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Equivalent skin effect model for time-domain analysis starting from electromagnetic simulator values

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Abstract

An equivalent circuit to reproduce the skin effect behavior in an arbitrary structure modeled as resistance-inductance has been developed modifying the Foster model. Taking the values of inductance and resistance in the frequency domain directly from electromagnetic analysis obtained by commercial software, the proposed model allows to implement an equivalent circuit that can be used with time-domain simulator. The developed circuit is very simple, thus reducing the computational times with respect to other approaches, but still preserves a good match with the electromagnetic simulation results.

1. Introduction

For many electronic applications knowing the frequency trend of components is fundamental to ensure the device of interest satisfy the Electromagnetic Compatibility (EMC) standard regulations. Compliance with international limits prevents interference and guarantees reliability on the operations of the device but it is also essential for companies to put the products on the market without incurring in penalties and fines. The need to know the frequency response of the object during the design phase and to take into account the effect of parasitic elements already during the initial simulations prevents a waste of costs and time for manufacturer and engineer by reducing the risk of iterating the production process after testing the physical prototype.

To perform an electromagnetic analysis many different tools exist with similar simulation workflow and format of the provided results. According to the structure you want to analyze the simulation setup are regulated choosing the boundary, the range of frequency and defining the type of port. Afterwards the electromagnetic simulation is performed and the results are obtained in the form of scattering (or S) parameters. The S-parameters describe the relation between incident and reflected waves at each port of a network in a certain range of frequency, therefore they provide a complete representation of the behavior of a device with an arbitrary number of port in frequency domain. Subsequently, in order to consider the effects of the identified parasitic elements, a co-simulation must be performed where in the original schematic in addition to the simple circuital components the parameters extracted from the electromagnetic simulation are taken into account, the waveforms are consequently affected also by the presence of non-idealities.

Since electromagnetic tools provide the results of the simulated network by S-parameters, with the described approach it is possible to have an accurate prediction in the frequency domain. However, these results can not be used directly for time-domain transient simulations. Hence, different modeling strategies have been proposed over the past few years to provide an accurate circuit that can be used in time-domain simulation and that is capable of reproducing the skin effect behavior which is the dominant non-ideality when the frequency starts to increase. Conventional methodologies to derive a circuit that can be used in time domain simulators require the knowledge of the geometry and materials properties of the analyzed object and they are only suitable for specific structure as round wires [1], on-chip spiral inductors [2], on-chip interconnects [3], and transmission lines [4], [5]. Other methods require the computation of the current and use analytical evaluation with long and complex mathematical procedure [6], [7]. One of the most used is the Kim and Neikirik method [8] that instead for a general case uses an iterative approximation [9], fixing the number of cells and choosing the value of components that best fit, the equivalent circuit can only be obtained when the minimum error is reached after an iterating procedure.

In some electromagnetic simulators it is also possible to obtain a direct on-the-fly conversion from S-parameters to an equivalent circuit suitable for time-domain simulation. An example is Q3D, an ANSYS tool, where the behavior of parasitic elements can be provided both as S-parameters or an equivalent circuit. The representation of parasitic inductance and resistance with the S-parameters is in frequency domain, therefore it can not be used for time domain analysis. Instead, the equivalent circuit is provided by the simulator in two ways. In the first case the equivalent circuit can be extracted at a single frequency point. This approach however, is not very useful when you want to see the behavior over a wide frequency range. In the second case a conversion in time domain directly from S-parameters is performed. However, to increase

accuracy to a tolerable level, a high order circuit must be generated. This can cause problem of convergence in the subsequently co-simulation performed with the rest of circuit with time-domain simulator.

In this work an alternative approach for generating an equivalent circuit suitable for time-domain simulation is proposed. With the presented model it is possible to reproduce the skin effect of the parasitic resistance and inductance for a wide range of frequency with an accurate precision and fast computation, for an arbitrary structure. In Section 2 the problem of the skin effect is explained. In Section 3 the procedure to derive the model is presented. In Section 4 the results are shown and compared with the expected values and the other existing models. Finally, the conclusions are drawn.

2. Skin-effect frequency behavior

To be able to perform a simulation in a broadband frequency range it is necessary to take into account the skin effect, which is the predominant cause that determines the variation of the parasitic resistance and inductance with the increase of frequency.

In order to better understand simulator results, we consider here that the skin depth relation used by Q3D simulator [10] is the following:

$$\delta = \sqrt{\frac{2}{\omega \sigma \mu_0 \mu_r}} \qquad (1)$$

where ω is the angular frequency equal to $2\pi f$, σ is the conductor's conductivity in S/m, μ_0 is the permeability of free space and μ_r is the conductor's relative permeability. Then according to the dimensions and property of the device under test the software divides the frequency range of simulations in three sections: a low frequency region (usually indicated as DC region), a transition region, and a high frequency one (AC), as illustrated in Figure 1.
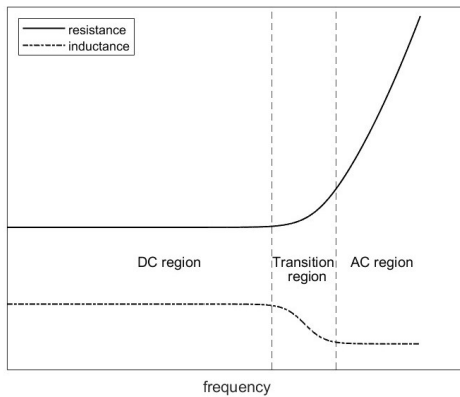

Figure 1: Resistance and inductance behavior due to the skin effect and division of the frequency range into DC region, transition region and AC region.

In AC region, as frequency increases, the current flow is concentrated more near the conductor surface. This results in a reduction of the effective cross-section and a decreasing of the skin depth. According to Q3D, considering an object of thickness d the starting frequency of AC region is computed as the frequency that generate a skin depth much smaller than d, obtaining a limit equal to: $f_{AC,limit} = \frac{9}{\sigma \pi d^2 \mu_0 \mu_r}$. A similar consideration is done to compute the upper limit of the DC region, obtaining the frequency that produces a skin depth greater than the device thickness: $f_{DC,limit} = \frac{1}{\sigma \pi d^2 \mu_0 \mu_r}$. In the DC region the inductance and resistance have a nearly constant behaviour up to $f_{DC,limit}$. Instead in the AC region the value of inductance decreases down to an approximately constant value, whereas the resistance grows proportionally to the square root of frequency. Between the DC and AC regions there is the transition region where resistance and inductance values are estimated based on DC and AC asymptotes and it spans about one decade of frequency. The equivalent circuit must be constructed in order to reproduce this evolution.

3. Skin-effect modeling in time domain

Some canonical ladder circuit as the Foster's one depicted in Figure 2a and Cauer's one depicted in Figure 2b can be used in network synthesis to reproduce the behavior of certain impedance, starting from the knowledge of its poles and zeroes.
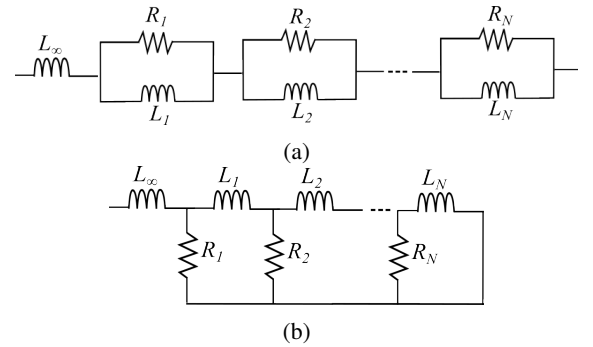

Figure 2: Ladder circuit for network synthesis with Foster 1st form (a) and Cauer 1st form (b).

According to Foster model the impedances that can be represented have the following form:

$$Z(s) = \sum_{i=1}^{N} \frac{A_i s}{s + \sigma_i} + A_\infty s \qquad (2)$$

where, as represented in Figure 2a: $A_\infty = L_\infty$, $A_i = R_i$ and $\sigma_i = \frac{R_N}{L_N}$.

However in our case the only available information concerns the frequency variation of resistance and inductance, the position of poles and zeroes is unknown therefore the impedance can only be reproduced based on

simulator data. The proposed circuit has been depicted in Figure 3 and it is a modified version of the Foster one. Without knowing a priori the position of the singularities but directly taking data from Q3D, it is able to provide the correct behavior of resistance and impedance in the starting region considering the impedance limit, to reproduce the increasing of resistance proportional to $\sqrt{f}$ with each cell element $R_i - L_i$, and at the same time to recover the value of the inductance with L_{par}. Considering the frequency space between the ending of DC region and the maximum frequency desired for the analysis, a new cell element is added for each decade of frequency. Therefore if N is the number of decade of interest after $f_{DC,limit}$ a number of cell equal to N must be inserted in the final equivalent circuit.
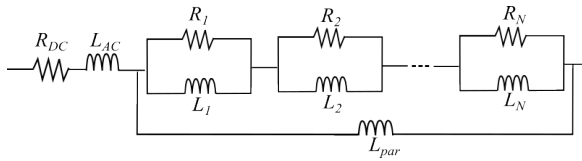

Figure 3: Representation of the proposed equivalent circuit to reproduce the skin-effect behavior.

The impedances that can be represented now are:

$$Z(s) = A_0 + \frac{A_p s \sum_{i=1}^{N}\left[A_i \prod_{\substack{j=1 \\ j \neq i}}^{N}(s+\sigma_j)\right]}{A_p \prod_{i=1}^{N}(s+\sigma_i) + \sum_{i=1}^{N}\left[A_i \prod_{\substack{j=1 \\ j \neq i}}^{N}(s+\sigma_j)\right]} + A_\infty s \tag{3}$$

where $A_0 = R_{DC}$, $A_\infty = L_{AC}$, $A_p = L_{par}$, $A_i = R_i$ and $\sigma_{i,j} = \frac{R_{i,j}}{L_{i,j}}$ as can be seen in Figure 3.

In order to get the desired value for inductance and resistance in the DC region the limits of the impedance for frequency that tends to 0 are computed and they are imposed to be equal to the simulator values extracted in that region:

$$\lim_{f \to 0} \text{Re}[Z(s)] = R_{DC} = R_{DC,sim} \tag{4}$$

$$\lim_{f \to 0} \text{Im}\left[\frac{Z(s)}{2\pi f}\right] = L_{AC} + \frac{L_{par} \sum_{i=1}^{N} L_i}{L_{par} + \sum_{i=1}^{N} L_i} = L_{DC,sim} \tag{5}$$

where Re and Im are the real part and the imaginary part operator, respectively.

The same procedure is applied to higher frequency to obtain the desired inductance value at the maximum frequency needed:

$$\lim_{f \to \infty} \text{Im}\left[\frac{Z(s)}{2\pi f}\right] = L_{AC} = L_{AC_{max},sim} \tag{6}$$

Conversely, to reproduce the progress of the resistance with the increasing of frequency, the real part of the impedance is approximated with its limit at frequency that tends to infinity. Subsequently for each decade of frequency after the ending of DC region, the computed limit is equate to the corresponding parameter extracted from the simulator, therefore the resistance of every $R_i - L_i$ cell is assigned, until reaching the cell $R_N - L_N$ that corresponds to the behavior of the last decade of frequency of interest:

$$\lim_{f \to \infty} \text{Re}[Z(s)] = R_{DC} + \sum_{i=1}^{N} R_i = R_{AC_N,sim} \tag{7}$$

In this way the desired resistance values have been set and to obtain them in correspondence of the right frequency points so that the frequency evolution of the real part of the impedance is correctly reproduced, the inductance values are chosen as follows:

$$L_N = \frac{k_N R_N}{2\pi f_N} \tag{8}$$

where k_N, is a degree of freedom, whose suggested value, for $N \neq 1$, is $k_N = 3$ since it was possible to notice experimentally that the position of the zeroes and poles falls in the correct place to ensure that the increase in the resistance curve is proportional to the square root of the frequency. Otherwise for the first cell the value of inductance has to be chosen not to have negative values. By setting $k_1 \geq 3$ we satisfy the following condition that guarantees having only real components:

$$L_1 + \sum_{i=2}^{N} L_i > L_{DC} - L_{AC} \tag{9}$$

Consequently constraints on each element of the circuit were defined, and by solving the mathematical impositions it is possible to directly obtain the values of the resistances and inductances that constitute the equivalent skin-effect circuit model.

4. Examples

The proposed method is now implemented into practical examples. Two structures are analyzed to observe the generic validity of the model that can be used for different fields of interest. In particular a PCB coil and a bonding wire of a package are taken into analysis.

First, the PCB coil also used in [11] is considered. It was printed on a standard $1.66\,\text{mm}$ board with FR4 substrate and a $35\,\mu\text{m}$ copper plating, and it has a theoretical value of $L = 1.47\,\mu\text{H}$. One type of characterization, presented in [11], is done with the use of a Vector Network Analyzer (VNA). It provides the scattering parameters with the measurement of the power of the incident and reflected waves. The value of inductance is therefore derived from the measured S-matrix and it is compared with the value obtained from the electromagnetic simulator, where the files that described its geometry and the assignment of

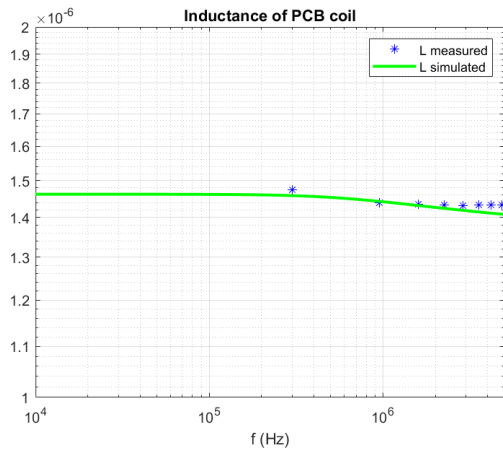material properties are loaded. The comparison is shown in Figure 4.

Figure 4: Comparison of inductance value of PCB coil between measured value obtained with VNA and simulated value obtained with the electromagnetic simulator Q3D.

As can be seen, the values obtained from the measurement are very similar to those obtained from the electromagnetic simulator, and they also well approximate the expected value. With the increase in frequency it is also possible to notice the decrease in the inductance value due to the skin effect, and to take this variation into account in a time-domain simulator a model that reproduces this trend must necessarily be used. Thus the model proposed in the previous section can be implemented taking the values from the electromagnetic software.

The electromagnetic simulation runs up to 100 MHz because before this value the skin effect is the predominant non-ideality, while after that frequency the resonance takes over. The upper limit of the DC region is approximately equal to 100 kHz, instead the lower bound of AC region is about 1 MHz as can be seen from the curves obtained from the electromagnetic simulation in Figure 5. Therefore, the required number of cells to reproduce this behavior is $\log(100\,\text{MHz}/100\,\text{kHz}) = 3$.

Subsequently after having performed the electromagnetic analysis the values of resistance and inductance in DC region and in each decade of frequency after the ending of DC region are extracted from the simulator. The conditions explained above are imposed and the equivalent circuit that is obtained has the following parameters: $R_{DC} = 0.9884\,\Omega$, $R_1 = 0.16\,\Omega$, $R_2 = 1.15\,\Omega$, $R_3 = 3.95\,\Omega$, $L_{AC} = 1.378\,\mu\text{H}$, $L_1 = 77\,\text{nH}$, $L_2 = 55\,\text{nH}$, $L_3 = 19\,\text{nH}$, $L_{par} = 0.19\,\mu\text{H}$.
The resulting resistance and inductance obtained from the computed equivalent circuit is then evaluated and compared to the target data obtained from the electromagnetic simulation. A comparison with the existing method of Kim and Neikirik following the iterative procedure for

a generic structure is also performed.
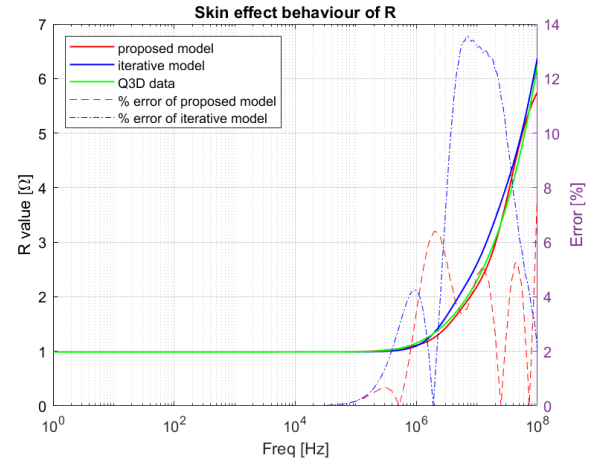
(a)
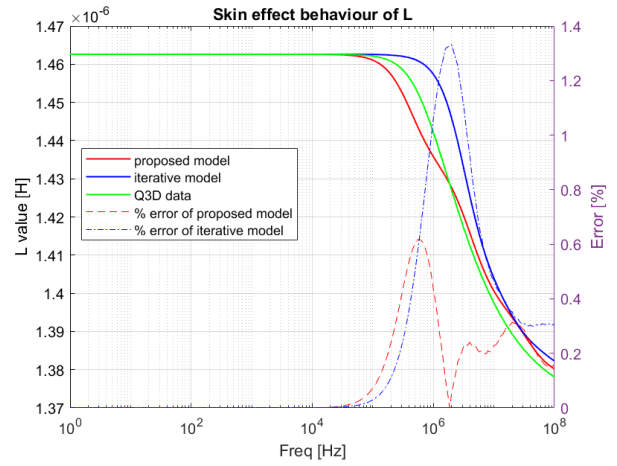
(b)

Figure 5: Comparison of resistance and inductance behavior of a PCB coil between proposed circuit, Kim and Neikirik iterative method and wanted value obtained from electromagnetic simulation.

The second case considered is a copper bonding wire with a diameter of 1.2 mil of a QFN package, to which the same procedure has been applied. In this case since the rising times of the signals that are involved are of the order of *ns* we are interested in parasitic resistance and inductance in a wide band of frequencies, therefore the simulation is performed up to 10 GHz. The limit of DC region is about 1 MHz while the lower bound of AC region is approximately equal to 10 MHz. Since now the decades of interest after the initial constant region are 4, the equivalent circuit has 4 cell elements with the following values: $R_{DC} = 0.03\,\Omega$, $R_1 = 5.2\,\text{m}\Omega$, $R_2 = 30\,\text{m}\Omega$, $R_3 = 0.1\,\Omega$, , $R_4 = 0.32\,\Omega$, $L_{AC} = 1.416\,\text{nH}$, $L_1 = 0.25\,\text{nH}$, $L_2 = 0.14\,\text{nH}$, $L_3 = 48\,\text{pH}$, $L_4 = 15\,\text{pH}$, $L_{par} = 0.69\,\text{nH}$. Also in this case the parasitic resistance and inductance of

the proposed circuit are compared in Figure 6 with that obtained with the iterative method and the desired values of the simulator.
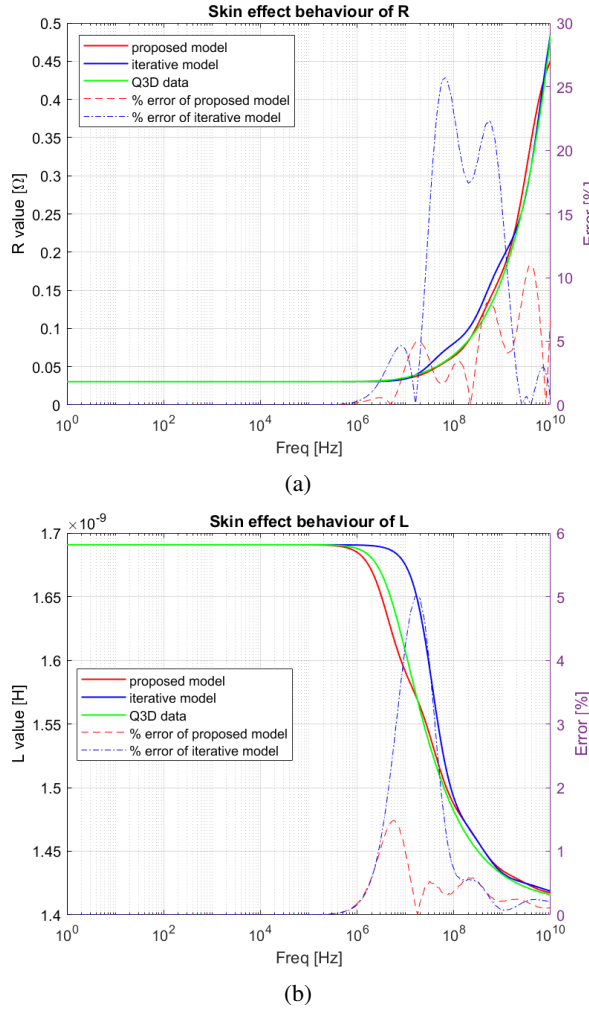

(a)

(b)

Figure 6: Comparison of resistance and inductance behavior of a copper wire of a package between proposed circuit, Kim and Neikirik iterative method and wanted value obtained from electromagnetic simulation.

As a result of the comparison, it can be noted that in both the examples over the entire range of frequencies the obtained curves almost approximate the target behavior of the electromagnetic simulation. By computing the relative error of the models with respect to the simulator values it is possible to see that the error of the proposed method is comparable with the one of the iterative procedure. Thus a circuit with equal simulation complexity constitute only of R and L elements is obtained but with a reduced computational time and less calculation effort, maintaining accuracy and showing a lower maximum error on the range of frequency of interest.

A comparison between the proposed circuit and the broadband one generated directly by Q3D is also discussed. To obtain the same precision of the presented model the circuit generated by the simulator, shown in Figure 7 must be of an order higher than 10, that leads into a circuit with very high complexity.
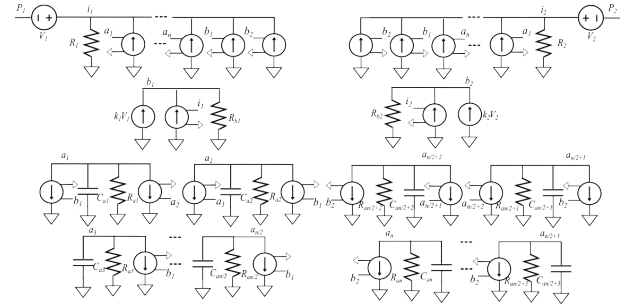

Figure 7: Broadband circuit generated from Q3D simulator to reproduce the variation of R and L due to the skin effect. To obtain the same precision of the presented model n must be greater than 10.

Furthermore it is not used stand alone but it is connected to the rest of the electrical circuit for the co-simulation in time domain simulator. This can lead to convergence problems and the simulation in the time domain may not be performed. These are problems that can be overcome with the use of the model discussed since it has a very reduced number of components but maintaining precision, and the co-simulation can be performed unchallenged.

5. Conclusions

An alternative method to reproduce the skin-effect of parasitic resistance and inductance with time-domain simulator has been realized. Directly solving mathematical equations the values of circuit components can be obtained, and the equivalent model can be derived simply and rapidly respect to the existing techniques. The equivalent circuit can be implemented without the need of knowing the position of the poles and zeros of the impedance, and without knowing the geometry and material properties of the analyzed object but can be applied on a generic structure. It fit well the simulated values and can be used for a wide range of frequencies.

6. Acknowledgements

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