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WHAT IS A "GOOD" ARGUMENTATION IN MATHEMATICS CLASSROOM?

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This paper investigates the argumentative processes produced by upper secondary school students during an educational activity designed with the dual goals of introducing classical concepts of probability theory through problem solving and promoting argumentative competence. We reckon that in this context the main function of argumentations is to support probabilistic thinking development. We are interested in investigating how this function influences the development of argumentative processes in the classroom. We analyse two classroom episodes in order to show how argumentative processes evolve in relation to the interventions of the teacher and the peers, and how the changes in the produced arguments reflect the function that argumentation is intended to serve.

INTRODUCTION

In the educational context, the teaching and learning of argumentation is receiving increasing attention and it is included in the official documents of several countries. In Italy, argumentation appears within the learning goals at every scholastic level, even with explicit reference to mathematics (Mariotti, 2022). A reflection on the aspects of argumentation that can be considered and assessed in the mathematical classroom is therefore necessary (Stylianides et al., 2016). This depends on the conceptualisation of argumentation. Many Mathematics Education researchers have investigated the topic of argumentation from different perspectives and there is no single definition of argumentation in the field (Hanna, 2020). As far as mathematics education is concerned, some research highlights the social side of argumentation and focuses on the aspects of the argumentation that influencing acceptance by the classroom. Stylianides (2007) notices that the statements used in an argument should be in line with certain standards of the current mathematical culture and, at the same time, it should be accepted within the conceptual reach of the classroom participants. Classmates and especially the teacher play a crucial role. Interlocutors – and therefore their beliefs, knowledge, and convictions – can influence the development of the argumentative process (Krummehuer, 1995) and the constitution of socio-mathematical norms (Yackel, 2001). Students-teacher cooperation is usually exercised through the use of language, which plays a key role in mathematics classroom, and the development of (written and/or spoken) texts (Ferrari, 2021). Argumentation processes can be developed in classrooms in relation to different activities and purposes (not only to convince). For example, argumentation can be taken into consideration in relation to conjecture (Pedemonte, 2007), or it could be used to make students' thinking visible (Cusi et al., 2017). In addition, it is often related to students' learning (Schwarz, 2009).

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We think that the different functions of the argumentative process in classrooms are related to the educational goals. This work is part of the research strand that considers argumentation primarily as a social process, based on production of texts. We focus on argumentations developed during an upper secondary school classroom sequence, designed with the double aim of introducing classical probability theory concepts through problem-solving and fostering argumentative competence. In this context, the development of argumentative processes and probabilistic thinking are interconnected. In particular, argumentations sustain the development of sensemaking in probability. The core of the paper is investigating how the argumentation function of sustaining learning influences the development of argumentative processes in the classroom.

THEORETICAL FRAMEWORK

In line with Ferrari (2019), we consider argumentation as an interactive process based on the production of (written and/or spoken) texts. Texts and, more generally, language play different functions in mathematical classroom. For example, Ferrari (2021) distinguishes functions that are related to everyday use and functions typical of mathematics education. His research makes use of tools of linguistic pragmatics, whose main interest is the study of different uses of language. As far as argumentation is concerned, the neuropsychologists Mercier and Sperber (2017) thoroughly study uses of argumentations and reasoning. In accordance with the authors, arguments are mostly intended for social consumption. People may present them to explain and justify themselves, to evaluate others' reasoning, or to convince those who think differently. Producing and sharing arguments generally make communication more advantageous, by making it more reliable. A good argumentation should display coherence relationships between the speaker's claim and the knowledge, convictions and system of beliefs held by the addressees, allowing the audience to evaluate these relationships on their own. The ways in which the coherence relationship can be displayed and evaluated depend on the context and on cultural aspects. Moreover, in Mercier and Sperber's (2017) theory, the role of the interlocutor/s is fundamental. As in the broader case of communication, the interactive nature of the dialogue and the interlocutor's responses allow to refine justifications and arguments, shaping the argumentation process. The interlocutor's reactions are particularly useful for two reasons: she/he can indicate whether she/he has understood, and she/he can actively guide the effort of the speaker. In our study we mainly focus on conversations developed in educational settings that are characterised by educational aims. We are hence interested in reactions that contain information about student's performance and understanding in mathematical classroom. In accordance with the framework of Hattie and Timperley (2007) we call such reactions feedback. The main purpose of feedback is to help learners shorten the distance between current understanding and performance and the intended educational goals. The effective feedback addresses the issues of clarifying the direction of the teaching and learning trajectory, of considering the progress that has been made towards the goal and of defining which activities are useful to make better progress. It could address multiple issues at once. Feedback works at four levels:

Task level (in relation to quality of the task implementation), Process level (concerning the process underlying a task), Self-regulation level (referring to the way students face the task and the achievement of the learning goals, including control and confidence), Self level (usually expressed as praise addressed to students).

In line with the theoretical framework and the context of the didactical sequence considered, we focus on argumentative processes mainly addressed to other classmates and responding to the function of supporting the learning processes occurring in the mathematics classroom. The question the article seeks to answer is: how do these argumentative processes evolve in relation to teacher and student feedback, and which ways to express the coherence relationships between the speaker's statement and the knowledge and belief system of the peers emerge?

METHODOLOGY

We collected data from a didactical sequence that was designed with the teacher and that was implemented during regular mathematics classroom activities of 19 students of 11th grade. Two main educational goals were involved: to introduce classical probability theory concepts through problem-solving and to foster collaborative argumentative processes. The last one is at the core of the paper. The didactical sequence design included two main phases. In the first phase students were engaged with introductory activities focused on the resolution of the classical problem of "division of the stakes", historically associated with the emergence of classical probability concepts in the Pascal-Fermat correspondence (Borovcnik & Kapadia, 2014). In the second phase, students were challenged with different problems, which allowed them to face some of the misconceptions typical of the field (Batanero, 2005). In this work we consider only argumentative processes developed during the first phase. This phase focused on the "division of the stakes" problem, which is:

Two players A and B play heads or tails with a fair coin. Each game, corresponding to each coin toss, is won by A if the outcome of the toss is heads and by B if the outcome is tails. A and B give 12 euros each. The stake is 24 euros. The player who first wins 6 rounds wins the game, and thus the entire stake. A always bets on "heads" and B on "tails". The game is interrupted at the score 1- 0 for A. How should the stakes be fairly divided i.e., that it gets both players to agree?

The problem was presented to the students before any theoretical probability concept related to it. Firstly, students faced the problem in small groups and shared their resolution to the whole class (step 1). Secondly, students returned in their small group to answer some teacher's questions that help them to analyse their and other groups' resolutions with a critical stance. The reflections were then shared during a collective discussion (step 2). Subsequently students were presented with some solutions given by mathematicians in the history of mathematics. We chose four resolutions: Pacioli's, Cattaneo's, Cardano's and Fermat's. Mathematicians' solutions were proposed for the problem in which the game was supposed to stop at a different score: 5- 3 for A. For the detail of the resolution and the ways in which they have been presented to students we refer to Paola (2019). Each of the four students' groups received a different resolution. Students then worked in small groups to understand the proposed resolution and they presented it at the rest of the classroom (step 3). Afterwards, in small groups' students reflected and critically analysed the proposed resolutions. Finally, during class discussion, students were asked to argue in order to sustain or to reject the proposed resolutions (step 4).

The lessons were video recorded, and the students' written productions were collected. Interactive argumentations have been the focus of a qualitative analysis.

PRESENTATION AND ANALYSIS OF TWO CLASSROOM'S EPISODES

In this section, we present and analyse data from the last two class discussions. Two episodes are described, in which students from the same group (G1) propose different arguments to support the same conclusion, and the related reactions of peers and the teacher. In addition, we briefly describe two representations introduced by another group (G2), that are crucial to analyse the second episode.

Episode 1

The first episode occurs during the class discussion in step 3. G1 study the mathematician's solution that was assigned to them and then present it to the classroom. Since they do not agree with the solution, they show another resolution they came up with. Below, we report Andrea's presentation of the G1 proposal and the subsequent reaction of one of his classmates.

Andrea: We calculated that, now I'll explain how, A has a chance of winning by 7/8 while B by 1/8. Why? Because we start from a situation of 5 to 3. And if we assume […] that B wins, tails would have to come up three times in a row. However, the probability that of getting tails three times in a row is one over two cubed. Why? Because the probability of getting heads or tails is always fifty percent, so one-half, and if this must happen three times in a row it will be one-eighth and therefore two cubed. Consequently, the probability of A winning is seven-eighths…

Enrico: I don't understand why, if it has to come three times heads, I have to multiply three times a half.

At this point in the discussion almost all students agree to divide the stakes proportionally to the probability of the two players winning. However, as declared by Enrico, it is not clear how to calculate these "probabilities of winning". Andrea's text is mathematically correct, and Andrea seems to be willing to explain and justify each step of his group solution. However, his text is based on some assumptions about probability computation that are not shared in the classroom. Enrico's question highlights this lack of understanding and agreement with the underlying assumption. Since Andrea is not able to explain why the result was 1/8, another G1-participant, Giacomo, tries to help. Giacomo is the only one in the classroom who had already encountered probability theory in previous scholastic segments. Firstly, Giacomo tries to explain the result showing all the possible outcomes of three tosses of a fair coin.

However, this does not explain why they used the multiplication, so Enrico raises the issue again.

Enrico: However, I don’t understand why the product…

Giacomo: Because at the algebraic level if you consider the probability of coming out tails to be equal to one half, if you toss the second coin, this [toss] is independent of flipping the first coin. Being two independent events, you have to multiply one half by one half.

Enrico: But why? I don't understand.

Giacomo’s justification is based on the knowledge of some classical probability computational algorithms, which are not known to the other students in the classroom. While Andrea's argument does not explain why the result of the calculus is 1/8, in Giacomo’s argument the justification is presented, but it is based on theory, which seems to be evoked in an almost authoritative way. According to Mercier and Sperber’s (2017) definition, both Andrea’s and Giacomo’s interventions are not good argumentations, since their peers are not able to evaluate the coherence relationship between the speaker’s claim and their knowledge, convictions and system of beliefs. At this point, the teacher decides to stop the discussion. He states that Giacomo’s assertion about the product of stochastic independent events sounds almost like a “guru’s recommendation”, and it is not useful to clarify the reason why. He concludes that the class needed to think about it, and he moves the discussion forward. The teacher could have accepted G1’s arguments and praised it as the correct resolution. On the contrary, he stops the conversation, without commenting on the correctness of the resolution. The teacher’s feedback is at the level of the process. He evaluates Enrico’s reaction as being suitable for the situation and he considers Giacomo’s intervention as not helpful to build common knowledge. Doing so, the teacher suggests that the function of a good argumentation is not only to support a resolution, but also to help students develop a common knowledge and understanding. The feedback is about the students’ performance (where the students are in their learning trajectory) and, at the same time, it is useful to clarify the teaching and learning goals.

G2’s representations

Later in the same lesson Enrico’s group (G2) presents Fermat’s resolution by means of a tree diagram represented in Figure 1. Fermat imagines that the players play the maximum number of games remaining and considers the possible outcomes. With a score of 5 to 3, there are three rounds left and eight possible outcomes, represented by the branches of the tree diagram at the top of Figure 1, where T is for heads and C is for tails. In seven out of eight cases A wins; that is, the probability of winning for A is 7/8 and for B is 1/8. Fermat’s resolution is based on a fiction (in any case, the two players play three rounds) that is not easy for students to accept. In fact, G2-students observe that the players would play three more rounds only if tails came up in the first two tosses, otherwise the game stops after one or two rounds. Then, they present another possible representation of the game (Figure 1, lower part). In this tree diagram each branch stops with the victory of one of the two players, and the probability of winning for B is calculated to be equal to 1/4.

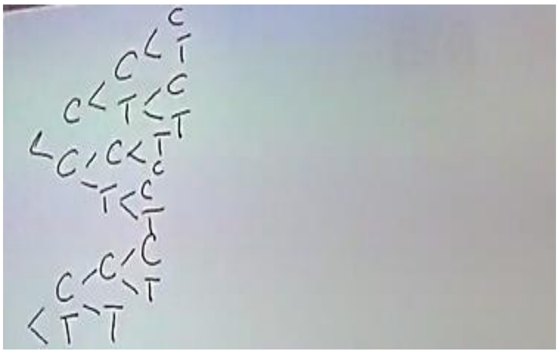

Figure 1: Tree diagrams of Fermat’s (at the top) and G2’s (below) resolutions.

G2’s representations express their point of view and conviction about the problem, and their reluctance to accept Fermat’s mathematical fiction. Their difficulty in grasping the not equiprobability in the second representation could be related to the *equiprobability bias* (Batanero et al., 2005). These representations are subsequently referred to by the teacher and peers and can be considered shared within the class.

Episode 2

The second episode takes place during step 4 of the activity. After small groups’ reflection students are asked to discuss collectively. At a certain point of the class discussion, G1 shares their reflection and conclusion. Sabrina summarises the group resolution. She states that also using the G2’s tree diagram it would be possible to calculate player B winning probability to be equal to 1/8 (and not 1/4). She supports her claim with the argument reported below. Her speech is intertwined with the construction of the pie chart illustrated in Figure 2, the steps for its construction have been included within the transcript in brackets.

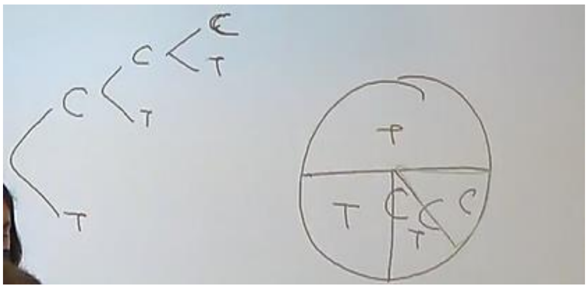

Figure 2: Group 1’s pie chart.

Sabrina: We tried to explain it with a pie chart. Here there is a 50% chance to get heads or tails, so the chart is like this (she draws a circle and divides it in half with a segment. She writes T in the upper half of the circle and C in the lower one). If heads wins, the game ends here (she points at the upper half of the circle), but if tails wins there is again a 50% chance that heads wins or tails wins (she divides the bottom half in two equal parts and writes T for heads and C for tails). Again, here the game ends and here it continues. And then again here it splits because if heads come up, heads

> wins, or tails, tails wins (she divides the circle quarter denoted by letter C in two parts), either way the game ends, but what we see here is 1/8.

All the students in the classroom, particularly G2’s students, claim they understand, and they agree with G1’s solution. Sabrina's argument refers to G2’s tree, which reflects their convictions and responds to their need to use representations more closely related to the game development. G1’s representation allows Sabrina to show the not-equiprobability of the possible outcomes by correlating the branches with different-sized slices of the pie chart, and without referring to unshared theoretical results. Pie charts are generally used to represent ratios and quantities and students had no difficulty in reading the result in such representation. According to Mercier and Sperber (2017), it can be considered a good argumentation, since it shows the consistency between G1’s and G2’s convictions and knowledge, and it allows students to evaluate the relationships on their own.

DISCUSSION AND CONCLUSION

Andrea’s and Sabrina’s arguments support the same conclusion. However, they express their conclusion in a completely different way. If the function of argumentation is to help collective sensemaking development of probability theory concepts, it is crucial to consider peers’ convictions and knowledge as a starting point. This allows to convince them by showing coherence relationships that they can evaluate on their own. Andrea’s and Giacomo’s arguments do not support the desired educational goal of developing probability concept sensemaking; therefore, they are rejected by some of their classmates and by the teacher. Enrico’s reaction to Andrea’s e Giacomo’s interventions could be interpreted as a sign of the fact that Enrico shares the learning goals. Conversely, Sabrina’s diagram seems to play a crucial role in conveying probability meanings. The pie chart strongly characterises Sabrina’s argument and allows her classmates to change their opinion. The analysis shows that goals shared and expressed by the feedback seem to be crucial for shaping the development of the collective argumentation, clarifying what can be accepted or not by the classroom at the moment. Considering the functions that the argumentative texts should fulfill in the classroom could help the teacher give feedback to students. Moreover, since the development of argumentation depends on context and interlocutors, adapting one's arguments to different interlocutors could be understood as a feature of argumentative competence. In conclusion, we remark how interactions are “elegant ways to divide cognitive labor” (Mercier and Sperber, 2017, p. 236), and how this interactive nature of the process could help students refine and enhance the arguments produced. However, the characteristics of a suitable argument depend on several factors, such as the didactical functions of argumentative processes, and the time in the teaching-learning process at which they take place. Further investigations are certainly needed. Firstly, the role of representations and, in general, the possible ways in which coherence relationships can be shown in mathematical argumentation are to be explored further. Secondly, considerations about possible ways to enhance students’ participation as an attentive audience are to be made. Finally, the peculiarity of the probability context is to be deepened, and other contexts could be taken into account.

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