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Integral Equations for Real-Life Multiscale Electromagnetic Problems

Editors: Francesca Vipiana [1] ; Zhen Peng [2]
View affiliations
Affiliations: 1: Department of Electronics and Telecommunications, Politecnico di Torino, Italy
2: Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, USA
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- **Front Matter**

  p. (1)

- **1 Introduction**

  p. 1 –3 (3)

  In the context of computational electromagnetics (CEM), surface integral equation (SIE) techniques based on the method of moments (MoM) [1] offer a potent tool that has become essential for simulating and engineering a diverse range of applications. These applications encompass advanced antenna design [2,3], radar cross-section (RCS) [4], stealth technologies [5], electromagnetic compatibility and interference (EMC/EMI) [6], and nanoscience applications [7], among others. SIE methods are particularly attractive when dealing with large-scale radiation and scattering issues. Unlike volumetric approaches that require the characterization of three-dimensional (3D) structures and embedding space, SIE methods necessitate the parameterization of two-dimensional (2D) boundary surfaces only. Although they result in dense and extensive matrix systems for large-scale problems, the utilization of iterative fast solvers, such as the multilevel fast multipole algorithm (MLFMA) [8,9], enables efficient resolution of such problems.

- **2 Surface integral equation formulations**

  p. 5 –73 (69)

  This chapter includes a brief review of fundamental material needed for understanding, formulating, and numerically solving surface integral equations appearing in electromagnetics. Using this material, we then develop the most common integral equations for time-harmonic problems involving linear, piecewise homogeneous, and isotropic materials. Methods for numerically solving the integral equations are developed and discussed, including approaches for numerically evaluating the singular and near-singular integrals that arise.

- **3 Kernel-based fast factorization techniques**

  p. 75 –123 (49)

  This chapter has focused on MLFMA as a representative kernel-based fast factorization technique. To construct a basis for further discussion, we first considered the conventional MLFMA, which is based on the plane-wave expansion of electromagnetic waves, at a formulation level. To solve multi-scale problems involving dense (uniform or non-uniform) discretizations of electrically large objects, alternative MLFMA versions are needed since the conventional MLFMA suffers from a low-frequency breakdown. We listed a variety of ways to implement low-frequency-stable MLFMAs, such as based on multipoles, inhomogeneous plane waves, coordinate shifts, and approximation techniques. We showed how MLFMA implementations can be used to solve extremely large problems via parallelization, while they can be applied to complex structures with different material properties, including plasmonic and NZI objects. Examples were given for solutions of densely discretized objects to demonstrate how MLFMA can handle such complicated problems that possess modeling challenges. Finally, problems with non-uniform discretizations that naturally arise in multi-scale simulations were considered. A rigorous implementation for stable, accurate, and efficient solutions of these problems requires a well-designed combination of a suitable formulation/discretization, an effective solution algorithm (MLFMA version), and a carefully designed clustering mechanism.

- **4 Kernel-independent fast factorization methods for multiscale electromagnetic problems**

  p. 125 –177 (53)

  In this chapter, the low-rank factorization methods for real-life multiscale simulations are proposed. The low-rank factorization methods are fully algebraic, the rank-deficient nature is exploited for the coupling matrix blocks produced by two well-separated groups. It is well known, the whole impedance matrix of method of moments (MoM) is full-rank, while the off-diagonal matrix blocks are low-rank. The off-diagonal matrix blocks are produced by the "far" coupling basis functions, which are over sampling than the Nyquist limit [1]. With the low-rank factorization methods, the impedance matrix can be approximated for fast evaluations of matrix-vector products in iterative solutions [2] or fast direct solvers [3,4].

- **5 Domain decomposition method (DDM)**

  p. 179 –229 (51)

  This chapter concerns the use of domain decomposition (DD) methods for the surface integral equation (SIE)-based solution of time-harmonic electromagnetic wave problems. DD methods have attracted significant attention for solving partial differential equations [1-16]. These methods are appealing due to their ability to obtain effective, efficient preconditioned iterative solution algorithms. They are also attractive because of their inherently parallel nature, an important consideration in keeping with current trends in computer architecture.

- **6 Multi-resolution preconditioner**

  p. 231 –276 (46)

  The purpose of this chapter is to provide the main guidelines for an efficient implementation of the multi-resolution (MR) preconditioner for the electromagnetic (EM) analysis of perfect electric conductor (PEC) structures of arbitrary 3-D shape via the method of moments (MoM) applied to the electric field integral equation (EFIE) and to the combined field integral equation (CFIE).

  The chapter is structured in four main parts. First, the generation of the MR basis functions as a linear combination of the standard basis functions is described. Second, the generation of a multi-level set of meshes, starting from the usual mesh, is reported: the MR functions are defined on each level of the generated set of meshes. These two parts are essential to implement the proposed preconditioner. Then, the third part is dedicated to the insertion of the MR preconditioner into the solution algorithm, together with the description of some implementation tricks. Finally, numerical results, where the MR preconditioner is applied to complex realistic 3-D structures, are reported and commented.

The expected property of the MR preconditioner is an improvement of the convergence rates of iterative solvers, with a limited computational cost for its generation and application. The proposed preconditioner can be applied to realistic structures with arbitrary topological complexity.

This chapter discussed optimal *h*-refinement preconditioning strategies, leveraging Calderón identities, for some of the most widespread IEs: the EFIE, CFIE, and PMCHWT. The treatment has been kept accessible by providing the main insights into the curing mechanisms and effectiveness of the schemes. Because the Calderón identities also provide a partial regularization of the low-frequency ill-conditioning and numerical issues affecting these equations, the coupling of the Calderón strategies with quasi-Helmholtz projectors - that make this partial stabilization complete - have also been presented. To ensure that the reader can make the most out of the techniques presented, we have consistently provided discretization strategies for the different schemes, with a particular focus on the PMCHWT for which several alternatives were introduced, along with numerical examples illustrating the effectiveness of the schemes.

In this chapter, we study an experimental formulation called decoupled potential integral equation (DPIE) [1]. The aim of this formulation is to obtain a method that is robust at all frequencies, in particular at low frequencies for multiply connected geometries. We also discuss experimental discretization methods that are high-order, adaptive and fast.

The computer simulation of electromagnetic (EM) phenomena has emerged as a powerful and indispensable tool in both the design and analysis stages of many engineering endeavors. In the time-harmonic regime, integral equation (IE) methods are widely used for such simulations [1–3]. Applications range from advanced antenna design [4,5], stealth technologies [6-8], integrated circuits [9,10] to optics and photonics [11,12]. One advantage of solving an integral equation is that both the analysis and unknowns reside only on the boundary surfaces of the targets. It often requires fewer unknowns to solve compared to a differential equation, where the unknowns scale volumetrically. More significantly, since surface-based modeling is used, it is easier to prepare analysis-suitable models from computer-aided design (CAD) geometries. However, the application of IE methods to the Maxwell equations leads to a dense, complex, and indefinite matrix equation. Techniques such as the fast multipole method [13,14], fast Fourier transform [15,16], interpolative decomposition [17,18], and $\mathscr{H}$-matrix compression [19,20] have been developed to compress dense IE matrices.