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A versatile offset operator for the discrete observation of coupled vibroacoustic systems / DE ROSA, S., Catapane, G., Casaburo, A., Magliacano, D., Petrone, G., Franco, F.. - (2022), pp. 22-25. (9th International Symposium on Scale Modeling Naples, Italy 02-04 March 2022).

*Availability:*

This version is available at: 11583/2989022 since: 2024-05-27T16:42:35Z

*Publisher:*

9th International Symposium on Scale Modeling

*Published*

DOI:

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# A VERSATILE OFFSET OPERATOR FOR THE DISCRETE OBSERVATION OF COUPLED VIBROACOUSTIC SYSTEMS

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## ABSTRACT

*There is an increasing attention about structural similitudes, since they allow to change the scales (sizes, materials, topologies, etc.) of any engineering system. Often, the associated predictions are still not fully reliable since the scaling laws can be incomplete and thus it is desired to have methods aimed at defining transformation matrices which link the outputs between two sets of points belonging to different linear systems. The recently defined VOODOO method has demonstrated that a transformation matrix can be defined by using the Betti theorem and it can be invoked for both deterministic and stochastic loads. In principle, with this transformation, two completely different systems can be connected, according to few rules. This paper provides the theoretical framework underlying the extension of VOODOO to the structural-acoustic coupling.*

## 1 INTRODUCTION

The investigation of structural similitude has received great attention in the last years, since the possibility of simplifying experimental tests and numerical simulations, with consequent money and time savings, is very appealing in many engineering fields. The main idea underlying all similitude methods is to design a scaled-up or down model of a full-scale structure, the prototype, in order to execute more manageable tests and analyses, then to reconstruct the response of the prototype from that of the model. Similitudes can be defined according to the type of scaled parameters (geometrical, kinematic, dynamic, material, etc.), as well as according to the complete fulfillment of a set of similitude conditions, leading to a complete similitude. On this basis, many similitude methods have been developed [1], such as STAGE (Similitude Theory Applied to Governing Equations) [2], SAMSARA (Similitude and Asymptotic Models for Structural-Acoustic Research and Applications) [3], and sensitivity analysis [4]. However, all these methods do not provide accurate reconstructions of the structural response when the set of similitude conditions is not completely satisfied, generating partial similitudes, or avatars. In order to overcome this drawback, VOODOO (Versatile Offset Operator for the Discrete Observation of Objects) method has been recently proposed [5]. Basically, the method allows to predict the response of the systems, may they be in complete or partial similitude, through the definition of a linear transformation matrix, the VOODOO matrix. The purpose of this work is to provide the theory behind the extension of VOODOO method to the coupling of acoustic and structural systems, both expressed in discrete coordinates.

## 2 THE LINEAR TRANSFORMATION FOR COUPLED SYSTEMS

### 2.1 The basic concept

It is herein assumed that in both the domains, model and prototype, the same number degrees of freedom (DOFs) can be investigated,  $N$ : the output functions are  $\mu(\omega)$  and  $\pi(\omega)$  for the model and prototype, respectively, being  $\omega$  the excitation frequency (Fig.1) [5].

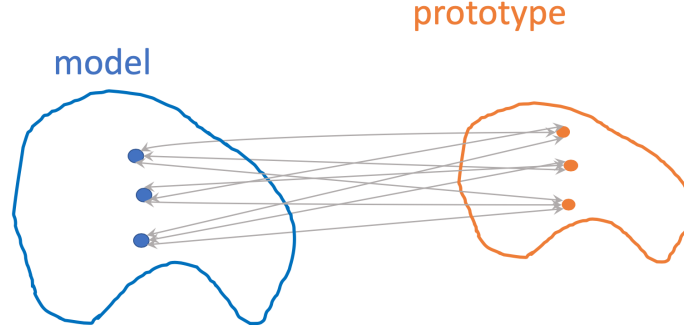


Figure 1: *Linear transformation for a  $N = 3$  case.*

It is thus supposed to have a transformation matrix,  $\mathbf{T}(\omega)$  such that the vectors of the output responses,  $\mathbf{y}$ , can be related:

$$\mathbf{y}_\mu(\omega) = \mathbf{T}(\omega) \mathbf{y}_\pi(\omega) \quad (1)$$

and

$$\mathbf{y}_\pi(\omega) = \mathbf{T}(\omega)^{-1} \mathbf{y}_\mu(\omega) \rightarrow \mathbf{y}_\pi(\omega) = \mathbf{\Theta}(\omega) \mathbf{y}_\mu(\omega) \quad (2)$$

The uppercase bold letters denote matrices and lowercase ones are for vectors. The vectors  $\mu$  and  $\pi$  are  $[N \times 1]$  the matrix  $\mathbf{T}$  is  $[N \times N]$  and  $\mathbf{\Theta}$  is its inverse.

### 2.2 The Coupled response

It is intended to simulate a coupled vibroacoustic system so that:

$$\mathbf{y}_\mu(\omega) = \begin{bmatrix} \mathbf{u}_\mu(\omega) \\ \mathbf{q}_\mu(\omega) \end{bmatrix} ; \mathbf{y}_\pi(\omega) = \begin{bmatrix} \mathbf{u}_\pi(\omega) \\ \mathbf{q}_\pi(\omega) \end{bmatrix} \quad (3)$$

where  $\mathbf{u}$  is the vector of the output displacements and  $\mathbf{q}$  is the vector of the output values for the velocity potential; this latter is related to the output pressures,  $p$ , through [6]

$$q(t) = \int p dt \Rightarrow j\omega q(\omega) = p(\omega) \quad (4)$$

being  $t$  the time variable and  $j$  the imaginary unit.

The coupled systems can be written by using the symmetrical formulation (named  $u - q$ ) since this guarantees that the product of the inputs and outputs are energies. Omitting the frequency dependence for the sake of brevity and noting the pedices  $s$  and  $a$  for the structural and acoustic degrees of freedom, one has respectively:

$$\mu : \begin{bmatrix} \mathbf{m}_{s,\mu} & 0 \\ 0 & \mathbf{m}_{a,\mu} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_\mu \\ \ddot{\mathbf{q}}_\mu \end{bmatrix} + \begin{bmatrix} 0 & \mathbf{A}_\mu \\ \mathbf{A}_\mu^T & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}}_\mu \\ \dot{\mathbf{q}}_\mu \end{bmatrix} + \begin{bmatrix} \mathbf{k}_{s,\mu}^* & 0 \\ 0 & \mathbf{k}_{a,\mu} \end{bmatrix} \begin{bmatrix} \mathbf{u}_\mu \\ \mathbf{q}_\mu \end{bmatrix} = \begin{bmatrix} \mathbf{F}_\mu \\ \mathbf{Q}_\mu \end{bmatrix} \quad (5)$$

$$\pi : \begin{bmatrix} \mathbf{m}_{s,\pi} & 0 \\ 0 & \mathbf{m}_{a,\pi} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{u}}_\pi \\ \ddot{\mathbf{q}}_\pi \end{bmatrix} + \begin{bmatrix} 0 & \mathbf{A}_\pi \\ \mathbf{A}_\pi^T & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{u}}_\pi \\ \dot{\mathbf{q}}_\pi \end{bmatrix} + \begin{bmatrix} \mathbf{k}_{s,\pi}^* & 0 \\ 0 & \mathbf{k}_{a,\pi}^* \end{bmatrix} \begin{bmatrix} \mathbf{u}_\pi \\ \mathbf{q}_\pi \end{bmatrix} = \begin{bmatrix} \mathbf{F}_\pi \\ \mathbf{Q}_\pi \end{bmatrix} \quad (6)$$

The  $\mathbf{A}$  is the equivalent nodal area matrix [7] and  $\mathbf{A}^T$  is its transpose. The *mass*,  $m$ , *stiffness* and *damping* matrices can be assembled by using the finite element approach and the vectors  $\mathbf{F}$  and  $\mathbf{Q}$  are the known input terms.

In this kind of coupled problems, it is fundamental to take note of the units in the S.I. as in the following table:

<i>structural domain</i>	<i>acoustic domain</i>
$\mathbf{m}_s = \text{kg}$	$\mathbf{m}_a = \text{m}^4 \text{kg}^{-1}$
$\mathbf{k}_s^* = \text{kg s}^{-2}$	$\mathbf{k}_a^* = \text{m}^4 \text{kg}^{-1}$
$\mathbf{F} = \text{kg m s}^{-2}$	$\mathbf{Q} = \text{m}^3 \text{s}^{-1}$
$\mathbf{u} = \text{m}$	$\mathbf{q} = \text{m}^{-1} \text{s}^{-1} \text{kg}$

Herein, the damping has been considered inside the definition of the stiffness matrices, so using  $\mathbf{k}_s^* = \mathbf{k}_s(1 + j\eta_s)$  and  $\mathbf{k}_a^* = \mathbf{k}_a(1 + j\eta_a)$  being  $\eta_s$  and  $\eta_a$  the loss factor values for the structural and acoustic degrees of freedom, respectively. As stated before,  $\mathbf{F} \mathbf{u} = \mathbf{Q} \mathbf{q} = \mathbf{J}$ .

### 2.3 The VOODOO transformation

The determination of the  $N^2$  members of the  $\mathbf{T}(\omega)$  is the main point: it can be derived by assuming a known distribution of the outputs for a unit excitation value at each point in both the model and the prototype.

It is useful to rewrite in compact form Eq.(5) and Eq.(6).

$$\mu : \begin{bmatrix} \mathbf{D}_{u,u}^{(\mu)} & \mathbf{D}_{u,q}^{(\mu)} \\ \mathbf{D}_{q,u}^{(\mu)} & \mathbf{D}_{q,q}^{(\mu)} \end{bmatrix} \begin{bmatrix} \mathbf{u}_\mu \\ \mathbf{q}_\mu \end{bmatrix} = \begin{bmatrix} \mathbf{F}_\mu \\ \mathbf{Q}_\mu \end{bmatrix} \quad (7)$$

$$\pi : \begin{bmatrix} \mathbf{D}_{u,u}^{(\pi)} & \mathbf{D}_{u,q}^{(\pi)} \\ \mathbf{D}_{q,u}^{(\pi)} & \mathbf{D}_{q,q}^{(\pi)} \end{bmatrix} \begin{bmatrix} \mathbf{u}_\pi \\ \mathbf{q}_\pi \end{bmatrix} = \begin{bmatrix} \mathbf{F}_\pi \\ \mathbf{Q}_\pi \end{bmatrix} \quad (8)$$

It can be assumed that the total number of degrees of freedom  $N$ , can be formed by  $N_s$  and  $N_a$  and this subdivision holds for both the  $\pi$  and  $\mu$  systems:  $\mathbf{D}_{u,u} = N_s \times N_s$ ;  $\mathbf{D}_{q,q} = N_a \times N_a$ ;  $\mathbf{D}_{q,u} = N_a \times N_s$ ;  $\mathbf{D}_{u,q} = N_a \times N_s$ .

The Eq.(1) can be rewritten as follows:

$$\begin{bmatrix} \mathbf{u}_\mu \\ \mathbf{q}_\mu \end{bmatrix} = \begin{bmatrix} \mathbf{T}_{u,u} & \mathbf{T}_{u,q} \\ \mathbf{T}_{q,u} & \mathbf{T}_{q,q} \end{bmatrix} \begin{bmatrix} \mathbf{u}_\pi \\ \mathbf{q}_\pi \end{bmatrix} \quad (9)$$

The  $N^2$  unknowns are the members of  $\mathbf{T}$  matrix and they can be obtained by assembling  $N$  algebraic systems each with  $N$  unknowns and equations.

The first subset of  $N_s$  equations can be obtained by using Eq.(1) and by imposing in turn a unit value of known term on each of the  $N_s$  degrees of freedom, say  $k$ :  $F_{\pi,k} = 1$ ,  $F_{\mu,k} = 1$ ,  $Q_{\pi,k} = 0$ ,  $Q_{\mu,k} = 0$ . The second subset of  $N_a$  equations can be obtained by imposing in turn a unit value of known term on each of the  $N_s$  degrees of freedom, say  $l$ :  $F_{\pi,l} = 0$ ,  $F_{\mu,l} = 0$ ,  $Q_{\pi,l} = 1$  and  $Q_{\mu,l} = 1$ . This procedure has to be repeated for all the structural and acoustic degrees of freedom.

It has to be noted that the Betti theorem can be invoked [8] so that for the  $p$  acoustic degree of freedom and the  $t$  structural one this relation holds:

$$\frac{q_p^{(F_t=1)}}{F_t = 1} = \frac{u_t^{(Q_p=1)}}{Q_p = 1} \quad (10)$$

By re-ordering the unknowns, the final matrix will be a block diagonal matrix,  $[N^2 \times N^2]$ , and the  $r$ -th block matrix on the diagonal,  $[N \times N]$ , will be as follows (considering Betti theorem):

$$\begin{bmatrix} \mathbf{u}_\pi^{(F_\pi=1)} & \mathbf{q}_\pi^{(F_\pi=1)} & 0 & 0 \\ \mathbf{u}_\pi^{(Q_\pi=1)} & \mathbf{q}_\pi^{(Q_\pi=1)} & 0 & 0 \\ 0 & 0 & \mathbf{u}_\pi^{(F_\pi=1)} & \mathbf{q}_\pi^{(F_\pi=1)} \\ 0 & 0 & \mathbf{u}_\pi^{(Q_\pi=1)} & \mathbf{q}_\pi^{(Q_\pi=1)} \end{bmatrix} \begin{bmatrix} \mathbf{T}_{u,u} \\ \mathbf{T}_{q,u} \\ \mathbf{T}_{u,q} \\ \mathbf{T}_{q,q} \end{bmatrix} = \begin{bmatrix} \mathbf{u}_\mu^{(F_\mu=1)} \\ \mathbf{u}_\mu^{(Q_\mu=1)} \\ \mathbf{q}_\mu^{(F_\mu=1)} \\ \mathbf{q}_\mu^{(Q_\mu=1)} \end{bmatrix} \quad (11)$$

In case of identical systems,  $\mathbf{T}$  is an identity matrix and its determinant assumes a unit value; even in the case of a replica (all the sizes scaled up or down with the same value)  $|\mathbf{T}| \neq 1$ . The determinant of  $\mathbf{T}$ , frequency dependent, could be used for analysing the degree of similarity between a model and a prototype.

### 3 CONCLUSIONS

The theoretical framework underlying the extension of VOODOO method to coupled structural-acoustic systems is herein provided. It allows to link two systems in similitude (where, in this case, this term must be interpreted in a very broad sense, because also systems with totally different geometries can be considered) and to reconstruct the response of one system from the response of the other, and vice versa, through a linear transformation matrix. The next steps must necessarily be the validation in on- and off-design conditions, in order to fully understand its potentialities and limits.

### ACKNOWLEDGMENTS

The authors acknowledge the support of the Italian Ministry of Education, University and Research (MIUR) through the project DEVISU, funded under the scheme PRIN-2107 – grant agreement No. 22017ZX9X4K006.

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