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On Modeling Collective Risk Perception via Opinion Dynamics

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Abstract

Modeling the collective response to an emergency is a problem of paramount importance in social science and risk management. Here, we leverage the social-psychology literature to develop a mathematical model tailored to such a real-world problem, grounded in the opinion dynamics theory. In our model, a network of individuals revises their risk perception by processing information broadcast by the institution and shared by peers, and accounts for heterogeneity in terms of individuals' trust in institutions, peers, and in their own risk sensitivity. Through a rigorous analysis of the model, we establish that the temporal average opinions of the individuals converge to a steady state and, under some assumptions, we are able to analytically characterize such a steady state, shedding light on how the individuals' heterogeneous risk sensitivity shapes the collective response. Numerical results and simulations are provided to illustrate and corroborate our findings.

Keywords: Agents networks, Opinion dynamics, Social dynamics

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1. Introduction

The development and analysis of mathematical models for social dynamics have witnessed an increasing interest in the systems and control community, providing novel theoretically-informed tools to understand and predict collective human behavior [1, 2, 3, 4, 5, 6, 7, 8, 9]. In particular, a key area of research focuses on studying opinion formation in social communities through the lens of opinion dynamics models [10, 11, 12, 13, 14, 15, 16]. These models, in which individuals' opinions evolve in time through a linear averaging process that accounts for the information exchanged with peers on a social network, have been used to explore a wide range of social phenomena, from the evolution of social power [17, 18], to the emergence of disagreement and polarization [19, 20, 21].

Concerning opinion formation, a problem of particular interest is to predict the emergent behavior of a population of individuals during an emergency [22]. In this situation, it is crucial to predict how the individuals of a population collectively respond to the information that public authorities and institutions broadcast on the nature of the risk of the event under consideration in order to avoid underestimating the risk or, on the other extreme, emergence of panic reactions.

Despite the importance of such problem, the literature presents few mathematical models of opinion formation tailored specifically to such a scenario. On the one hand, classical mathematical models focus on an abstract representation of opinion dynamics [14]; on the other hand, social-psychological efforts are mostly concerned with unveiling the individual-level risk interpretation process [23, 24, 25, 26, 27], typically overlooking how such individual-level mechanism propagates at the population-level. In [28, 29, 30], different agent-based models tailored to capturing the emergence of collective risk perception about an emergency have been proposed and used to perform numerical simulations. However, the complexity of such models hinders rigorous analytical studies, calling for the development of new analytically-treatable mathematical models for collective risk perception.

Here, we fill in this gap by proposing a novel analytically-treatable model for collective risk perception, which is grounded in the theory of opinion dynamics [13, 31]. In particular, inspired by [29] and building on the social-psychology literature [23, 24, 25, 26, 27], we consider a network of interacting individuals who are forming their opinion on the risk of a given hazard. Specifically, individuals are exposed to two different sources of information: an evaluation of the risk which is officially broadcasted by the institutions and a local risk perception shared by peers on a dynamical social influence network [24, 25]. Consistent with the social-psychology literature on risk interpretation [26], individuals recursively revise their risk perception by processing these different information sources through their own risk sensitivity [27].

Our main contribution is threefold. First, we propose the mathematical model for collective risk perception and we demonstrate that it can be cast as a generalized version of the well-known Friedkin–Johnsen opinion dynamic model [11] on a time-varying network. However, the complexity of the network formation process hinders its direct analysis using standard opinion dynamics techniques [31]. Second, we prove that, while individuals' opinions in general tend to keep oscillating, their temporal average converge under some mild assumptions on the social network structure. Third, under some assumptions, we analytically characterize the steady-state temporal average opinion. Such theoretical result allows us to shed light on how individuals' risk sensitivity shapes the collective risk perception, showing that a small amount of individuals with high risk sensitivity could lead to overreactions and panic. Although abstract, these results can provide useful insights on risk communication and perception, thus contributing to the important and timely issue of disaster risk reduction [32].

The rest of the paper is organized as follows. In Section 2, we present the model. In Section 3, we prove some general properties of the model, including convergence. In Section 4, we characterize the steady-state temporal average opinions. Section 5 concludes the paper and outlines future research.

1.1. Notation

We gather here the notation used in the paper. We denote the set of nonnegative and strictly positive integer numbers by $\mathbb{N}$ and $\mathbb{N}_+$, respectively. A vector $\boldsymbol{x}$ is denoted with bold lower-case font, with ith entry x_i and $\boldsymbol{x}^\top$ denoting its transpose; a matrix $\boldsymbol{A}$ is denoted with bold upper-case font, with jth entry of the ith row A_{ij}. Given a stochastic event E, we denote its probability by $\mathbb{P}[E]$; given a random variable x, we denote its expectation by $\mathbb{E}[x]$.

2. Model

We consider a population of $n \in \mathbb{N}_+$ individuals, denoted by the set $\mathcal{V} = \{1, \dots, n\}$. Individuals are connected through a time-invariant network $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ that captures social influence between the individuals of the population. In particular, the directed edge $(i, j) \in \mathcal{E}$ if and only if i can be influenced by the opinion of j. For any individual $i \in \mathcal{V}$, we denote by $\mathcal{N}_i := \{j : (i, j) \in \mathcal{E}\}$ the set of (out)-neighbors of i, that is, the set of individuals who can directly influence the opinion of i, and by $d_i := |\mathcal{N}_i|$ the (out)-degree of the individual. Here, we assume that the set of neighbors of each individual is fixed. However, as we shall see in the following, it is not said that each individual interacts with all their neighbors at every time step. This will eventually induce a time-varying structure that describes the temporal evolution of social interactions.

Each individual $i \in \mathcal{V}$ is characterized by an opinion $x_i(t) \in [0, 1]$, which represents individual i's *risk perception* on the emergency at discrete time $t \in \mathbb{N}$, with initial opinion $x_i(0) \in [0, 1]$. Opinions are gathered into a vector $\boldsymbol{x}(t) = [x_1(t), \dots, x_n(t)]^\top$, which represents the state of the network at time t.

Each individual $i \in \mathcal{V}$ is characterized by three parameters:

1. *risk sensitivity* $\rho_i \in \{-1, 0, +1\}$,
2. *trust in institutions* $\tau_i \in [0, 1]$, and
3. *trust in peers* $\mu_i \in [0, 1]$,

with $\tau_i + \mu_i \leq 1$. Note that $1 - \tau_i - \mu_i$ can be interpreted as a measure of the persistence of the individual.

Opinions of the individuals evolve in time in accordance with observations from the social-psychology literature on risk interpretation, which provides evidence of the fact that individuals do not directly take the information broadcasted by the institution, but they process it using information from peers and their own risk sensitivity [23, 24, 25, 26, 27]. Grounded on such literature, we define a two-step update mechanism. First, the individuals gather information from the institutional source and from peers, and process such information, according to a weighted average dynamics, regulated by the parameters representing the individuals' trust in institutions and in peers, respectively. Second, the individuals revise their opinion by using such information gathered, and further processing it, based on their own risk sensitivity. Such a two-step mechanism is illustrated in Fig. 1, and all the model variables and parameters are summarized in Table 1.

In the following, we formally define these dynamics and explicitly derive the set of equations that governs the model. For simplicity, we will denote the intermediate step of the opinion in the revision from $x_i(t)$ to $x_i(t+1)$ after the first step as $z_i(t)$.

2.1. Step I: Information gathering

At each time step $t \in \mathbb{N}_+$, each individual $i \in \mathcal{V}$ receives information from the institutions about the nature of the risk. Specifically, the institution broadcasts a (constant) signal $\iota \in [0, 1]$, which quantifies the *nature of the risk*. Such a signal should be interpreted as a normalized quantity, so that $\iota = 0$ means no risk and $\iota = 1$ corresponds to maximal risk.

At the same time, individuals share information with their peers, consistent with the evidence coming from the social-psychological literature on risk management [24, 25]. Specifically, at each time-step $t \in \mathbb{N}_+$, each individual $i \in \mathcal{V}$ interacts with a peer j, selected uniformly at random among their neighbors $\mathcal{N}_i$, independently of the past. The neighbor j decides to share with i their opinion

Table 1: Model variables and parameters.

symbol	meaning
n	number of individuals
$\mathcal{N}_i$	(out-)neighbors of individual i
d_i	number of (out-)neighbors of individual i
ρ_i	risk sensitivity of individual i
τ_i	trust in institutions of individual i
μ_i	trust in peers of individual i
ι	information broadcast by the institutions
$f_i(\cdot)$	sharing probability function of individual i
$x_i(t)$	opinion of individual i at time t
$y_i(t)$	temporal average opinion of individual i up to time t

with state-dependent probability equal to $f_j(x_j(t))$, where $f_j : [0,1] \to [0,1]$ is a function termed *sharing probability function* that maps the opinion of individual j to their tendency to communicate it. This function captures the fact, well-known in the social-psychology literature, that people tend to transmit information that is in accordance with their risk perception [27].

To represent such an information sharing process, we use a time-varying network $\mathcal{G}_t = (\mathcal{V}, \mathcal{E}_t)$. If at time $t \in \mathbb{N}$ individual i interacts with j, and j decides to share their opinion, then we add the link (i,j) to the edge set $\mathcal{E}_t$. We define the adjacency matrix of the communication network as a $n \times n$ time-varying matrix $\boldsymbol{A}(t)$, with off-diagonal entries $A_{ij}(t) = 1$ if $(i,j) \in \mathcal{E}_t$ and $A_{ij}(t) = 0$ otherwise. The diagonal entries are defined as $A_{ii}(t) = 1 - \sum_{j \in \mathcal{V} \setminus \{i\}} A_{ij}(t)$. Note that, at each time, exactly one entry per each row of $\boldsymbol{A}(t)$ is nonzero: this is the jth entry if i receives information from j, or diagonal entry if i does not receive information from the network at time t.

Then, individual i revises their opinion by averaging their current opinion $x_i(t)$ with the information they receive from the different sources of information (i.e., ι and, possibly, $x_j(t)$), using the weights given by the trust in institutions τ_i and in peers μ_i, respectively, obtaining the following convex combination:

$$z_i(t) = (1-\mu_i-\tau_i)x_i(t) + \mu_i \sum_{j\in\mathcal{V}} A_{ij}(t)x_j(t) + \tau_i\iota, \tag{1}$$

which reduces to $z_i(t) = (1-\tau_i)x_i(t)+\tau_i\iota$, when no information is received from the network, i.e., if $A_{ii}(t)=1$.

2.2. Step II: Opinion processing through risk sensitivity

After having revised their opinion on the basis of the information gathered from external sources (institutions and peers), individuals further process their opinion through their own risk sensitivity. Specifically, following [29], we assume that each individual $i \in \mathcal{V}$ updates their opinion as

$$x_i(t+1) = \begin{cases} \frac{1}{2}(1+z_i(t)) & \text{if } \rho_i = +1, \\ z_i(t) & \text{if } \rho_i = 0, \\ \frac{1}{2}z_i(t) & \text{if } \rho_i = -1, \end{cases} \tag{2}$$

which can be conveniently re-written as a linear combination:

$$x_i(t+1) = \Big(1-\frac{1}{2}|\rho_i|\Big)z_i(t) + \frac{1}{4}|\rho_i|(1+\rho_i). \tag{3}$$

We conclude this section by observing that the entire two-step opinion update mechanism can be cast in a compact form as the linear averaging dynamics on a (weighted) time-varying network, which is summarized in the following statement.

Proposition 1. *For each and every $i \in \mathcal{V}$, the opinion update mechanism reads*

$$x_i(t+1) = (1-\lambda_i)\sum_{j\in\mathcal{V}} \tilde{A}_{ij}(t)x_j(t) + \lambda_i u_i, \tag{4}$$

where

$$\tilde{A}_{ij}(t) = \begin{cases} \dfrac{\mu_i}{1-\tau_i}A_{ij}(t) & \textit{if } j \neq i, \\ 1 - \dfrac{\mu_i}{1-\tau_i}\big(1-A_{ii}(t)\big) & \textit{if } j = i, \end{cases} \tag{5a}$$

$$\lambda_i = \frac{1}{2}|\rho_i|(1-\tau_i)+\tau_i, \tag{5b}$$

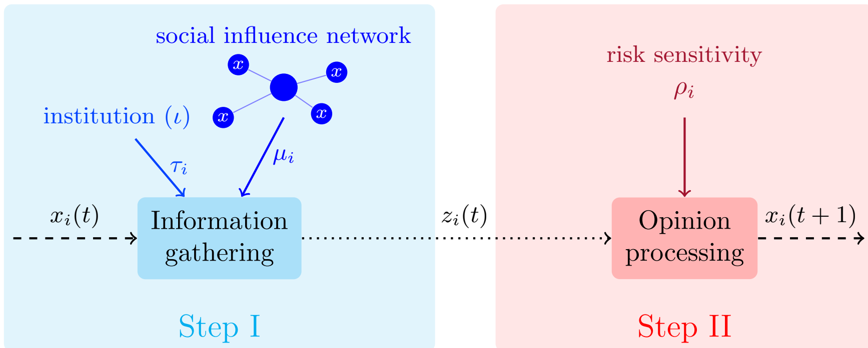


Figure 1: Schematic of the two-step opinion update mechanism.

$$u_i = \frac{\left(1-\frac{1}{2}|\rho_i|\right)\tau_i\iota + \frac{1}{4}|\rho_i|(1+\rho_i)}{\frac{1}{2}|\rho_i|(1-\tau_i)+\tau_i}. \tag{5c}$$

Proof. By substituting Eq. (1) into Eq. (3), we obtain

$$\begin{aligned} x_i(t+1) &= \Big(1-\tfrac{1}{2}|\rho_i|\Big)\Big((1-\mu_i-\tau_i)x_i(t) \\ &\quad +\mu_i \textstyle\sum_{j\in\mathcal{V}} A_{ij}(t)x_j(t)+\tau_i\iota\Big)+\tfrac{1}{4}|\rho_i|(1+\rho_i) \\ &= \Big(1-\tfrac{1}{2}|\rho_i|\Big)\Big(\big(1-\tau_i-\mu_i(1-A_{ii}(t))\big)x_i(t) \\ &\quad +\mu_i \textstyle\sum_{j\in\mathcal{V}\setminus\{i\}} A_{ij}(t)x_j(t)+\tau_i\iota\Big)+\tfrac{1}{4}|\rho_i|(1+\rho_i), \end{aligned} \tag{6}$$

which, after simplification and proper-re-writing of the coefficients, yields Eqs. (4)–(5). □

Remark 1. *From Proposition 1, we observe that Eq. (4) can be interpreted as a Friedkin–Johnsen opinion dynamics model on a time-varying network [11]. However, it is important to notice that the complexity of the network formation process (which is inherently state-dependent) does not allow to directly apply the theoretical findings for Friedkin–Johnsen models, which have been developed in a time-invariant framework [13], and then extended to time-varying (but not state-dependent) scenarios [31]. This makes the study of the model nontrivial.*

3. Convergence Results

In this section, we prove some general properties of the model. Our main contribution is a characterization of the asymptotic behavior of the model. Specifically, we prove that, while individuals' opinion will tend to keep oscillating, their temporal average converges to a steady state. Before obtaining such a result, we start by observing that the model is always well-defined, that is, that the opinions will always remain within their domain.

Lemma 1. *The set $[0,1]^n$ is positively invariant for the model in Eq. (4), that is, if $\boldsymbol{x}(0) \in [0,1]^n$, then $\boldsymbol{x}(t) \in [0,1]^n$, for all $t \in \mathcal{N}$.*

Proof. We proceed by induction. At $t = 0$, $x_i(0) \in [0,1]$ for all $i \in \mathcal{V}$ by assumption. Now, assume that $x_i(t) \in [0,1]$, for all $i \in \mathcal{V}$. Then, from Eq. (5a), we observe that all the entries of $\tilde{\boldsymbol{A}}$ are nonnegative and each row sums to 1. Hence, Eq. (4) states that $x_i(t+1)$ is a convex combination of the states $x_j(t)$, and u_i. Hence $x_i(t+1) \geq \min\{\min_{j\in\mathcal{V}} x_j(t), u_i\} \geq 0$, being $u_i \geq \frac{\tau_i}{\tau_i+1}\iota \geq 0$; and $x_i(t+1) \leq \max\{\max_{j\in\mathcal{V}} x_j(t), u_i\} \leq 1$, being $u_i \leq 1 - \frac{\tau_i}{\tau_i+1}(1-\iota) \leq 1$. □

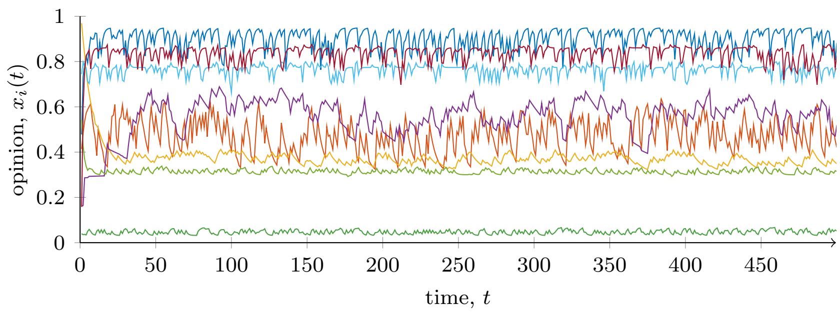
In general, the opinion of each node, $x_i(t)$, may not necessarily converge to a steady state value, but it can oscillate, due to the stochastic nature of the process that regulates the opinion exchange mechanism. See, e.g., the simulations in Fig. 2a. However, we can define the temporal average opinion of agent $i \in \mathcal{V}$ as

$$y_i(t) := \frac{1}{t+1}\sum\nolimits_{s=0}^{t} x_i(s). \tag{7}$$

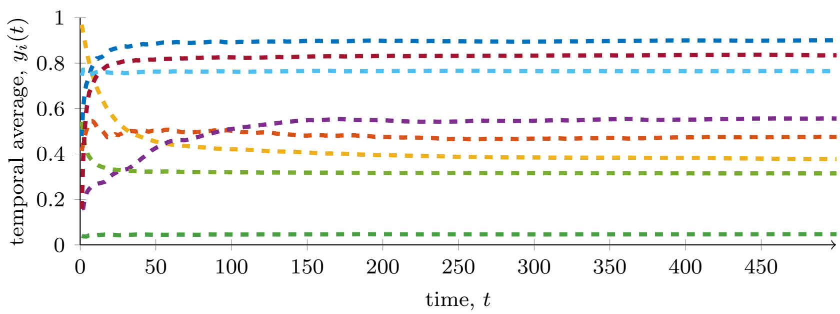
From Fig. 2b, one can observe that the temporal average opinion vector $\boldsymbol{y}(t) = [y_1(t),\dots,y_n(t)]^\top$ seem to converge. This phenomenon resembles the emergent behavior of gossip consensus dynamics with stubborn agents [33, 34]. However, in our model, oscillations are due to heterogeneity in how individuals process information, rather than due to the presence of stubborn individuals. In the rest of this section, we will prove a convergence result to provide analytical support to such claim. We start by showing that the dynamics is ergodic.

Proposition 2. *The process $\boldsymbol{x}(t)$ with update mechanism in Eq. (4) is ergodic.*

Proof. The proof follows from the compact formulation of the ORE model in Eq. (4), which satisfies the assumptions in Theorem 1 in [35]. In fact, we observe that, at each time step $t \in \mathbb{N}_+$, the network $\mathcal{G}_t$ is generated independent of



(a) Opinion



(b) Temporal average opinion

Figure 2: Numerical simulation depicting the temporal evolution of (a) opinions and (b) temporal average opinions for $n = 8$ individuals on a complete backbone network. Parameters τ_i and μ_i are selected uniformly at random in $[0, 1/2]$, ρ_i in $\{-1, 0, +1\}$, and initial condition $x_i(0)$ in $[0, 1]$, for each $i \in \mathcal{V}$ independently of the others.

previous time steps, implying that $(\boldsymbol{A(t)})_{t\in\mathbb{N}_+}$ and, ultimately, the $(\tilde{\boldsymbol{A}}\boldsymbol{(t)})_{t\in\mathbb{N}_+}$, are sequences of independent matrices. □

Corollary 1. *Since the process $\boldsymbol{x}(t)$ is ergodic, it holds that if the mean dynamics $\mathbb{E}[\boldsymbol{x}(t)]$ converges to a steady state $\bar{\boldsymbol{x}}$, then the temporal average opinion vector converges to the steady state of the mean dynamics, i.e., $\lim_{t\to\infty} \boldsymbol{y}(t) = \bar{\boldsymbol{x}}$.*

Based on Corollary 1, we study the mean dynamics, i.e., the evolution of $\mathbb{E}[\boldsymbol{x}(t)]$, in order to draw conclusions on the temporal average opinion. We start by explicitly deriving the update rule for the mean opinion dynamics.

Proposition 3. *For each and every $i \in \mathcal{V}$, the expected opinion evolves as*

$$\mathbb{E}[x_i(t+1)] = \big(1-\lambda_i\big)\sum_{j\in\mathcal{V}} W_{ij}(\boldsymbol{x}(t))x_j(t) + \lambda_i u_i, \tag{8}$$

with

$$W_{ij}(\boldsymbol{x}(t)) = \begin{cases} \frac{\mu_i}{d_i(1-\tau_i)} f_j(x_j(t)) & \text{if } j \in \mathcal{N}_i, \\ 1 - \frac{\mu_i}{d_i(1-\tau_i)} \sum_{j\in\mathcal{N}_i} f_j(x_j(t)) & \text{if } j = i, \\ 0 & \text{otherwise}, \end{cases} \tag{9}$$

and λ_i and u_i from Eq. (5b) and Eq. (5c), respectively.

Proof. First, we compute the probability that i receives information from $j \in \mathcal{N}_i$ at time t, as

$$\mathbb{P}[A_{ij}(t)=1] = \mathbb{P}[i \text{ contacts } j]\mathbb{P}[j \text{ shares}] = \frac{f_j(x_j(t))}{d_i}. \tag{10}$$

Using Eq. (10), we compute the probability that i receives information not only from the institution, but also from the network, at time t, as

$$\mathbb{P}\left[A_{ii}(t)=0\right] = \sum_{j\in\mathcal{N}_i} \mathbb{P}[A_{ij}(t)=1] = \frac{1}{d_i}\sum_{j\in\mathcal{N}_i} f_j(x_j(t)). \tag{11}$$

Hence, using Eq. (1), Eq. (10), and Eq. (11), we compute the expected value of the opinion of individual i after the information exchange step, by conditioning on the values of the ith row of matrix $\boldsymbol{A}(t)$, as follows:

$$\begin{aligned}
\mathbb{E}[z_i(t)] &= \mathbb{P}\left[A_{ii}(t)=1\right]\big((1-\tau_i)x_i(t)+\tau_i\iota\big) \\
&\quad+\sum_{j\in\mathcal{N}_i}\mathbb{P}\left[A_{ij}(t)=1\right]\big((1-\mu_i-\tau_i)x_i(t)+\mu_i x_j(t)+\tau_i\iota\big) \\
&= \Big(1-\frac{1}{d_i}\sum_{j\in\mathcal{N}_i}f_j(x_j(t))\Big)\big((1-\tau_i)x_i(t)+\tau_i\iota\big) \\
&= \sum_{j\in\mathcal{N}_i}\frac{1}{d_i}f_j(x_j(t))\big((1-\mu_i-\tau_i)x_i(t)+\mu_i x_j(t)+\tau_i\iota\big) \\
&= \Big(1-\frac{\mu_i}{d_i}\sum_{j\in\mathcal{N}_i}f_j(x_j(t))-\tau_i\Big)x_i(t)+\frac{\mu_i}{d_i}\sum_{j\in\mathcal{N}_i}f_j(x_j(t))x_j(t)+\tau_i\iota.
\end{aligned} \tag{12}$$

Finally, we combine Eq. (12) and Eq. (3), obtaining an equation that determines the expected value of the opinion at time $t+1$, as a function of the current opinion of the individual, of their neighbors, and the model parameters:

$$\begin{aligned}
\mathbb{E}[x_i(t+1)] = &\Big(1-\frac{|\rho_i|}{2}\Big)\Big(1-\frac{\mu_i}{d_i}\sum_{j\in\mathcal{N}_i}f_j(x_j(t))-\tau_i\Big)x_i(t) \\
&+\Big(1-\frac{1}{2}|\rho_i|\Big)\frac{\mu_i}{d_i}\sum_{j\in\mathcal{N}_i}f_j(x_j(t))x_j(t) \\
&+\Big(1-\frac{1}{2}|\rho_i|\Big)\tau_i\iota+\frac{1}{4}|\rho_i|(1+\rho_i),
\end{aligned} \tag{13}$$

which can be conveniently re-written as Eq. (8), yielding the claim. □

Finally, we are ready to prove that, under some reasonable assumptions on the network of interactions and on the function f, the expected opinions and, ultimately, the temporal average opinions converge to a steady state.

Assumption 1. *Assume that the network $\mathcal{G}=(\mathcal{V},\mathcal{E})$ is strongly connected, $f_i(x)>0$ for all $x>0$ and $i\in\mathcal{V}$, $\iota>0$, and $\tau_i>0$, for all $i\in\mathcal{V}$.*

We would like to comment that Assumption 1 is not very restrictive and is consistent with real-world scenarios. In fact, imposing $f_i(x)>0$ for all $x>0$ implies that individuals have a nonzero probability of discussing about the possible disaster if they perceive some risk; $\iota>0$ means that there exists a risk (which is the key motivation of the model); and $\tau_i>0$ means that the signal broadcasted by the institution has at least some impact (possibly very marginal) on individuals' risk perception, consistent with the social-psychology literature on the topic [26, 27]. Finally, in many scenarios, social interactions are undirected (i.e., for any pair of individuals $i, j \in \mathcal{V}$, if information can be shared from i to j, then it can also be shared from j to i. In this case, the strong connectivity assumption is either always satisfied, or the population can be partitioned into isolated connected communities, where each community can be treated on its own as a strongly connected population.

Theorem 1. *Under Assumption 1, the temporal average opinion vector $\boldsymbol{y}(t)$ defined in Eq. (7) under the opinion update in Eq. (4) converges almost surely to a steady state, i.e., $\lim_{t\to\infty} \boldsymbol{y}(t) = \bar{\boldsymbol{x}} \in [0,1]^n$.*

Proof. First of all, we observe that if $\tau_i = 1$ or $\mu_i = 0$, then an individual's opinion is not influenced by others, so $x_i(t) = x_i(0)$ for all $t \geq 0$, yielding the claim for individual i. Let now focus on the individuals with $\mu_i \neq 0$ and $\tau_i \neq 1$.

We start proving that, under Assumption 1, the mean dynamics of the ORE model $\mathbb{E}[x_i(t)]$ from Proposition 3 converges almost surely to a steady state, that is, $\lim_{t\to\infty} \mathbb{E}[x_i(t)] = \bar{x}_i \in [0,1]$. To obtain such convergence result, we consider the mean dynamics in Eq. (8), with the expression of $W_{ij}(x(t))$ reported in Eq. (9). First of all, we observe that, the update rule in Eq. (4) establishes a lower-bound on $x_i(t)$. In fact, since from Lemma 1, $x_i(t) \geq 0$, then we can further refine the bound by establishing that $x_i(t) \geq \lambda_i u_i \geq \frac{1}{2}\tau_i \iota$. We define the uniform bound

$$\alpha := \min_{i\in\mathcal{V}} \frac{\mu_i}{d_i(1-\tau_i)} f_i\Big(\frac{1}{2}\tau_i \iota\Big). \tag{14}$$

Under Assumption 1, we observe that $\frac{1}{2}\tau_i\iota > 0$, which implies that also $f_i(\frac{1}{2}\tau_i\iota) > 0$. Hence, $\alpha > 0$. From Eq. (9), we observe that we can derive the following time-invariant bound on the weight for each link: $W_{ij}(\boldsymbol{x}(t)) \geq \alpha$, for all $i \in \mathcal{V}$, $j \in \mathcal{N}_i$. Thus, the time-varying graph with weights W is strongly connected, being $\mathcal{G}$ strongly connected. Hence, the mean dynamics in Eq. (8) is a Friedkin-Johnsen model on a strongly connected time-varying network, so $\mathbb{E}[\boldsymbol{x}(t)]$ converges [11, 13].

Finally, the fact that the mean dynamics $\mathbb{E}[\boldsymbol{x}(t)]$ converges almost surely to a steady state $\bar{\boldsymbol{x}}$ (proved in the above), combined with the fact that the process is ergodic (Proposition 2) implies that $\lim_{t\to\infty} \boldsymbol{y}(t) = \lim_{t\to\infty} \mathbb{E}[\boldsymbol{x}(t)] = \bar{\boldsymbol{x}}$ (Corollary 1), which yield the claim. $\square$

4. Steady state characterization

In the previous section, we proved that, under some mild assumptions, the temporal average opinion of the individuals converges to a steady-state value. In general, the characterization of such a steady state is nontrivial due to the complexity of Eq. (8), which yields a system of n coupled nonlinear recursive equations — one for each individual, where the inherent nonlinearity comes from the fact that the term $W_{ij}(\boldsymbol{x})$ (which couples the equations) is state-dependent. In this section, we consider a specific implementation of the model, for which we can analytically compute such a quantity, with a specific focus on the role of risk sensitivity. To perform such analysis, we make the following assumptions.

Assumption 2. *Let $\mathcal{G}$ be a complete network, that is, $\mathcal{N}_i = \mathcal{V}$, for all $i \in \mathcal{V}$. Moreover, let us assume that the parameters are uniform across the individuals, that is, $\tau_i = \tau \in (0, 1]$ and $\mu_i = \mu$, and that the functions f_i are uniform across the individuals and coincide with the identity function, that is, $f_i(x_i) = x_i$. We also assume $\iota > 0$.*

In such a setting, we introduce the following notation. Let $\eta_+ := \frac{1}{n}|\{i : \rho_i = +1\}|$, $\eta_- := \frac{1}{n}|\{i : \rho_i = -1\}|$, and $\eta_0 := \frac{1}{n}|\{i : \rho_i = 0\}|$ be the fraction of population with high, low, and neutral risk sensitivity, respectively. It clearly holds $\eta_+ + \eta_- + \eta_0 = 1$.

Theorem 2. *Under Assumption 2, the asymptotic value of the temporal average opinion of individual $i \in \mathcal{V}$ under the opinion update in Eq. (4) satisfies*

$$\lim_{t\to\infty} y_i(t) = \begin{cases} \bar{y}_+ & \text{if } \rho_i = +1\,, \\ \bar{y}_0 & \text{if } \rho_i = 0\,, \\ \bar{y}_- & \text{if } \rho_i = -1\,, \end{cases} \tag{15}$$

where $(\bar{y}_+, \bar{y}_0, \bar{y}_-) \in [0,1]^3$ *is solution of*

$$\begin{aligned}\bar{y}_+ =& \frac{1}{2}\Big(1-\mu\big(\eta_+\bar{y}_+ + \eta_0\bar{y}_0 + \eta_-\bar{y}_-\big) - \tau\Big)\bar{y}_+ \\ &+ \frac{1}{2}\mu\big(\eta_+\bar{y}_+^2 + \eta_0\bar{y}_0^2 + \eta_-\bar{y}_-^2\big) + \frac{1}{2}\tau\iota + \frac{1}{2},\end{aligned} \tag{16a}$$

$$\begin{aligned}\bar{y}_0 =& \Big(1-\mu\big(\eta_+\bar{y}_+ + \eta_0\bar{y}_0 + \eta_-\bar{y}_-\big) - \tau\Big)\bar{y}_0 \\ &+ \mu\big(\eta_+\bar{y}_+^2 + \eta_0\bar{y}_0^2 + \eta_-\bar{y}_-^2\big) + \tau\iota,\end{aligned} \tag{16b}$$

$$\begin{aligned}\bar{y}_- =& \frac{1}{2}\Big(1-\mu\big(\eta_+\bar{y}_+ + \eta_0\bar{y}_0 + \eta_-\bar{y}_-\big) - \tau\Big)\bar{y}_- \\ &+ \frac{1}{2}\mu\big(\eta_+\bar{y}_+^2 + \eta_0\bar{y}_0^2 + \eta_-\bar{y}_-^2\big) + \frac{1}{2}\tau\iota.\end{aligned} \tag{16c}$$

Proof. First, we observe that, according to Theorem 1, the temporal averages of individuals' opinion converge to a steady state $\bar{\boldsymbol{x}}$, which is the steady state of the mean dynamics. Then, we observe that ergodicity of the process guarantees that the steady states of the mean dynamics do not depend on the initial condition. Based on this observation, a symmetry argument can be used to guarantee that $\bar{x}_i = \bar{x}_j$ if $\rho_i = \rho_j$, being all the other parameters equal and the network fully connected, that is, Eq. (15) holds. At this stage, we observe that, at the equilibrium, under Assumption 2, the following two equalities hold true:

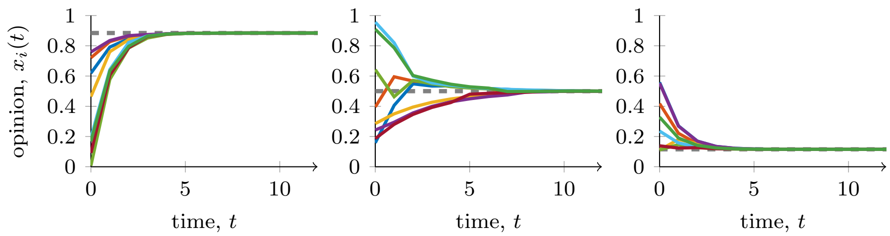
$$\frac{1}{d_i}\sum_{j\in\mathcal{N}_i} f(\bar{x}_j) = \frac{1}{n}\sum_{j\in\mathcal{V}} \bar{x}_j = \eta_+\bar{y}_+ + \eta_0\bar{y}_0 + \eta_-\bar{y}_- \tag{17}$$

and

$$\begin{aligned}\frac{1}{d_i}\sum_{j\in\mathcal{V}} f(\bar{x}_j)\bar{x}_j &= \frac{1}{n}\sum_{j\in\mathcal{N}_i}\bar{x}_j^2 \\ &= \frac{1}{n}\sum_{j:\rho_i=+1}\bar{y}_+^2 + \frac{1}{n}\sum_{j:\rho_i=0}\bar{y}_0^2 + \frac{1}{n}\sum_{j:\rho_i=-1}\bar{y}_-^2 \\ &= \eta_+\bar{y}_+^2 + \eta_0\bar{y}_0^2 + \eta_-\bar{y}_+^2.\end{aligned} \tag{18}$$

Finally, we write the equilibrium condition for the mean dynamics, starting from Eq. (13), and we substitute Eq. (17) and Eq. (18) into such expression, obtaining Eq. (16). □

Theorem 2 provides a powerful tool to characterize the steady-state temporal average opinion of the network. In general, given the parameter of the



(a) $\eta_+ = 1$ (b) $\eta_0 = 1$ (c) $\eta_- = 1$

Figure 3: Numerical simulation of the ORE model with $n = 8$ individuals on a complete backbone network, with $\iota = 0.5$, $\tau_i = \mu_i = 0.3$, for all $i \in \mathcal{V}$, initial condition $x_i(0)$ selected uniformly at random in $[0, 1]$, for each $i \in \mathcal{V}$ independently of the others, and (a) $\rho_i = +1$, (b) $\rho_i = 0$, (c) $\rho_i = -1$, for all $i \in \mathcal{V}$. The gray dashed lines are the predicted consensus state from Proposition 4.

model, the solution of the three coupled quadratic equations in Eq. (16) can be easily computed using a numerical solver. On the other hand, determining the analytical solution may be, in general, challenging, due to the complexity of the equations. In the rest of this section, we will use Theorem 2 to analytically characterize the steady-state temporal average opinion for some specific scenarios where analytical treatment is possible. Then, we will complement the study by means of numerical simulations.

4.1. Homogeneous population

First, we consider the scenarios of a homogeneous population, where all the individuals have positive, neutral, or negative risk sensitivity, i.e., setting $\eta_+ = 1$, $\eta_0 = 1$, or $\eta_- = 1$, respectively. In these scenarios, we are able to prove almost sure convergence of the opinion of each individual to a consensus, and characterize its expected value, as detailed in the following. Our theoretical results are confirmed by simulation results in Fig. 3.

Proposition 4. *If Assumption 2 holds and the entire population has the same risk sensitivity, then the ORE model in Eq. (4) almost surely converges to a* *consensus, that is,* $\lim_{t\to\infty} x_i(t) = x^*$ *with:*

$$\mathbb{E}[x^*] = \begin{cases} \iota + \dfrac{1-\iota}{1+\tau} & \text{if } \eta_+ = 1, \\ \iota & \text{if } \eta_0 = 1, \\ \iota - \dfrac{\iota}{1+\tau} & \text{if } \eta_- = 1. \end{cases} \tag{19}$$

Proof. First of all, we prove almost sure convergence using Theorem 3.3 from [36]. The proving argument involves the definition of an augmented network with an additional node (which we can label as 0) with $\mu_0 = \tau_0 = 0$, and initial opinion equal to

$$x_0(0) = \frac{\left(1 - \frac{1}{2}|r|\right)\tau\iota + \frac{1}{4}|r|(1+r)}{\frac{1}{2}|r|(1-\tau) + \tau} \tag{20}$$

with $r = 1$ if $\eta_+ = 1$, $r = 0$ if $\eta_0 = 1$, and $r = -1$ if $\eta_- = 1$. Note that, being $\mu_0 = \tau_0 = 0$, then it holds true that $x_0(t) = x_0(0)$, for all $t \geq 0$. The entire model can be reformulated as a De Groot model on a time-varying (state-dependent) network [13, 31] with node 0 as a globally reachable node at every time t. Hence, Theorem 3.3 from [36] guarantees almost sure convergence to a consensus, which yields the first part of the claim.

Since $x_0(0) = x_0(t)$, for all $t \geq 0$, necessarily the value of the expected consensus state coincides with the state of the stubborn node of the augmented network, $x_0(t)$. Finally, by substituting $r \in \{+1, 0, -1\}$ into Eq. (20), we obtain Eq. (19). □

Remark 2. *From Proposition 4, we observe that, for uniform populations, the system converges to a consensus, whose expected value can be computed. In the absence of any risk sensitivity biases, the consensus coincides with the actual information sent out by the institution* $x^* = \iota$*. Positive or negative risk sensitivity would instead lead to an overestimation or a underestimation of the risk, respectively, as can be observed in Fig. 3.*

Remark 3. *Note that the trust in peers (i.e., parameter* μ*) does not play a role in determining the asymptotic consensus state, but it may affect the speed of convergence. As a consequence, one could relax the assumption that such a quantity is homogeneous across the population in Assumption 2.*

4.2. Role of heterogeneous risk sensitivity

Here, we want to investigate the role of individuals with high risk sensitivity in shaping the emergent behavior of the population. We start by considering a polarized scenario, in which half of the population has low risk sensitivity and half of the population has high risk sensitivity. In this scenario, we can analytically prove that the presence of individuals with high risk sensitivity would lead to an overestimation of the risk. Then, numerical solution of the equations in Eq. (16) is used to provide further evidence to our claim.

Proposition 5. *If Assumption 2 holds, $\iota = 1/2$, $\eta_+ = \eta_- = 1/2$, and $\mu = 1-\tau$ then the temporal average opinion of each individual in the ORE model in Eq. (4) almost surely converges to a steady state with mean opinion $< \bar{y}_i >:= \frac{1}{n}\sum_{i\in\mathcal{V}} \bar{y}_i \geq 1/2$, with strict inequality holding if $\tau < 1$.*

Proof. In this scenario, the equilibrium equations in Eq. (16) reduce to the following coupled quadratic equations:

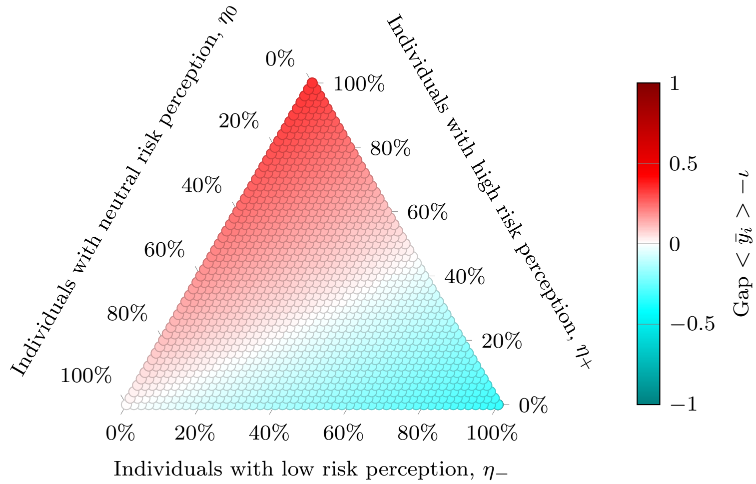
$$\bar{y}_+ = \frac{1}{2}(1-\tau)\Big(\bar{y}_+ - \frac{1}{2}\bar{y}_- y_+ + \frac{1}{2}\bar{y}_-^2\Big) + \frac{1}{2}\tau + \frac{1}{2}, \tag{21a}$$
$$\bar{y}_- = \frac{1}{2}(1-\tau)\Big(\bar{y}_- - \frac{1}{2}\bar{y}_+ y_- + \frac{1}{2}\bar{y}_+^2\Big) + \frac{1}{2}\tau, \tag{21b}$$

where Eq. (16b) is omitted, being $\eta_0 = 0$. Let us define $\xi = \frac{\bar{y}_+ + \bar{y}_-}{2}$ and $\zeta = \frac{\bar{y}_+ - \bar{y}_-}{2}$ as the average and half-difference between the two mean opinions. We observe that the steady state with mean opinion $< \bar{y}_i >:= \frac{1}{n}\sum_{i\in\mathcal{V}} \bar{y}_i = \xi$. Hence, the problem reduces to prove that $\xi > 1/2$. By computing the sum and the difference between the two equations in Eq. (21) and recalling the definition of ξ and ζ, we derive

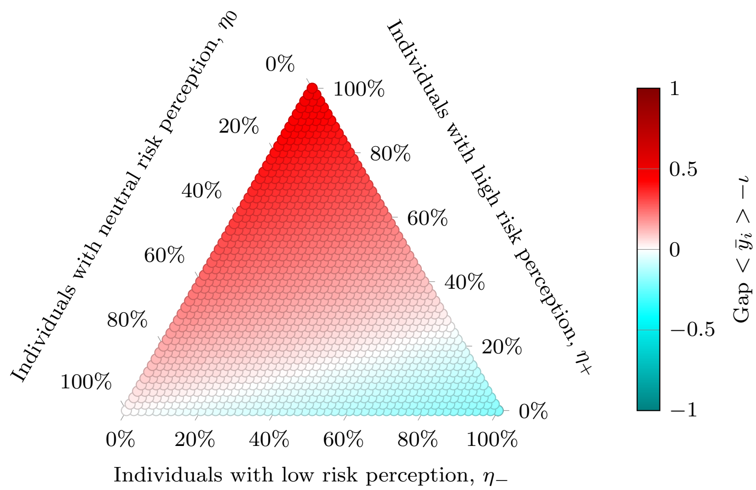
$$\xi = \frac{1}{2}(1-\tau)(1-\xi)\xi + \frac{1}{2}(1-\tau)(\xi^2+\zeta^2) + \frac{1}{4}\tau + \frac{1}{4}, \tag{22a}$$
$$\zeta = \frac{1}{2}(1-\tau)(1-\xi)\zeta + \frac{1}{4}. \tag{22b}$$

From Eq. (22b), we explicitly compute

$$\xi = \frac{1-2\zeta(1+\tau)}{2\zeta(1-\tau)}. \tag{23}$$



(a) $\iota = 0.5$



(b) $\iota = 0.3$

Figure 4: Average final opinion for different fractions of individuals with high, neutral, and low risk sensitivity, computed by solving numerically Eq. (16). In (a), $\iota = 0.5$, in (b), $\iota = 0.3$. Common parameters are $n = 100$, and $\tau = \mu = 0.5$. The opinion of each individual is sampled uniformly at random in $[0, 1]$, independently of the others.

Our objective is to verify that $\xi > 1/2$. Using Eq. (23), a necessary and sufficient condition for having $\xi > 1/2$ is that

$$\xi > \frac{1}{2} \iff 1 - 2\zeta(1+\tau) > \zeta(1-\tau) \iff \zeta < \frac{1}{3\tau+1}. \tag{24}$$

To check this condition, we need to compute the solution of Eq. (22) for the variable ζ. To this aim, we substitute Eq. (23) into Eq. (22a) and, after all the algebraic simplifications, we obtain the following third-order equation:

$$\phi(\zeta) = 2(1-\tau)^3\zeta^3 + (\tau^2 + 4\tau + 3)\zeta - (1+\tau) = 0. \tag{25}$$

It is straightforward to check that the function $\phi(\zeta)$ is monotonically increasing in ζ for any $\tau \in [0, 1]$. In fact, it holds

$$\phi'(\zeta) = 6(1-\tau)^3\zeta^2 + \tau^2 + 4\tau + 3 > 0 \tag{26}$$

that $\phi(0) < 0$ and $\phi(1) > 0$. Therefore, Eq. (25) has only one real solution, which lies in $[0, 1]$. However, despite this solution can be analytically computed (being the unique real solution of a third-order equation), its complexity hinders the possibility to readily check whether it is less than $\frac{1}{3\tau+1}$. However, we can compute

$$\begin{aligned} \phi\Big(\frac{1}{3\tau+1}\Big) &= \frac{\Big(2(1-\tau)^3 + (\tau^2+4\tau+3)(1+3\tau)^2 - (1+3\tau)^3(1+\tau)\Big)}{(3\tau+1)^3} \\ &= \frac{4 + 6\tau + 22\tau^2 - 14\tau^3 - 16\tau^4}{(3\tau+1)^3}, \end{aligned} \tag{27}$$

which is strictly positive for any $\tau < 1$. Therefore, being $\phi(\zeta)$ strictly monotonically increasing, its unique zero must satisfy $\zeta < \frac{1}{3\tau+1}$, implying that $\xi > 1/2$, which yields the claim. □

This theoretical result suggests that the presence of individuals with high risk sensitivity may be critical in determining a collective overreaction to the information broadcast by the institution. Our hypothesis is that even a minority of individuals with high risk sensitivity could be sufficient to steer the mean final opinion towards an overestimation of the risk.

To provide evidence to support such hypothesis, we leverage Theorem 2 by numerically solving Eq. (16) for a wide range of different values of the parameters η_+, η_0, and η_-. Our results, reported in Fig. 4a, show that the region in which the risk is overestimated (red) is larger than the one in which it is underestimated (cyan), suggesting that people with high risk sensitivity play a dominant role in determining the final average opinion of the entire population. For instance, from the plot we observe that if only 10% of the population has high risk sensitivity, then the risk will be overestimated as far as the fraction of population with low risk sensitivity is less than 15%. This phenomenon is even more visible when institution communicates that the risk is small. For instance, in Fig. 4b, we observe that for $\iota = 0.3$, if only 10% of the population has high risk sensitivity, then the risk will be overestimated as far as the people with low risk sensitivity is less than 35%. When at least 27% of the population has high risk sensitivity, then the risk is always overestimated. The direct analytical verification of such hypothesis in more general scenarios requires nontrivial efforts due to the nonlinearity of the equations in Theorem 2 and is thus beyond the scope of this paper and left for future research.

5. Conclusion

In this paper, we proposed a model for collective risk perception grounded on the theory on the mathematical theory of opinion dynamics [11, 13, 31] and on the the social-psychology literature on risk perception [23, 24, 26, 27]. Through the analysis of the model, we proved convergence of the temporal average opinions on the risk of a given event. Then, under some homogeneity assumption, we provided a characterization of the steady-state temporal average opinions which have allowed us to provide analytical insight into the impact that few individuals with high risk sensitivity may have in determining collective overreactions.

The promising preliminary results presented in this paper pave the way for several avenues of future research. First, our theoretical analysis should be extended along several directions, including investigating the speed of convergence of the temporal average opinions and their transient behavior (see, e.g.,[37]), and generalizing our characterization of the steady-state, beyond the limitations of Assumption 2, e.g., toward unveiling the impact of the network structures and heterogeneity across the population on the system's emergent behavior. Second, effort should be placed in extending the model to incorporate further real-world features, such as the presence of media which may bias the information provided by the institution [38], and the possible occurrence or non-occurrence of the event and understand how this impact the collective risk perception. Third, in order to make this model relevant in the real world, validation and parametrization using experimental and survey data on risk perception will be performed as part of our future research.

CRediT authorship contribution statement

Lorenzo Zino: Conceptualization, Methodology, Formal analysis, Investigation, Software, Validation, Visualization, Writing — Original Draft

Francesca Giardini: Conceptualization, Methodology, Writing — Review & Editing, Supervision

Daniene Vilone: Conceptualization, Methodology, Writing — Review & Editing, Supervision

Ming Cao: Conceptualization, Methodology, Resources, Writing — Review & Editing, Supervision

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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