DYNAMIC ASSESSMENT OF THICK SANDWICH BEAMS USING A MIXED-REFINED ZIGZAG THEORY: EXPERIMENTAL VALIDATION USING LASER DOPPLER VIBROMETRY

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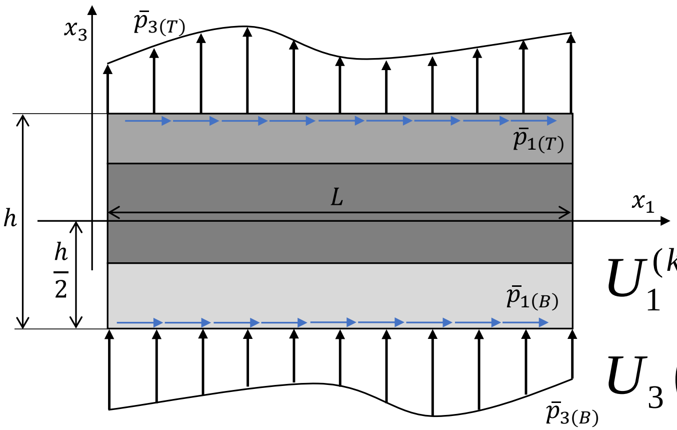
Outline

- Objectives
- Numerical mixed-RZT beam model
- Experimental set-up & sandwich specimens
- Numerical-experimental comparisons
- Concluding remarks & future perspectives

Objectives

- **To investigate** the dynamic behaviour of thick sandwich structures;
- **To include** in the RZT-based models for thick beams a more accurate prediction of the transverse shear and normal stress deformability;
- **To develop** a new simple beam element for numerical analyses;
- **To validate** the developed numerical model with new experimental data

Numerical model: the Beam {3,2}-mixed Refined Zigzag Theory B-RZT$^{(m)}_{\{3,2\}}$



Displacement field

$$U_1^{(k)}(x_1,x_3;t) = u(x_1;t) + x_3\theta(x_1;t) + \mu^{(k)}(x_3)\psi(x_1;t)$$

$$U_3(x_1,x_3;t) = w^{(0)}(x_1;t) + x_3 w^{(1)}(x_1;t) + x_3^2 w^{(2)}(x_1;t)$$

Global First-Order contribution

Higher-order contribution transverse stretching effect

Local third-order zigzag contribution

$$\mu^{(k)}(x_3) = \left(-x_3^2\chi_0 - x_3^3\omega_0 + \varphi^{(k)}(x_3)\right)$$

$$\omega_0 = \frac{2\left(\beta^{(1)} + \beta^{(N)} + 2\right)}{3h^2};\ \ \chi_0 = \frac{1}{2h}\left(\beta^{(N)} - \beta^{(1)}\right)$$

Third-order zigzag function properties:

- Vanish conditions on the outer beam surfaces;
- Satisfy the continuity displacements conditions at the interfaces;
- Guarantees a partial enforcement of the transverse shear stress continuity at the interfaces;
- Takes into account the effect of material and thickness variations

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Numerical model: the Beam {3,2}-mixed Refined Zigzag Theory[1] B-RZT$_{\{3,2\}}^{(m)}$

- Assumed fields:
  - Transverse normal stress:
    - Third-order power series expansion of the thickness coordinate;
    - Bottom/Top traction conditions

$$\sigma_{33}^{a}\left(x_1,x_3\right)=\sigma_0^z(x_1)+x_3\sigma_1^z(x_1)+x_3^2\sigma_2^z(x_1)+x_3^3\sigma_3^z(x_1)$$

  - Transverse shear stress:
    - Obtained from the integration of the **Cauchy's equations**

$$\sigma_{11,1}^{(k)}(x_1,x_3)+\tau_{13,3}^{(k)}(x_1,x_3)=0$$

    - Introduction in the assumed field a new set of strain variables $\begin{Bmatrix} e \\ k \\ k^{\psi} \end{Bmatrix}$

$$\varepsilon_{11}^{(k)}\left(x_1,x_3\right)=u_{,1}(x_1)+x_3\theta_{,1}(x_1)+\mu^{(k)}(x_3)\psi_{,1}(x_1)\rightarrow$$

$$\rightarrow e(x_1)+x_3k(x_1)+\mu^{(k)}(x_3)k^{\psi}(x_1)$$

[1] Sorrenti M. Refined zigzag models for the response of general multilayered composite and sandwich structures: numerical and experimental investigations. PhD Thesis. Politecnico di Torino, 2023.

Numerical model: the Beam {3,2}-mixed Refined Zigzag Theory B-RZT$^{(m)}_{\{3,2\}}$

$$\delta\Pi_{\text{int}} + \delta\Pi_{HR} + \delta\Lambda - \delta\Pi_{\text{in}} = 0$$

- Mixed Variational statement's contributions[2]:
  - d'Alembert Principle
  - Hellinger-Reissner functional
  - Penalty functional

$$\delta\Pi_{\text{int}} = \int_V \left[\left(\delta\varepsilon_{11}^{(k)}\sigma_{11}^{(k)} + \delta\gamma_{13}^{(k)}\tau_{13}^{a} + \delta\varepsilon_{33}\sigma_{33}^{a} \right) \right] dV \qquad \delta\Pi_{\text{in}} = -\int_V \rho^{(k)}\mathbf{U}^T\ddot{\mathbf{U}}\,dV$$

$$\delta\Pi_{HR} = \int_V \left[\delta\tau_{13}^{a}\left(\gamma_{13}^{(k)} - \gamma_{13}^{(k)a}\right) + \delta\sigma_{33}^{a}\left(\varepsilon_{33} - \varepsilon_{33}^{(k)a}\right) \right] dV$$

$$\delta\Lambda = \frac{1}{\eta}\int_V \left[\left(\delta u_{,1} - \delta e\right)\left(u_{,1} - e\right) + \left(\delta\theta_{,1} - \delta k\right)\left(\theta_{,1} - k\right) + \left(\delta\psi_{,1} - \delta k^{\psi}\right)\left(\psi_{,1} - k^{\psi}\right) \right] dV$$

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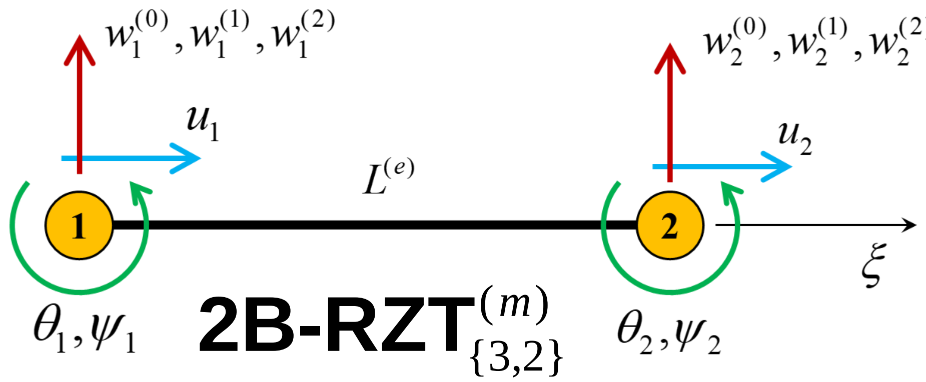
[2] Sorrenti M, Gherlone M. A new mixed model based on the enhanced-Refined Zigzag Theory for the analysis of thick multilayered composite plates. Composite Structures 2023;311:116787. https://doi.org/10.1016/j.compstruct.2023.116787.

A 2-node element based on the {3,2}-mixed Refined Zigzag Theory



- Main characteristics:
  - Use of linear Lagrangian shape functions;
  - Satisfaction of the **Babuska-Brezzi condition**[3,4] (mixed-formulation);
  - Static condensation of the additional strains' dofs (**from 18 to 12**);
  - Numerically accurate (when compared with analytical and 3D solutions[1]).

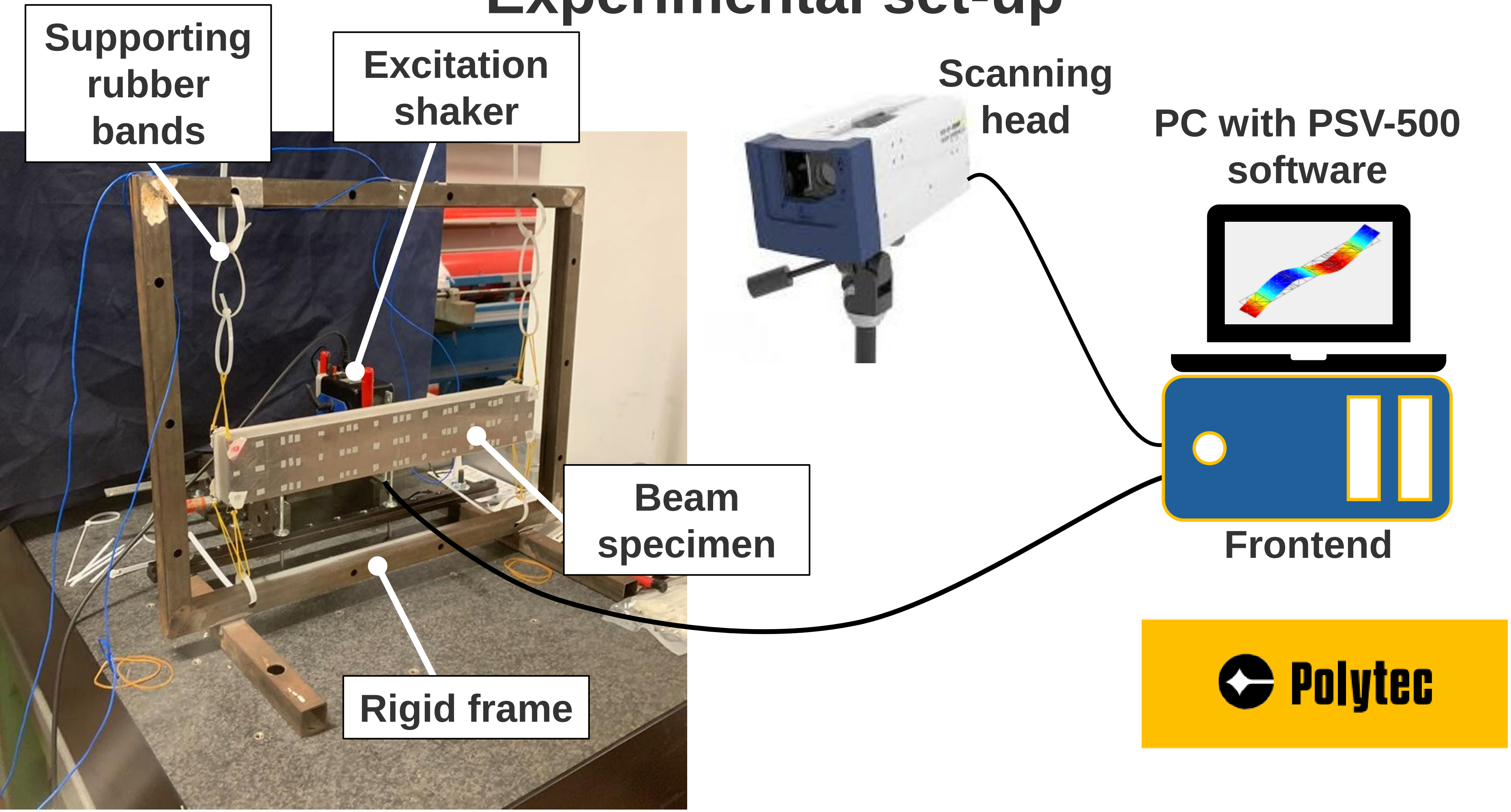
[3] Babuška I. The finite element method with Lagrangian multipliers. Numer Math 1973;20:179–92. https://doi.org/10.1007/BF01436561.
[4] Brezzi F. On the existence, uniqueness and approximation of saddle-point problems arising from lagrangian multipliers. RAIRO Analyse Numérique 1974;8:129–51. https://doi.org/10.1051/m2an/197408R201291.

Sandwich beam specimens

- ERGAL face-sheets;
- Rohacell® WF-110 foam core;
- AF-163-2K adhesive layer;
- Thick sandwich beams (**L/h<20**);
- Manufactured by Politecnico di Milano – DAER;
- Tested in LAQ-AERMEC at DIMEAS (Politecnico di Torino)

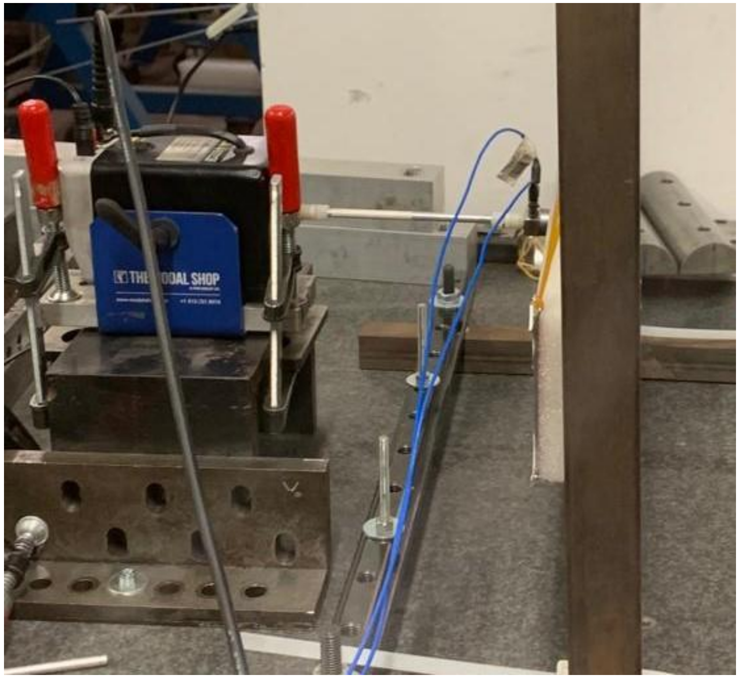
Specimen ID	L	B	h	h_f	L/h	h_c/h_f
D01	500.00	80.00	43.32	2.00	11.54	19.66
D02	500.00	80.00	41.18	1.00	12.14	39.18
D03	500.00	80.00	33.53	2.00	14.91	14.77
D04	500.00	80.00	31.54	1.00	15.85	29.54

Experimental set-up

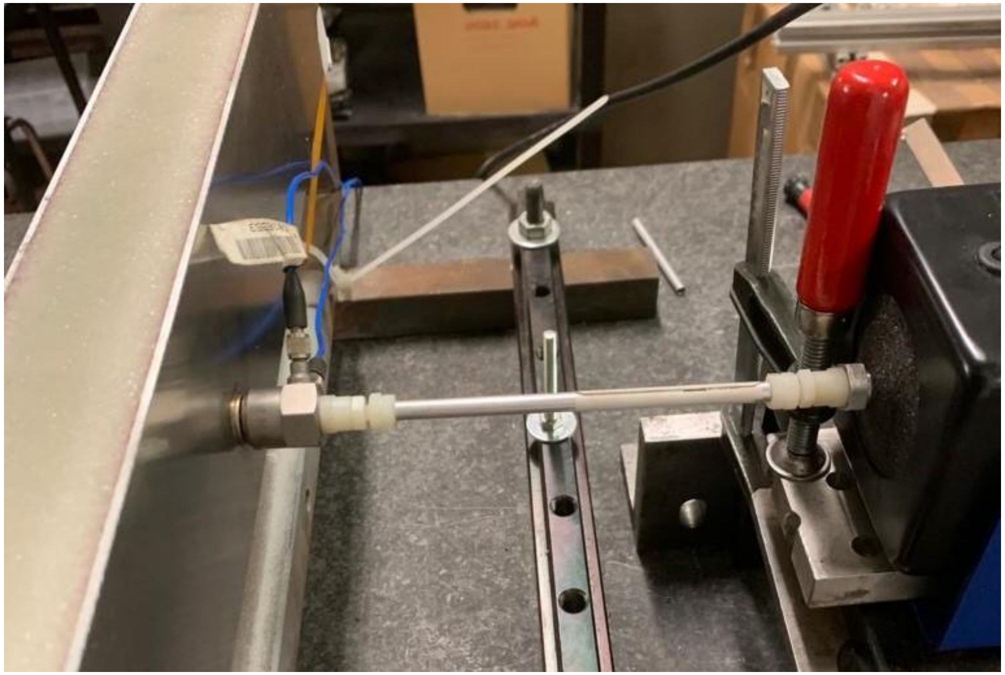


Experimental set-up

Excitation shaker K2007E01
(up to 9kHz and 67 N)

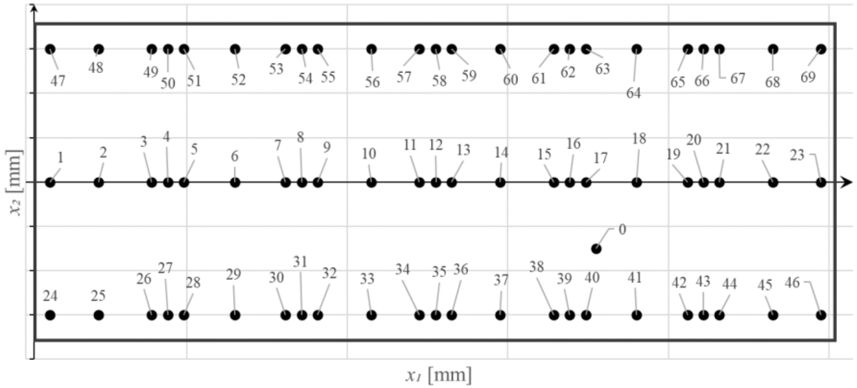


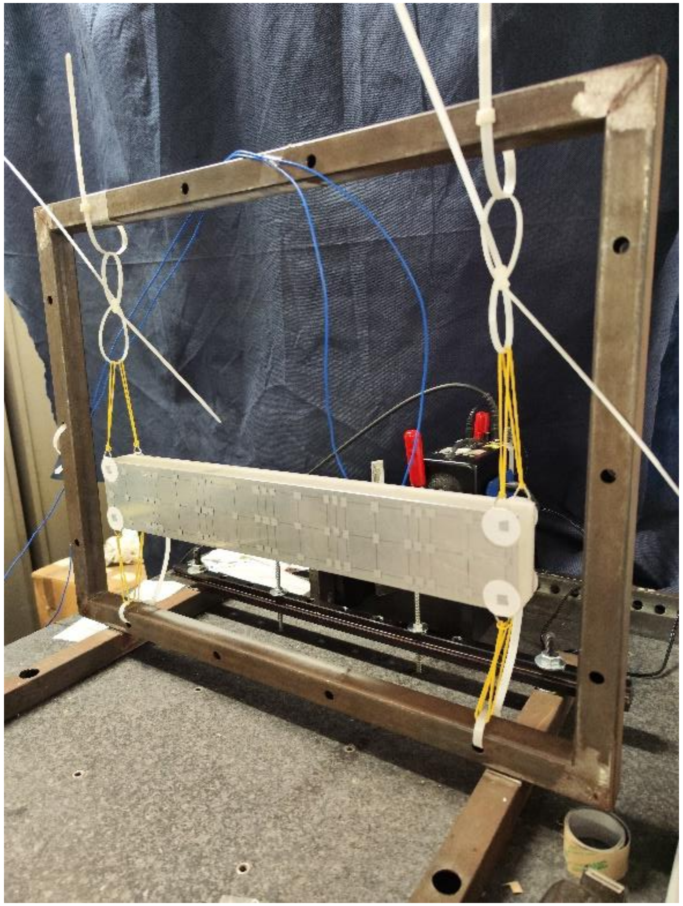
Stinger and Impedence Head
(PCB Piezotronics, Inc. – model 288D01)



Experimental set-up

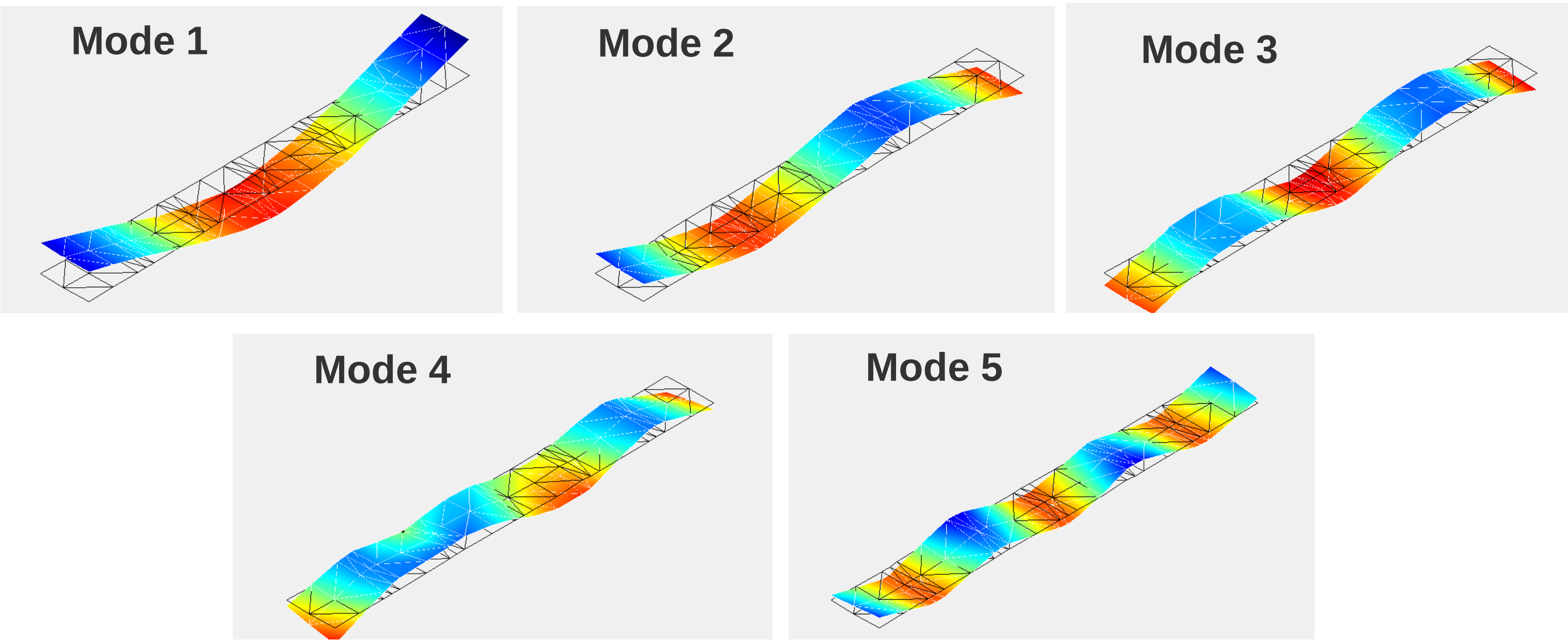
- Beam and shaker configuration
  - 69 scanning points + 1 acceleration (applied force point);
  - Shaker excitation → **periodic chirp** (from 400 Hz to 4000 Hz), constant amplitude;
  - Complex based averaging of 40 FRFs for each scanning point;
  - Only **flexural modes** have been considered in the comparison.





Numerical-experimental validations

Selected modes: flexural



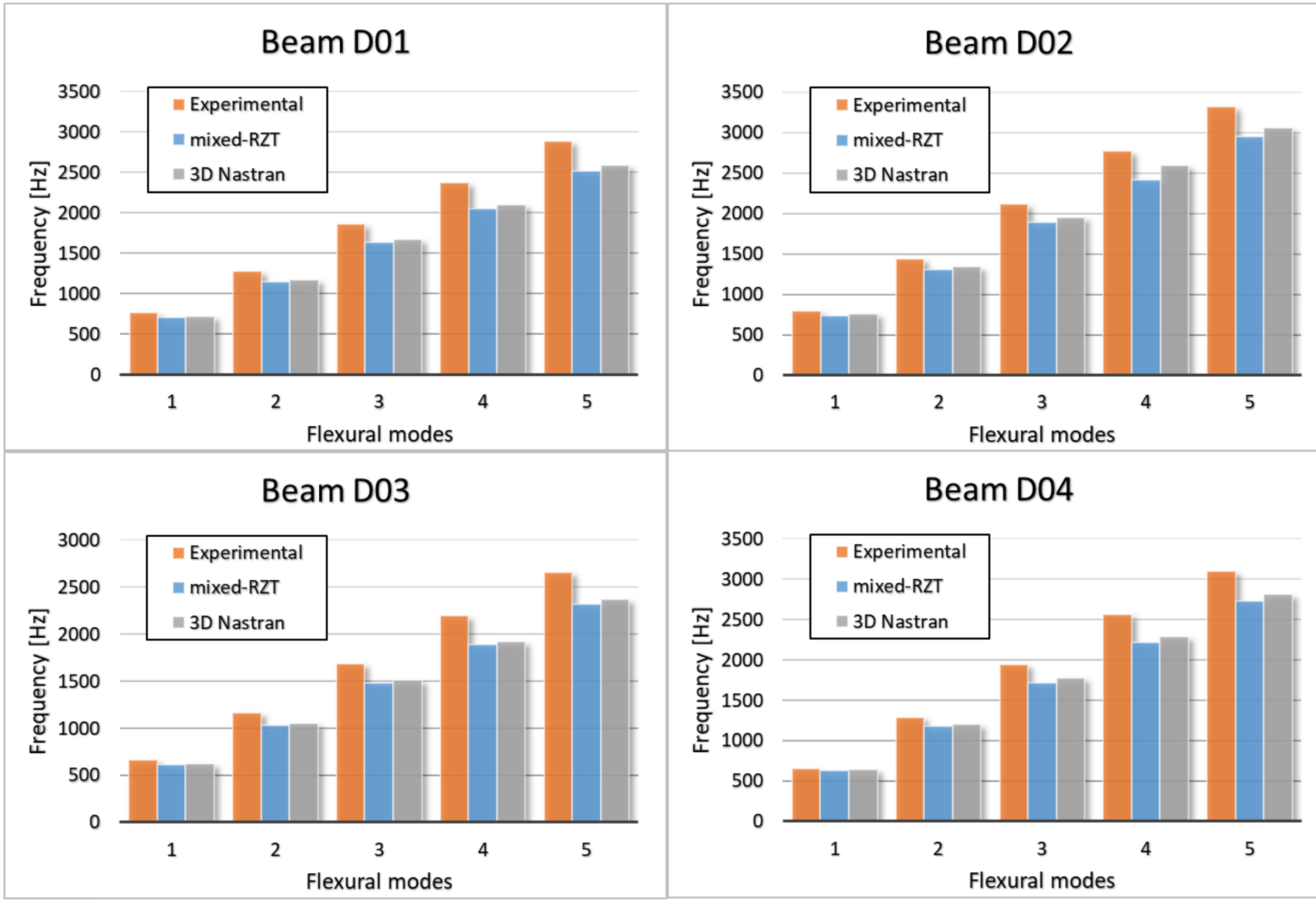
Beam D03

The modal shapes reconstructed with PSV software for beam D03 are similar with those obtained for other sandwich specimens

Numerical-experimental validations

4069 mixed-RZT beam elements

3D Nastran models >50000 HEXA8 solid elements[5]



[5] Carvalho S. Dynamic characterization of sandwich beams using a LASER Vibrometer. Master Thesis. Politecnico di Torino, 2023.

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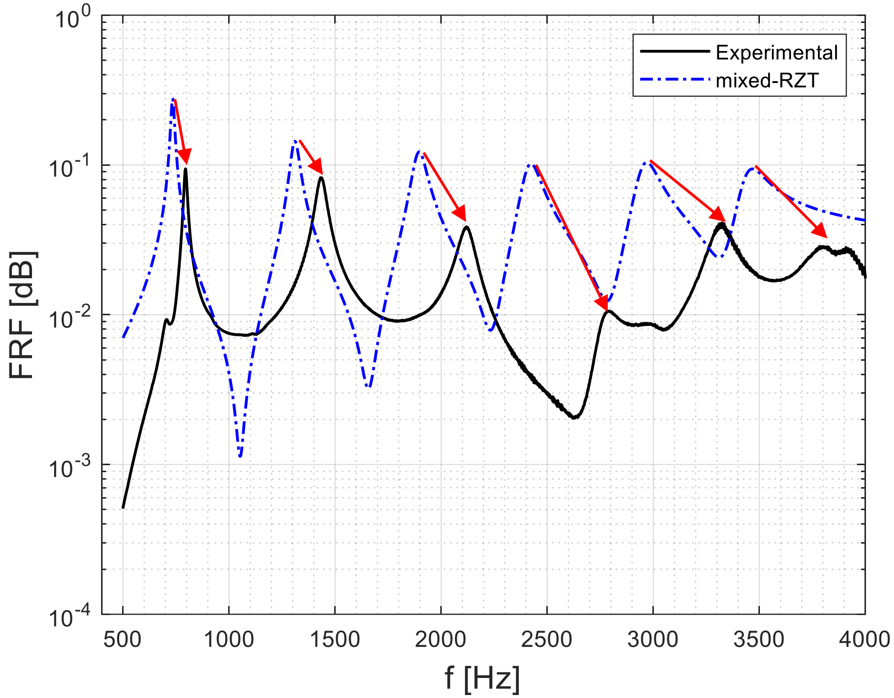
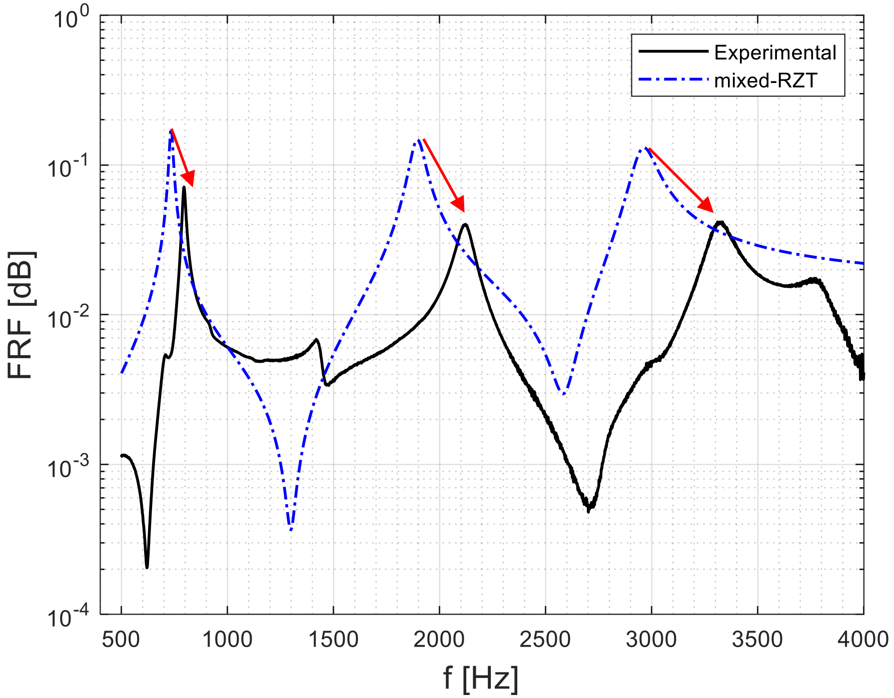
Numerical-experimental validations

Point 1 – Beam D02



All flexural frequencies and modes are observable

Point 12 – Beam D02



Only 1°, 3° and 5° modes are observable. Point 12 is a node of anti-symmetric modes (2° and 4°).

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Conclusion remarks and future perspectives

- A new mixed-RZT model for sandwich beams has been developed;
- This model addresses a new mixed-formulation used for developing a simple two-node beam element with:
  - Thickness stretching effect;
  - More accurate description of the transverse shear and normal stresses;
- New experimental data for thick sandwich beams;
- The use of LDV technique enabled:
  - More scanning points than conventional hammer test;
  - No influence of the measure systems (e.g. no concentrated mass such as accelerometers);
  - Wider frequency bandwidth (up to 4kHz);
- **Differences in the estimated frequencies with mixed-RZT not higher than 15%;**

Conclusion remarks and future perspectives

- Coherence between the 3D Nastran and the mixed-RZT one;
- Consistency between the numerical data and the experimental ones;
- Experimental results suggest a stiffer behaviour;
- Future perspectives to improve the mixed-RZT numerical model:
  - Inclusion of the stringer stiffness;
  - Inclusion of the stiffness and constraints introduced by the rubber bands;
  - Develop mixed-plate elements to take into account the torsion effect (not negligible in the experiment);
- To test sandwich beams with laminated face-sheets;
- To use the experimental/numerical modal data (e.g. modal shapes) for Structural Health Monitoring purposes in low-order models.

Thank you for your attention

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