Nonlinear Noise Estimation using Linear Least Squares-based Longitudinal Power Monitoring

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Abstract *A novel method for closed-form nonlinear noise estimation relying on linear least squares-based longitudinal power monitoring is presented and experimentally validated in a C-band transmission over an EDFA-amplified 5-span 60-km SMF link.*

Introduction

Optical performance monitoring plays a crucial role in ensuring reliable operation and effective management of today's optical networks. In this framework, flexibility and high capacity are key aspects to meet modern communication requirements, and their fulfillment should rely on low-complexity and cost-effective monitoring solutions[1].

Among the recently proposed techniques, longitudinal power monitoring (LPM)[2],[3] has proved to be effective in a wide range of applications, given its major advantage of solely leveraging the information already available in standard coherent receivers. Not only does it allow for distance-resolved power profile estimation of an optical signal, but it has also been successfully employed to estimate additional system parameters, such as polarization-dependent loss[4],[5], fiber types[6] and gain spectra of optical amplifiers[7]. Thus, LPM represents a potential solution for the identification and localization of possible sources of impairment that could degrade the system quality of transmission (QoT)[8].

Its potential applications, though, are not limited to those mentioned above. One of the fundamental figures of merit for system design and optimization is the generalized signal-to-noise ratio (GSNR)[9], that encompasses the contributions from both amplified spontaneous emission (ASE) noise and Kerr-induced nonlinear interference (NLI), modeled as an AWGN source. Hence, the segregation of each noise contribution allows the optimization of the optical network, leading to more efficient and low-margin transmissions.

In recent years, this subject has been addressed resorting to various methodologies. The main proposals involve artificial neural networks[10],[11], perturbation-based frequency-domain estimations[12] and proper manipulation of the statistics of the noise affecting the received signal[13]. However, all these approaches require either extensive knowledge of transmission and link parameters or large training datasets. Moreover, they have been tested in relatively simplified scenarios.

In this context, linear least squares (LLS)-based LPM[2] represents a potential solution to overcome the aforementioned limitations. Since Kerr-induced NLI is a power-dependent effect, the signal power profile made available by LLS-based LPM can be exploited to get an estimate of the nonlinear SNR.

This type of solution was suggested in[7] and a first implementation proposal resorts to the split-step Fourier method (SSFM) to perform NLI estimation[14]. In this paper, an alternative solution is proposed and experimentally validated, based on the reconstruction of the first-order perturbation of the Kerr effect according to the enhanced regular perturbation (eRP1) approximation[15]. In addition to limited ASE noise impact and no need for specific knowledge of the system, it also provides a closed-form solution for NLI estimation.

Methodology

The total SNR after propagation over an optical link can be defined as:

$$\mathrm{SNR} = (\mathrm{SNR}_{\mathrm{TRX}}^{-1} + \mathrm{OSNR}^{-1} + \mathrm{SNR}_{\mathrm{NL}}^{-1})^{-1} \quad (1)$$

where $\mathrm{SNR}_{\mathrm{TRX}}$ accounts for transceiver noise, OSNR for ASE noise and $\mathrm{SNR}_{\mathrm{NL}}$ for Kerr-induced NLI. The objective of this paper is nonlinear noise estimation, which consists in the estimation of the last term, namely $\mathrm{SNR}_{\mathrm{NL}}$.

For this purpose, the proposed solution relies on the LLS-based LPM algorithm and the least squares optimization problem it attempts to solve[2]:

$$\hat{\gamma}' = \underset{\gamma'}{\mathrm{argmin}} \|\boldsymbol{A}_1 - \boldsymbol{A}_{\mathrm{ref},1}\|^2 \quad (2)$$

In Eq. (2), $\gamma' = \frac{8}{9}\gamma\boldsymbol{P}$ is the optimization target and includes the contributions of the fiber nonlinear

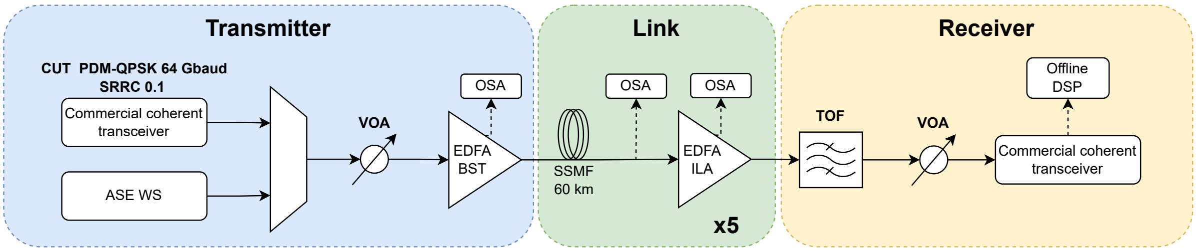

Fig. 1: Experimental setup. WS: wave shaper; VOA: variable optical attenuator; BST: booster; OSA: optical spectrum analyzer; ILA: in-line amplifier; TOF: tunable optical filter.

coefficient γ and the power profile $\boldsymbol{P}$ of the optical signal. $\boldsymbol{A}_1$ and $\boldsymbol{A}_{\mathrm{ref},1}$ represent the first-order Kerr perturbation according to the eRP1 approximation of the received and reference signals, respectively. Note that the latter can be expressed in matrix form as $\boldsymbol{A}_{\mathrm{ref},1} = \boldsymbol{G}\boldsymbol{\gamma'}$ to isolate the contribution of $\boldsymbol{\gamma'}$, as reported in[2].

Fundamentally, the algorithm yields the optical power profile that minimizes the difference between $\boldsymbol{A}_1$ and $\boldsymbol{A}_{\mathrm{ref},1}$ in the least-square sense. This implies that applying $\boldsymbol{G}$ to $\hat{\boldsymbol{\gamma}}'$ leads to an estimate of $\boldsymbol{A}_1$, that is the part of the signal carrying information on NLI:

$$\hat{\boldsymbol{A}}_1 = \boldsymbol{G}\hat{\boldsymbol{\gamma}}' \tag{3}$$

It is then straightforward, under the assumption that the power spectral density (PSD) is flat, to compute the nonlinear SNR as the difference in logarithmic units between the PSDs of the reference signal $\boldsymbol{A}_{\mathrm{ref}}$ and $\hat{\boldsymbol{A}}_1$:

$$\mathrm{SNR}_{\mathrm{NL}}^{\mathrm{dB}} = G_{A_{\mathrm{ref}}}^{\mathrm{dB}} - G_{\hat{A}_1}^{\mathrm{dB}} \tag{4}$$

Both Eq. (3) and Eq. (4) provide several advantages. In the first place, they are closed-form formulas. This eliminates the need for SSFM to estimate the nonlinear noise. Moreover, the impact of ASE noise is limited since it only affects $\hat{\boldsymbol{\gamma}}'$ in Eq. (3) and can be mitigated resorting to successive profile averaging and/or increasing the number of samples employed in the LPM algorithm[2]. Lastly, the required knowledge of the system is in principle restricted to the cumulated chromatic dispersion (CD), which is generally available in coherent transceivers for CD compensation.

The most significant drawback of this solution, though, is the fact that LPM only accounts for self-channel interference (SCI). The reason is that no information on interfering channels is available at the receiver side. This implies that, in a WDM scenario, a correction to Eq. (4) is required to include the contribution of cross-channel interference (XCI). The possible solutions depend on the level of knowledge of the system. In this work, the proposed correction is based on a convenient approximation of the normalized NLI efficiency presented in[16]:

$$\Gamma_{\mathrm{NLI}} \cong \frac{1.5 \cdot 10^{24}}{E_{\mathrm{ph}}^2} \frac{B_{\mathrm{opt}}^{1/4} \cdot \gamma^2}{K_{\mathrm{S}} \cdot \alpha_{\mathrm{dB}} \cdot D^{5/6}} \tag{5}$$

where $\mathrm{B}_{\mathrm{opt}}$ is the overall occupied optical bandwidth and, in the case of homogeneous configuration of the WDM comb, can be written as $\mathrm{B}_{\mathrm{opt}} = \mathrm{N}_{\mathrm{ch}}\Delta f$. This formula allows to derive a correction factor that only requires the knowledge of a single additional system parameter, i.e., the number of WDM channels N_{ch}. The ratio of Eq. (5) evaluated in a multi- and single-channel scenario yields a scaling factor to correct the result yielded by Eq. (4):

$$G_{\mathrm{XCI}} = \frac{\Gamma_{\mathrm{NLI}}^{\mathrm{N_{ch}}}}{\Gamma_{\mathrm{NLI}}^{1}} \cong \mathrm{N}_{\mathrm{ch}}^{1/4} \tag{6}$$

In conclusion, adding Eq. (6) to Eq. (4) yields the final formula for nonlinear noise estimation:

$$\mathrm{SNR}_{\mathrm{NL}}^{\mathrm{dB}} = G_{A_{\mathrm{ref}}}^{\mathrm{dB}} - G_{\hat{A}_1}^{\mathrm{dB}} - G_{\mathrm{XCI}}^{\mathrm{dB}} \tag{7}$$

Experimental setup and results

The proposed scheme was validated in the setup shown in Fig. 1. In this experiment, a wavelength-division multiplexing (WDM) transmission is performed using a commercial transceiver to generate the channel under test (CUT). The CUT is a PDM-QPSK 64-Gbaud signal, shaped by a square-root raised-cosine (SRRC) filter with roll-off 0.1 and centered at $f = 193.3\,\mathrm{THz}$. An ASE noise source followed by a WaveShaper (WS) was used to generate the interfering WDM channels. The resulting WDM comb consists of 18 channels spaced by $100\,\mathrm{GHz}$ over the C band.

The signal is amplified with a first erbium-doped fiber amplifier (EDFA) acting as booster (BST) and propagated through an optical link consisting of 5 standard single-mode fiber (SSMF) spans with a nominal length of $60\,\mathrm{km}$. Each span is terminated by an in-line EDFA (ILA) working in automatic gain control mode and compensating for the previous span loss. Note that the monitoring ports of EDFAs and the $90/10$ splitters placed at the end of each span are connected to an optical spectrum analyzer (OSA) to monitor the power spectral density (PSD) evolution of the propagating WDM signal.

At the receiver side, the CUT is filtered by a tunable optical filter (TOF) centered at the CUT center frequency and attenuated by a VOA to adjust its power before entering the commercial transceiver. Here, it gets sampled at 96 GSa/s by the analog-to-

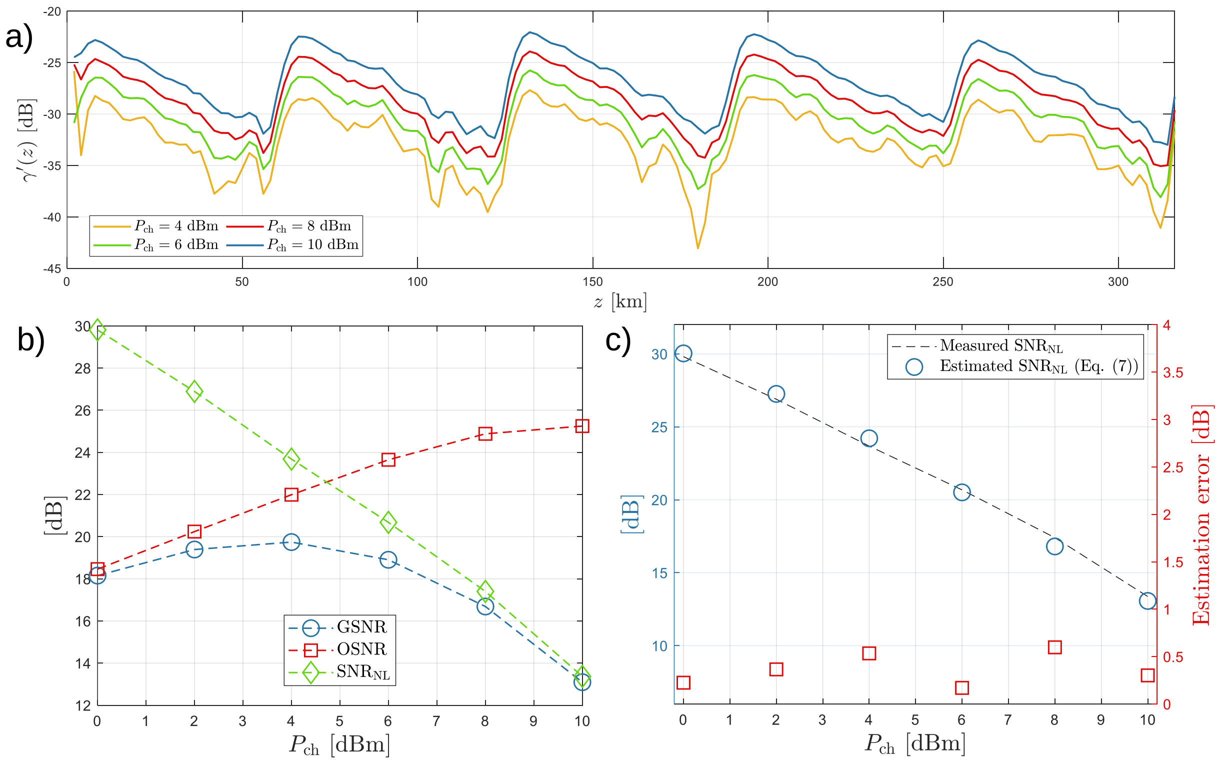

Fig. 2: a) Resulting power profiles after 100-profile averaging for P_{ch} from 4 dBm up to 10 dBm. b) Measured GSNR, OSNR and $\mathrm{SNR_{NL}}$. c) Comparison between measured (dashed black) and estimated (blue) $\mathrm{SNR_{NL}}$ with corresponding absolute estimation error (red).

digital converter (ADC) and the acquired samples are downloaded for offline processing.

After front-end corrections and resampling at a rate of 2 sample-per-symbol, they enter the coherent digital signal processing (DSP), where CD compensation (CDC), frequency offset compensation (FOC), LMS-based 2×2 MIMO fractionally-spaced equalization and Viterbi-Viterbi (4^{th} power) carrier phase recovery (CPR) are performed. The output of the CPR stage is then extracted, CD is reloaded and the signal is fed to the LPM algorithm. The number of samples used as input is 2^{17} and the spatial resolution is set to $\Delta z = 2\,\text{km}$.

In order to span over a wider range of $\mathrm{SNR_{NL}}$ values, the per-channel power P_{ch} has been varied between 0 dBm and 10 dBm, with a step of 2 dBm, by using a variable optical attenuator (VOA) placed before the BST EDFA.

For each tested condition, 100 profiles have been averaged to improve the SNR of the estimated power profiles. A few examples of results are displayed in Fig. 2(a). These profiles are finally used in the computation of Eq. (3) and Eq. (7) for the estimation of $\mathrm{SNR_{NL}}$.

To validate the proposed method, the nonlinear SNR is also measured experimentally. In particular, the overall SNR is retrieved from the commercial transceiver for each per-channel power level. The OSNR and the $\mathrm{SNR_{TRX}}$ are instead estimated from the power spectral densities measured by the OSA and a set of back-to-back data acquisitions, respectively. Hence, $\mathrm{SNR_{NL}}$ is easily obtained from Eq. (1). The results are reported in Fig. 2(b). Note that the $\mathrm{GSNR} = (\mathrm{SNR}^{-1} - \mathrm{SNR_{TRX}}^{-1})^{-1}$ is displayed instead of the SNR.

The measured values of $\mathrm{SNR_{NL}}$ are finally compared to those estimated by Eq. (7), as shown in Fig. 2(c). The corresponding absolute estimation error is also reported.

In general, the results obtained with the proposed method show good agreement with the measured values. The estimation error is always below $0.6\,\text{dB}$ and characterized by a root mean square error (RMSE) of approximately $\sim 0.4\,\text{dB}$. Moreover, the simple correction factor derived in Eq. (6) proves to be effective in mitigating the estimation bias induced by XCI in the considered transmission system. Future work should explore the limitations of this simple solution in more complex scenarios, such as heterogeneous or Raman-amplified links.

Conclusions

A novel closed-form nonlinear noise estimation algorithm relying on LLS-based LPM is proposed and experimentally validated. It proves to be effective in the considered transmission scenario, yielding an RMSE value of approximately $\sim 0.4\,\text{dB}$ computed over all the tested system conditions. Besides, the proposed method provides the advantage of limiting the impact of ASE noise and requiring a reduced knowledge of the link and system parameters.

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