regional decision support methodology we are proposing could be applied in other contexts, and different decision-aiding problems could be considered, such as, for instance, corporate facility location/timing problem.
- The proposed methodology could be integrated to include the opinions of several DMs and could be adapted in a group context decision-making process.
- Applications of the methodology to large-scale planning could be developed.
- Theoretical advances to consider much longer time periods, concerning also intergenerational issues could be dealt with.
- Several elements of the methodology could be subject to sensitivity analysis to test their robustness, such as the discount rate adopted or the number of solutions to show to the DM.
- The elicited value functions could also be tested with other methodologies, such as simulating the presence of a DM or with human artificial intelligence methods (e.g. Angilella et al., 2016; Corrente et al., 2024)

Finally, we wish to point out that a specific interest is related to the DOR methodology, which can be tested on several diversified decision problems to verify its advantages in real-world decision problems.

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