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(Article begins on next page)

# Dakar Mosque gridshell: exploring the benefits of the improved multibody rope approach through postbuckling analysis

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## Abstract

This paper presents a comparative study of a gridshell roof structure designed for the Islamic Cultural Center in Dakar, Senegal. The structure, conceived by Fragomeli & Partners, emulates the shape of a dune to seamlessly integrate with the surrounding environment. To define an optimal structural shape while preserving the architectural intent, two form-finding methodologies were employed. The first method, the Multibody Rope Approach (MRA), enables the calculation of the funicular structural shape in response to the imposed load configuration. The second method, Improved MRA (i-MRA), introduces slight geometric variations to the funicular shape to reduce the diversity of structural elements required for construction. This work investigates the impact of the geometric distortions introduced by i-MRA on structural stability. Linear buckling modal analysis has been conducted on the two generated geometries, taking into account various loading conditions and structural element slenderness. Through incremental load Geometrical Non-linear with Imperfection Analyses (GNIAs) the equilibrium paths are examined to analyze the postbuckling behaviour of the gridshells. In the final step, the elastoplas-

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tic constitutive law is incorporated to assess also the impact of material non-linearity. The models are examined through incremental load Geometrical and Material Non-linear Analyses (GMNIAs). The results indicate that the i-MRA geometry exhibits a more even stress distribution and better qualitative postbuckling behaviour.

*Keywords:* Gridshell, Multi-body Rope Approach, Postbuckling, Snap through instability, Nonlinear bifurcation

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## 1. Introduction

Architecture and structural engineering are closely intertwined, with architecture influencing and being influenced by structural design. Over time, these disciplines have evolved hand in hand, responding to the ever-changing needs and aspirations of society. The dynamic relationship between architecture and structure has played a pivotal role in shaping a wide range of structural typologies and forms [1]. This holds in particular for the spatial structures in which the shape is strictly interconnected with the stress distribution. In this context, form-finding methodologies were introduced during the 1960s and 70s to establish the structural geometry as a function of the load field. These techniques have found extensive application, especially in the design of shells and gridshells [2]. Concerning structural stability, gridshells can be considered as single-layer reticulated shells. Architectural audacity and complexity require safety analyses for structural integrity beyond the standard procedures [3]. In this regard, the IASS (International Association for Spatial Structures) published comprehensive guidelines to buckling load evaluation of metal structures [4]. To incorporate all the non-univocal aspects behind buckling collapse, different analyses and approaches are indicated to assess the final design critical load. The elastic buckling load  $Q_{EB}$

20 is usually evaluated through a Geometrically Nonlinear with Imperfection  
 21 Analysis (GNIA) or Geometrically and Material Nonlinear with Imperfec-  
 22 tion Analysis (GMNIA). The loading patterns, and the related displacement  
 23 coordinate  $\delta_i$  for which to evaluate the equilibrium path, are selected through  
 24 the the linear eigenvalue extraction for the  $n$  design combination load factors  
 25  $\Lambda_n$ . The generic Euler load  $Q_{CRi} = \Lambda_i q_i$ , being  $q_i$  the total applied load in  
 26 the  $i - th$  combination, and the maximum modal displacement are then used  
 27 as a set parameters for the GNIA. A linear static analysis of the structure  
 28 loaded by  $Q_{CR}$  provides the critical displacement  $\delta_{CR}$ , measured at the max-  
 29 imum node displacement as a generalized coordinate. In a load-displacement  
 30 graph, the straight line from the origin to the point  $(Q_{CR}, \delta_{CR})$  represents  
 31 an upper-bound for all the possible equilibrium paths, as shown in Figure 1  
 32 (recent authors refer to as energy barriers [5], [6], [7]). A geometrically non-  
 33 linear analysis without imperfections would follow a lower stiffness path (blue  
 34 dotted lines in Figure 1). This is due to the negative term of the geometric  
 35 stiffness matrix, leading to the bifurcation point (marked by the blue triangle  
 36 in Figure 1) associated with the global or local buckling branch. The bifur-  
 37 cated branches represent the only equilibrium space of the system, therefore  
 38 the resulting equilibrium path must lay within those if the control parameters  
 39 of the analysis are increased. A valley in the equilibrium path indicates the  
 40 presence of a snap-through instability. In this scenario, the system reaches  
 41 to the *next* stable position (undergoing significant displacements) on the pos-  
 42 itive slope portion of the valley at the same load level of the limit point. The  
 43 presence of the imperfections (continuous blue line in Figure 1) further re-  
 44 duces the system stiffness, since they are usually introduced from a geometric

45 deviation proportional to a critical eigenshape (or a combination of them).  
 46  $Q_{EBi}$  is therefore the limit point of the computed  $i$  equilibrium path using  
 47  $\Lambda_i$  as a load control analysis factor and a selected imperfections pattern. As  
 48 illustrated in Figure 1b),  $Q_{EB}$  may result higher than  $Q_{CR}$  as a result of a  
 49 hardening overall postbuckling branch. In such cases, the postbuckling be-  
 50 havior is classified as benign [8], since to an increase of the external load,  
 51 the stable nature of the bifurcation branch does not change (as it occurs for  
 52 the Euler beam). The structure collapse would then take place in the form  
 53 of a material failure or in correspondence with a local buckling secondary  
 54 branch (dark grey curves in Figure 1). The interaction between local and  
 55 global instabilities is a largely studied topic and of the central importance of  
 56 reticulated structures [9], [10], [11], [12], [13]. These local buckling branches  
 57 have always a negative slope portion in the global generalized coordinates  
 58 plane, due to effect of geometric nonlinearity, as evidenced by Gioncu [14],  
 59 [15], [16]. Thompson theorems ([17], [18]), [19] allow the composition of the  
 60 final equilibrium path starting from discrete branching analysis. Because the  
 61 resultant from a positive and negative slope is a negative one, the overall bi-  
 62 furcated paths in Figure 1 are always unstable (dotted blue line). The effect  
 63 of the imperfections can be measured through the erosion in the load curve  
 64 between the limit point and the branch intersection cusps (more than one  
 65 local buckling branch normally exists). Intuitively, sharper are these cusps  
 66 deeper is the load erosion, meaning that the structure is more sensitive to  
 67 geometrical imperfections. Moreover, to a vertical erosion corresponds an  
 68 increase in the path curvature in proximity of the limit point, and therefore,  
 69 a greater pre-buckling flexibility. Finally, a cusp bifurcation may occur as

70 the first equilibrium path bifurcation as in Figure 1a). In this case, the post-  
71 buckling is identified as catastrophic, or ultra-catastrophic when both the  
72 general coordinates reverse [8], [20]. In term of collapse mechanism, the limit  
73 point translates into the well known snap-through buckling phenomenon [21],  
74 [22], [23], [24]. Then, both the equilibrium paths in Figure 1 share such col-  
75 lapse mechanism, but with a profound difference in how it is triggered. A  
76 good indicator is the ratio  $\frac{Q_{EB}}{Q_{CR}}$ : when it is lower than 1, the corresponding  
77 equilibrium path exhibits both a higher imperfection sensitivity and a higher  
78 flexibility than the structures that have  $\frac{Q_{EB}}{Q_{CR}} > 1$ . In shell design, significant  
79 knockdown factors must be introduced to keep the structure at load levels  
80 safely below  $Q_{CR}$  [25], [26]. Conversely, when  $\frac{Q_{EB}}{Q_{CR}} > 1$ , there is still room  
81 for postbuckling behaviour improvement through local buckling prevention  
82 strategies that retard the overall collapse [27], [28]. The occurrence of local-  
83 ized yielding increases the displacement response of the structure, therefore  
84 the elasto-plastic buckling load  $Q_{EP}$ , evaluated through a GMNIA, can be  
85 notably lower than  $Q_{EBi}$ . This is particularly relevant for flexible structures  
86 and with the presence of stress spikes. A good indicator for this vulnerability  
87 is the yielding load  $Q_Y$ : the level of external load at which the first yielding  
88 stress is reached in any of the gridshell elements (observed by Kato in [29]).  
89 Formal differences in the equilibrium path are given by three main factors,  
90 that drive the overall stability behaviour: geometry, imperfections and con-  
91 nections stiffness [30]. Concerning the former, comparisons between classical  
92 spatial configurations have evidenced that geometries with high membrane  
93 regime of internal forces (i.e. domes) exhibit smaller flexibility towards ver-  
94 tical loads but increased imperfection sensitivity [31]. At the same time,

95 they also display a formal behavior transition when the shallowness ratio  
96 is varied, contrary to bending driven geometries like barrel vaults. Such  
97 change resembles the fundamental observations for continuous shell buckling  
98 by Von Karman [32], and more recently summarized in [33], and for flat  
99 arch stability [34],[35]. Anyway, reticulated systems suffer local instability  
100 problems more than their continuous counterparts, such as connection fail-  
101 ure or member (local) buckling, that can lead to progressive collapse of the  
102 whole compartment [36], [37], [13], [38]. Freeform geometries have instead a  
103 less predictable behavior, because the force fluxes have not a straightforward  
104 path to the bearings [39]. Mesh pattern plays a significant role in direction-  
105 ing such fluxes, and stability of freeform gridshells can be altered by it within  
106 the same median surface [40]. Slenderness of the members is also a part of  
107 the geometrical arrangement of a gridshell, and its effect on the equilibrium  
108 path is both nonlinear and non monotonic when varied [41],[42]. However,  
109 section members have often architectural constraints in shape and size, and  
110 in general chosen towards a dead load minimization. This three level set  
111 of geometrical features (median surface, mesh, slenderness members) concur  
112 hierarchically in generating the tangential stiffness matrix along the equilib-  
113 rium path [43], [44].

114 In this paper we tried to use qualitative studies of the equilibrium path shape  
115 as a powerful tool for the conceptual design of a gridshell, testing two differ-  
116 ent geometries generated through a Multibody Rope Approach (MRA) [45]  
117 and its improved version (iMRA) [46]. The MRA is a form-finding method-  
118 ology designed to define structural geometry [47] by dynamically solving a  
119 structural model consisting of falling masses connected by inextensible ropes

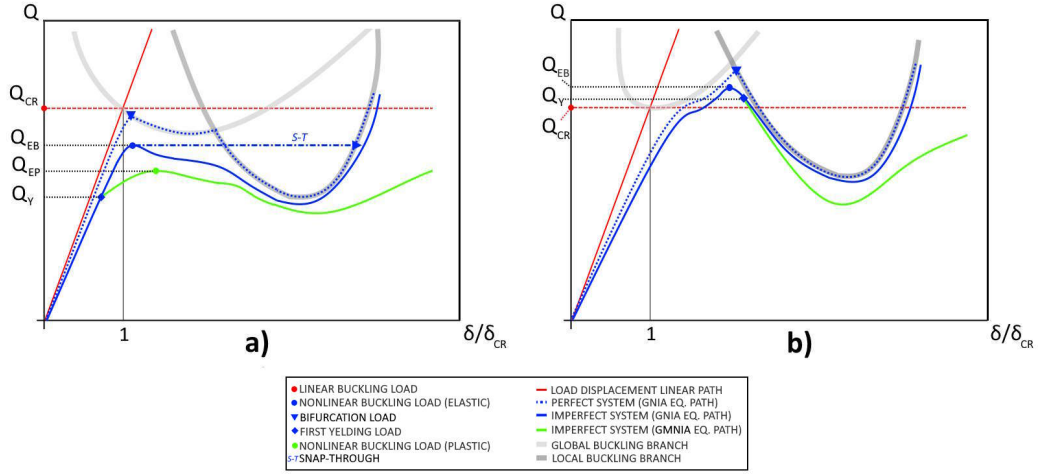


Figure 1: a) Equilibrium path of a reticulated shell with ultra-catastrophic post-buckling behavior. b) Benign (stiffening) postbuckling behavior.

120 [48, 49]. The i-MRA represents the enhanced version of the MRA [50]. The  
 121 method is specifically designed for the form-finding and the optimization of  
 122 the construction process of free-form gridshells [46]. A detailed study is con-  
 123 ducted on the postbuckling behaviour of the geometries produced by these  
 124 approaches [51]. In particular, we tested the influence of member slender-  
 125 ness through GNIA and GMNIA and several imperfection magnitudes for  
 126 separate load patterns. We instead kept a unique initial tessellation and  
 127 hypothesized a perfect rigid quadrangular connection.

## 128 2. Form-finding techniques

129 The Multibody Rope Approach (MRA) is a specialized method designed  
 130 specifically for determining the funicular shape of free-form gridshells. In  
 131 this form-finding method, the shape of the structure is derived by modelling  
 132 a multi-body physical system. This system consists of nodal masses inter-

133 connected by slack ropes, each with a length equal to  $l_{rope}$ . The structural  
 134 shape can be calculated by solving the dynamic model of the system 1 and  
 135 applying D'Alembert's principle to determine the equilibrium of the system.

136 The solution can be obtained as:

$$\vec{u}(t) = C_1 e^{-2\omega_n \zeta} + C_2 + \frac{C_3}{2\omega_n \zeta} t \quad (1)$$

137 Where  $C_1$ ,  $C_2$  and  $C_3$  are coefficients that can be calculated the system  
 138 initial conditions.

$$C_1 = -\frac{2\omega_n \zeta \dot{\vec{u}}(t-\Delta t) - C_3}{(2\omega_n \zeta)^2} \quad (2)$$

$$C_2 = -\frac{(2\omega_n \zeta)^2 \vec{u}(t-\Delta t) + 2\omega_n \zeta \dot{\vec{u}}(t-\Delta t) - C_3}{(2\omega_n \zeta)^2} \quad (3)$$

$$C_3 = \vec{p}_i + \sum_{j=1}^{n_i} \left\{ k \cdot \vec{F}_{rope,ji} \right\} \quad (4)$$

139 In Equation 1,  $\vec{u}(t)$  represents the position of the nodal masses, where  
 140  $t$  signifies the temporal instant at which the equation is computed. Addi-  
 141 tionally,  $\omega_n = \sqrt{\frac{k}{m}}$  and  $\zeta = \frac{c}{2\omega_n m}$  correspond to the natural frequency and  
 142 critical damping of the system, respectively. These two quantities can be  
 143 readily obtained by knowing the stiffness of the ropes  $k$ , the mass of the  
 144 nodes  $m$ , and the damping coefficient of the system  $c$ .

145 In the equation 4, the vector  $p_i$  represents the applied external force on  
 146 a generic node  $i$ , while the forces transmitted by the ropes connected to the  
 147 nodes are denoted by  $\vec{F}_{rope,ji}$ . It is important to note that the reaction of

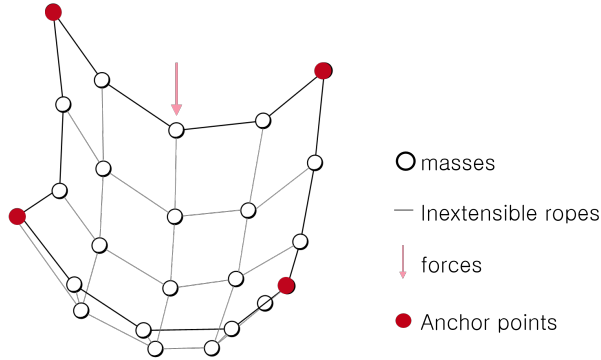


Figure 2: Dynamic model of falling bodies employed in the MRA and in the i-MRA.

148 the ropes can only be exerted when they are under tension. Thus, the forces  
 149  $\vec{F}_{rope, ji}$  can be derived as follows:

$$\begin{cases} F_{rope} = 0 & \text{if } l < l_{rope} \\ F_{rope} = k(l - l_{rope}) & \text{if } l \geq l_{rope} \end{cases} \quad (5)$$

150 Starting with a basic mesh, the final solution is calculated through a step-  
 151 by-step process. The conditions at each step rely on the solution from the  
 152 previous one.

153 In the final solution, structural components can be classified into two sets  
 154 based on their length. Elements shorter than  $l_{rope}$  are denoted as *loose ele-*  
 155 *ments* and those with a length matching  $l_{rope}$  are labelled as *target elements*.

156 The i-MRA method stands as an advancement over the MRA, seeking  
 157 not only to optimize geometry but also to improve the automation of the  
 158 construction process by minimizing the variety of structural element typolo-  
 159 gies [46]. In particular, two key techniques are employed to achieve this  
 160 optimization. The first involves grouping the structural elements and assign-

161 ing different slack coefficients to each group. In this case, starting from the  
 162 solution derived by the MRA, a new equilibrium configuration is calculated  
 163 using the subsequent formulation for the forces transmitted by the ropes,  
 164 denoted as  $\vec{F}_{rope,ji}$ .

$$\left\{ \begin{array}{ll} F_{rope} = 0 & \text{if } l < l_{rope,2} \\ F_{rope} = k(l - l_{rope,2}) & \text{if } l_{rope,2} < l \leq \gamma(l_{rope,1} - l_{rope,2}) + l_{rope,2} \\ F_{rope} = 0 & \text{if } \gamma(l_{rope,1} - l_{rope,2}) + l_{rope,2} < l < l_{rope,1} \\ F_{rope} = k(l - l_{rope,1}) & \text{if } l \geq l_{rope,1} \end{array} \right. \quad (6)$$

165 In Equation 6, the terms  $l_{rope,1}$ ,  $l_{rope,2}$ , ...,  $l_{rope,n}$  represent the lengths of  
 166 the ropes belonging to various groups.

167 Through an iterative process, a funicular configuration is determined  
 168 where the majority of the ropes, representing the structural elements, are ten-  
 169 sioned. then, the final model undergoes slight geometric variations through  
 170 the application of a repulsive force field on the nodal masses. This causes the  
 171 nodes to move away from each other and also tensions the remaining slack  
 172 ropes. The application of this method involves incorporating into the ex-  
 173 ternal forces  $p_i$  an additional contribution  $q_i$ , which represents the repulsive  
 174 forces as defined in Equation 7.

$$q_i = -k_{rep}(l_{rope} - l_{ij}) \quad (7)$$

175 The combination of these techniques enables the grouping of structural  
 176 elements by length and, consequently, minimizes the number of beam ty-  
 177 pologies required for the completion of the gridshell.



Figure 3: Islamic cultural center in Dakar, Senegal.

### 178 **3. Case Study: The Dakar Mosque**

179 The case study outlined in this paper focuses on the roof of the Islamic  
180 Cultural Center in Dakar, Senegal. The roof of the mosque, depicted in  
181 Figure 3, is designed by the architectural firm Fragomeli+Partners [52] in  
182 collaboration with Wafai Architecture. The distinctive roof design draws  
183 inspiration from the desert dunes, harmonizing with the surrounding envi-  
184 ronment.

#### 185 *3.1. Geometry Outline*

186 The design of the Dakar mosque roof consists of a steel gridshell span-  
187 ning meters 63 meters by 56 meters. The shape is defined with the aim of  
188 creating a free span, resting solely along its perimeter, eliminating the need  
189 for intermediate pillars, as depicted in Figure 4.

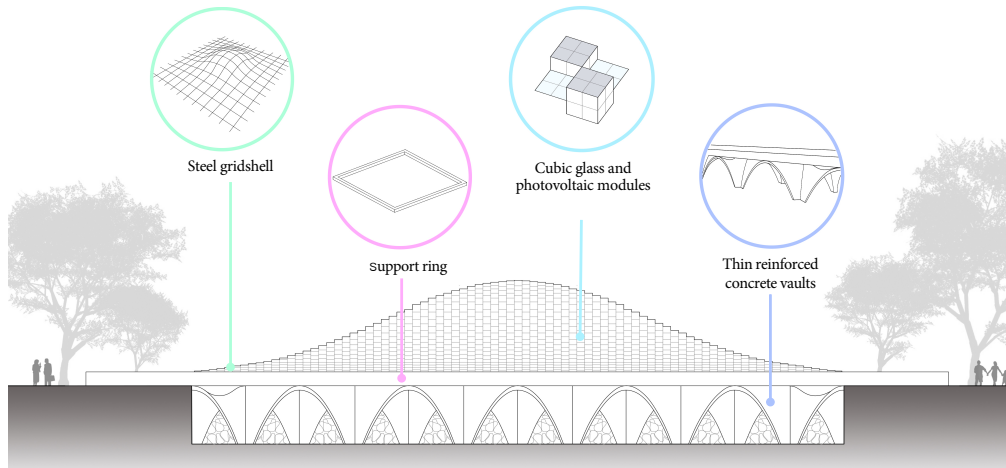


Figure 4: The concept behind the structural design of the Dakar Mosque.

190 Such geometry is primarily influenced by architectural design, introducing  
 191 specific constraints, including a distinctive curvature that distinguishes it  
 192 from conventional vaulted structures.

193 These constraints have significant implications for both the quantitative  
 194 and qualitative aspects of structural behaviour characteristics. In general,  
 195 the form-finding process involves applying the loads that act on the structure  
 196 permanently to the numerical model. This allows for defining a structural  
 197 form that is a funicular shape with respect to the loads that predominantly  
 198 act on the structure during its service life. In the specific case of the roof  
 199 of the Dakar mosque, to meet the architectural requirements, a load field  
 200 different from the one actually experienced by the structure was required  
 201 (as depicted in Figure 5). This deviation of the force field from the real  
 202 one allows for achieving the curvature inversion regions that characterize the  
 203 shape of the desert dune requested by the architects. At the same time, this  
 204 change defines a structural form that is no longer funicular with respect to

205 the actual permanent loads. Therefore, the structural form with curvature  
206 change is less-than-optimal compared to the funicular form.

207 In this paper, two distinct form-finding methodologies were employed to  
208 establish the shape of the roof. The form-finding methodologies are used to  
209 define optimal geometries, notwithstanding the above mentioned constraints.

210 In particular, the application of the Multibody Rope Approach (MRA)  
211 [48] and the improved Multibody Rope Approach (i-MRA) [46] enables the  
212 derivation of a geometry that achieves a structurally feasible configuration  
213 while preserving the initial architectural design.

214 A parametric model of the structure was defined to derive the dynamic  
215 hanging net model subject to the load system depicted in Figure 5. By  
216 applying the MRA and i-MRA techniques to solve the equilibrium of the  
217 system, two configurations were obtained, both aligned with the vision of  
218 the architectural designer.

219 The Dakar mosque case presents a clear distinction in the geometries re-  
220 sulting from the utilization of the MRA and i-MRA form-finding methods.  
221 The MRA technique results in a configuration with a structural hierarchy  
222 where the main bearing structure is the central arch (Figure 6a). The struc-  
223 tural elements required to construct the MRA shape are distinguished by 19

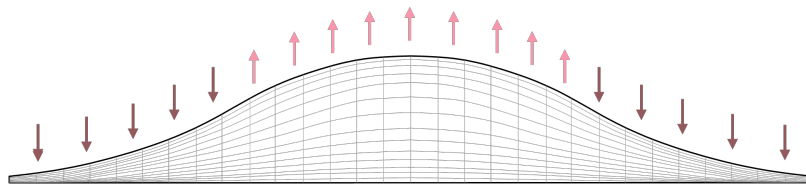


Figure 5: Force field applied to define architectural design.

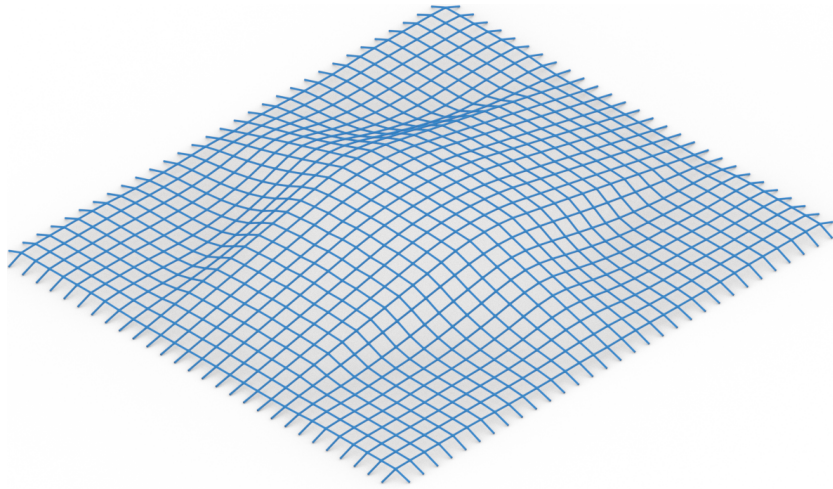
224 different lengths.

225 In comparison, the i-MRA approach creates a smoother and regular ge-  
226 ometry (Figure 6b). In this configuration, there are no self-evident structural  
227 hierarchies.

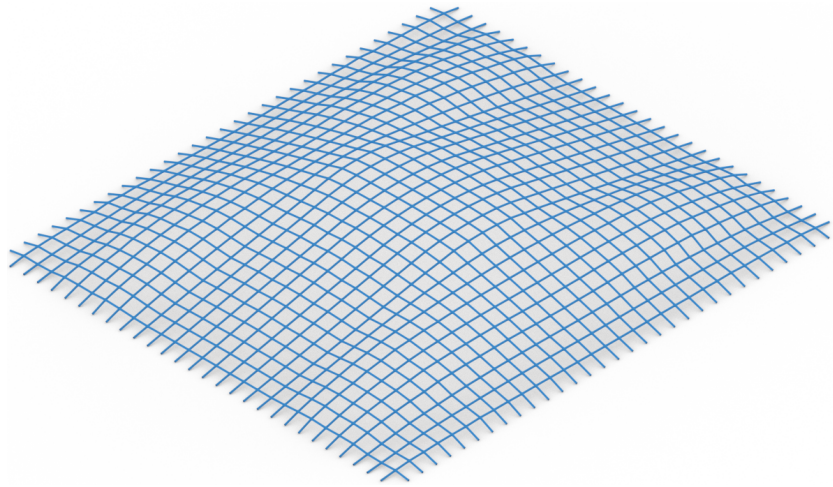
228 The i-MRA method, being designed to maximize the tensioned ropes in  
229 the hanging net model, promotes a greater degree of uniformity in the final  
230 geometry. Thus, a larger number of elements are engaged in supporting the  
231 applied loads. Consequently, only 8 element typologies define the i-MRA  
232 geometry.

233 The distribution of various types of structural elements for the two dis-  
234 tinct structural configurations is illustrated in Figure 7 through histograms.  
235 Specifically, the MRA method involves establishing a target length, which,  
236 in this case, was set at 2.00m. Implementing this method results in ob-  
237 taining 954 elements, constituting 53% of the total elements, with a length  
238 precisely matching the set target. Conversely, the remaining 47% of struc-  
239 tural elements exhibit lengths different from the target. These elements are  
240 characterized by 18 distinct lengths, ranging from a minimum of 1.78m to a  
241 maximum of 2.02m.

242 In the case of using i-MRA, it is possible to identify various target lengths.  
243 This allows grouping structural elements into a smaller number of types. In  
244 the specific case of the examined gridshell, four different target lengths were  
245 set at 1.85m, 1.90m, 1.95m, and 2.00m. In this scenario, it can be observed  
246 that 99% of structural elements have a length precisely matching one of  
247 these targets. This enabled the grouping of structural elements, reducing the  
248 number of different types from 19 to just 8.



(a) MRA



(b) i-MRA

Figure 6: Geometries obtained with the form-finding techniques.

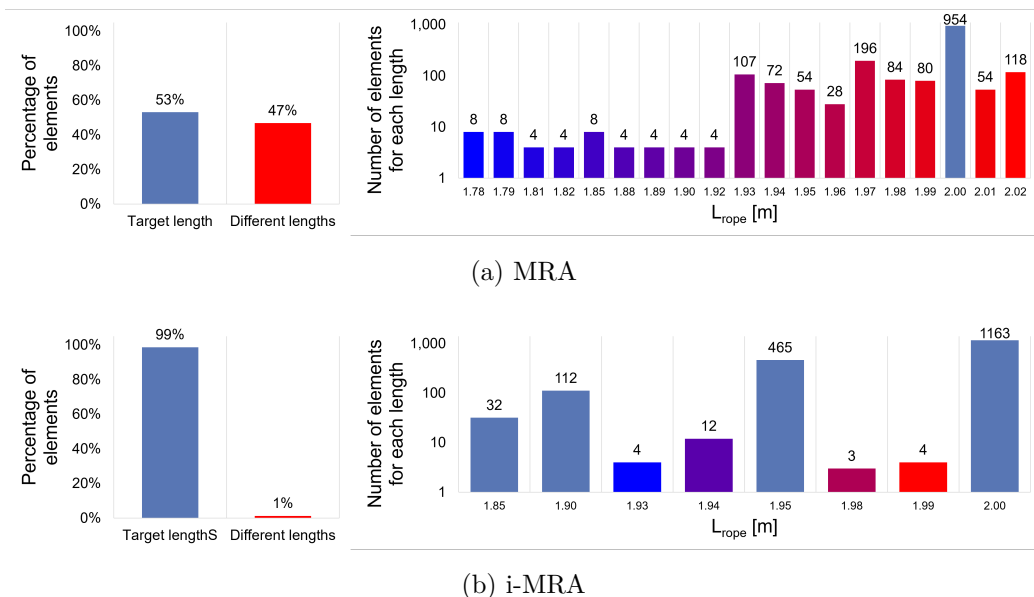


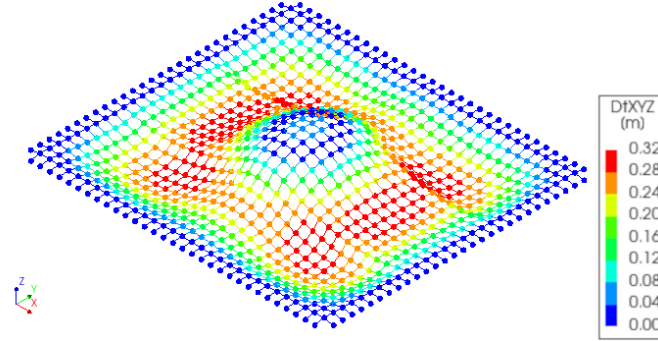
Figure 7: Structural element typologies distribution in the two structural configurations.

249 *3.2. Preliminary analyses*

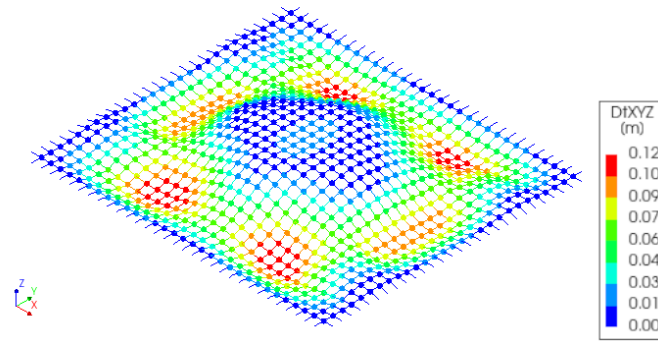
250 We employed a finite element model of the gridshell by using quadratic  
 251 three-node beam elements to investigate some fundamental structural prop-  
 252 erties. S355 steel circular hollow cross-section (CHS 19.1x8.0 mm) has been  
 253 used and the base perimeter has been modelled as a series of full constraints.

254 The structural elements constituting both structures are fully constrained  
 255 at their extremities. Choosing this type of constraint in a quadrangular mesh  
 256 gridshell is essential for establishing a hyperstatic structural scheme. Indeed,  
 257 opting for doubly hinged structural elements would result in a statically  
 258 undetermined structural scheme.

259 In order to compare the two different shapes and evaluate the effects of  
 260 geometric variations introduced by the i-MRA method, static and dynamic  
 261 analyses have been conducted using the commercial finite element software



(a) MRA



(b) i-MRA

Figure 8: Displacement due to self-weight.

262 Diana(R) (Dianafea bv, The Netherlands) [53]. Superstructure weight (solar  
 263 glass panels) has been modelled as concentrated nodal loads, in addition to  
 264 elements self-weight, considered as distributed.

265 In Figure 8 the displacement profiles obtained for the two geometries  
 266 have been reported. for both cases, the maximum displacements occur at  
 267 the curvature inversion regions of the roof. These areas represent struc-  
 268 tural weaknesses in terms of the response to vertical loading. Moreover, the

269 displacements are minimal near the supports and the central section of the  
270 structure, aligning with the expected behaviour. This emphasizes the rigidity  
271 of the central dome-like section compared to the curvature inversion zones.  
272 The behaviour validates the well-established effectiveness of vaulted struc-  
273 tures in supporting vertical loads. Indeed, the larger displacements occur in  
274 areas where the roof deviates further from the dome-like geometry.

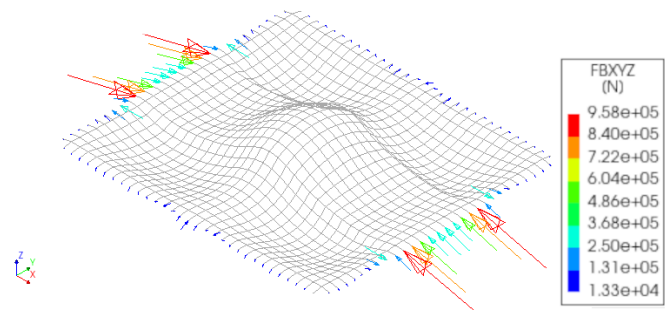
275 It is worth noting that the MRA shape exhibits a maximum displacement  
276 that is more than twice the value observed in the improved i-MRA case.  
277 However, even in the most extreme scenario, the maximum displacement  
278 remains well below 1/200th of the main roof span. Thus, the deformations  
279 observed in both cases are within acceptable limits and do not compromise  
280 the overall stability and functionality of the roof structure.

281 Figure 9 illustrates a comparative analysis of the reaction forces acting  
282 on the rigid supports.

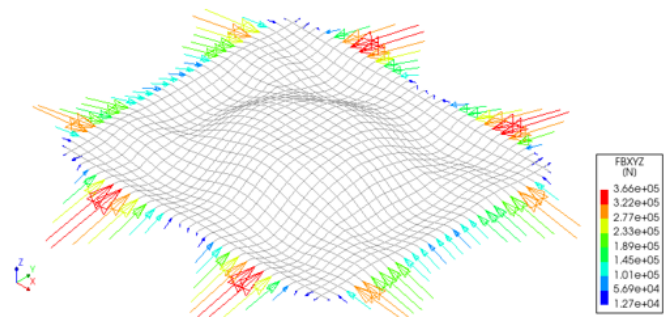
283 A clear pattern emerges in the MRA-derived geometry. It is evident  
284 the presence of an arching behaviour along the main span direction, within  
285 a narrow central zone of the roof. Significantly lower reaction forces are  
286 observed on the supports not directly influenced by the central arch. This  
287 hierarchy in the reaction distribution implies that the central arch is mainly  
288 responsible for supporting the vertical loads.

289 In contrast, the support reactions are more equally distributed and bal-  
290 anced in the i-MRA shape.

291 Arching mechanisms are present in both horizontal directions, indicating  
292 bidirectional structural behaviour. The result is a more efficient load-sharing  
293 capacity of the second geometry.

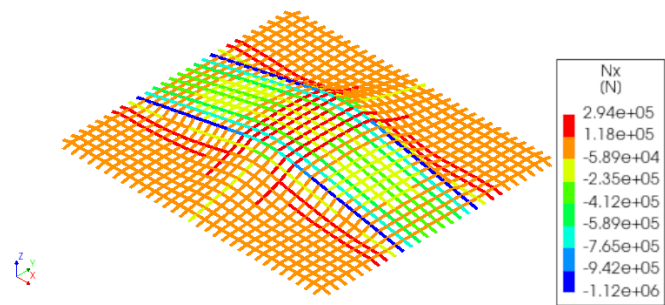


(a) MRA

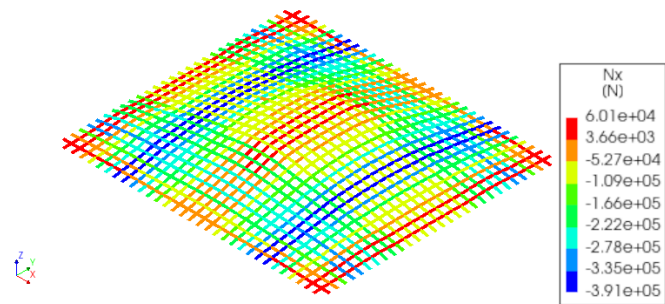


(b) i-MRA

Figure 9: Bearing reactions due to self-weight.



(a) MRA



(b) i-MRA

Figure 10: Axial force in the structural elements due to self-weight.

294 The axial force and bending moment generated by dead loads are depicted  
295 in Figure 10 and 11, respectively. An obvious difference between the two  
296 designs emerges considering the internal force patterns.

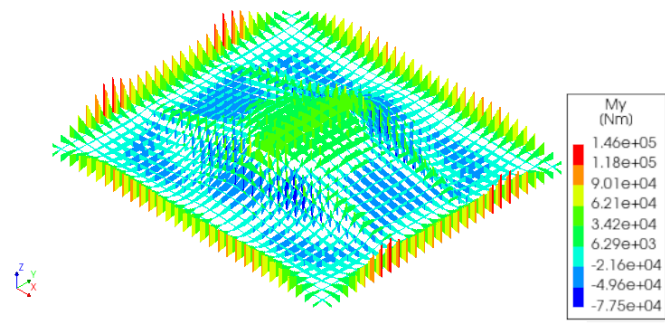
297 In the MRA structure, higher compressive forces are concentrated in the  
298 central arch. In contrast, the lateral elements are either lightly loaded or  
299 subjected to tension.

300 The colour-scaled graph effectively visualizes these distinct structural hi-  
301 erarchies. Indeed, the compressed central arch appears clearly distinguished  
302 from the secondary elements.

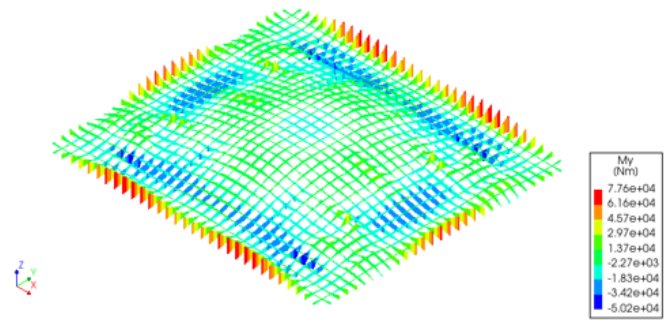
303 By opposition, the i-MRA geometry demonstrates a more balanced load  
304 distribution. Compressed arches are present in both directions. This results  
305 in a significant reduction in the extent of tensile regions compared to the  
306 MRA case. Moreover, in the MRA geometry, the maximum compressive  
307 axial forces are three times higher than those observed in the i-MRA case.  
308 Similarly, the maximum tensile axial forces are almost five times greater.

309 Figure 11 shows that the structural elements experience bending moments  
310 in both the studied geometries.

311 As expected, the magnitude of bending moments within the gridshell  
312 is relatively small. Nevertheless, it is worth noting that the i-MRA con-  
313 figuration demonstrates improved performance in this regard. Specifically,  
314 the maximum bending moment observed in the MRA case is approximately  
315 twice the corresponding bending moment in the i-MRA geometry. This dis-  
316 crepancy indicates that the i-MRA approach achieves a more balanced dis-  
317 tribution of bending moments, leading to a more efficient and optimized  
318 configuration.



(a) MRA



(b) i-MRA

Figure 11: Bending moment in the structural elements due to self-weight.

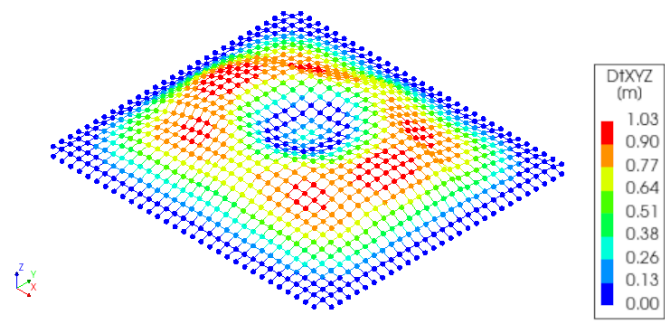
319        Considering the static analyses, it can be concluded that the i-MRA con-  
320        figuration leads to a more balanced distribution of forces. Thus, the i-MRA  
321        method demonstrates its ability to reduce structural hierarchies and enhance  
322        the overall performance of the structure.

323        Finally, the comparison between the two geometries in terms of modal  
324        frequencies and modal shapes is reported. The free vibration analysis of the  
325        gridshells shows that they have similar eigenvalues (frequencies), as reported  
326        in Table 1. However, the first eigenshapes reported in Figure 12 demonstrate  
327        two different configurations. In particular, the eigenshape associated with  
328        the MRA geometry exhibits higher deformation in the curvature inversion  
329        region. In contrast, the i-MRA configuration reveals four distinct structural  
330        zones where deformations are symmetrically distributed around the central  
331        axes.

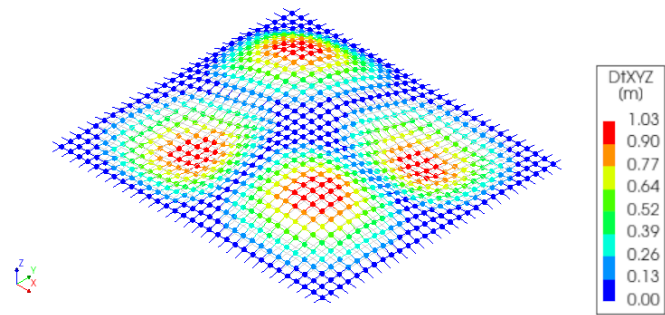
332        Furthermore, the i-MRA shape demonstrates a generalized frequency shift  
333        of approximately -0.1 Hz compared to the MRA configuration. Thus, the  
334        geometric variations introduced by the i-MRA method induce a slight mod-  
335        ification in the dynamic response, in favour of a higher overall stiffness.

<b>Mode</b>	<b>MRA</b>	<b>i-MRA</b>	<b>Mode</b>	<b>MRA</b>	<b>i-MRA</b>
1	1.03E+00	9.09E-01	11	2.11E+00	1.85E+00
2	1.07E+00	1.06E+00	12	2.21E+00	2.03E+00
3	1.08E+00	1.14E+00	13	2.25E+00	2.25E+00
4	1.18E+00	1.27E+00	14	2.52E+00	2.26E+00
5	1.81E+00	1.52E+00	15	2.75E+00	2.28E+00
6	1.83E+00	1.53E+00	16	2.82E+00	2.45E+00
7	1.88E+00	1.64E+00	17	2.89E+00	2.73E+00
8	1.92E+00	1.72E+00	18	2.96E+00	2.74E+00
9	2.05E+00	1.81E+00	19	3.04E+00	2.84E+00
10	2.11E+00	1.83E+00	20	3.06E+00	2.85E+00

Table 1: Eigen-frequencies obtained by the dynamic modal analysis.



(a) MRA



(b) i-MRA

Figure 12: Mode 1 eigen-shapes obtained with the dynamic modal analysis.

#### 336 4. Equilibrium Paths

337 Moving ahead to a deeper structural analysis, the behaviour of the two  
338 configurations of the roof with respect to stability is now investigated. The  
339 finite element program LUSAS [54] has been used and the structures have  
340 been modelled by adopting finite elements with a formulation based on the  
341 modified Timoshenko hypothesis for thick beams to the continuum theory.  
342 In the adopted formulation, the deformation due to shear is considered and  
343 the cross-sections remain plane and undistorted under deformation but do  
344 not remain normal to the beam axis. In the model, each structural element is  
345 divided into 4 different finite elements adopting a 4 nodal mesh. Analogous  
346 static schemes and material characteristics to previously described analyses  
347 have been employed.

348 In order to study the effects of the elements slenderness on the instability  
349 phenomena, three different cross-sections were investigated. The slenderness  
350 ratio  $\lambda = \sqrt{\frac{AL^2}{J}}$  is defined as a function of the cross-section area  $A$ , structural  
351 elements length  $L$  and bending inertia  $J$ . In the case studies, a standardized  
352 length was employed for the definition of the slenderness ratio. This adopted  
353 length is the predominant dimension among the elements in both structures.  
354 Specifically, this length aligns with the target length used in the MRA, es-  
355 tablished at  $L = 2.00m$ . The dimensions of the tubular cross-section have  
356 been chosen to investigate three possible slenderness ratios with  $\lambda$  values of  
357 50, 100, and 150.

358 Three different magnitudes of imperfection have been evaluated. The  
359 purpose is to assess the effects of geometric variations introduced during the  
360 construction phase on the instability phenomena.

361 In particular, the geometry is deviated from the initial one considering  
362 the shape of the first buckling mode of the structure. The buckled geometry  
363 is scaled to investigate three possible maximum deviations. Operating this  
364 way, the obtained results provide an index about the imperfection sensitivity  
365 of the initial geometry, being the eigenshapes intrinsically connected to the  
366 geometric stiffness matrix  $K_G$ . The first scenario represents the maximum  
367 accepted tolerance during the construction phase, equal to 1 cm. In the  
368 remaining scenarios, the maximum deviations have been defined as a function  
369 of the structure free span. The reason is to consider potential flaws due to  
370 the complexity of realizing such an advanced structure. The, the maximum  
371 deviations are set equal to  $L/1000 = 6cm$  and  $L/500 = 12.5cm$ .

372 Finally, two load patterns are investigated. In the first loading condition,  
373 vertical loads are uniformly distributed as concentrated forces acting on all  
374 the structural nodes. This configuration represents the most common loading  
375 condition during the structure service life. Actually, the structure is always  
376 subject to self-weight and permanent.

377 The second scenario focuses on a situation where just half of the structure  
378 is subjected to vertical loads. This condition is relevant as it represents  
379 an asymmetrical load scenario that can significantly impact the structural  
380 stability through force flows imbalance.

#### 381 *4.1. Linear Buckling*

382 In this section, the results of the linear eigenvalue buckling analysis of  
383 the two structural geometries under two different loading conditions are re-  
384 ported. The finite element models have been realized as described in Section  
385 4 by using the software LUSAS [54]. Buckling state is reached through the

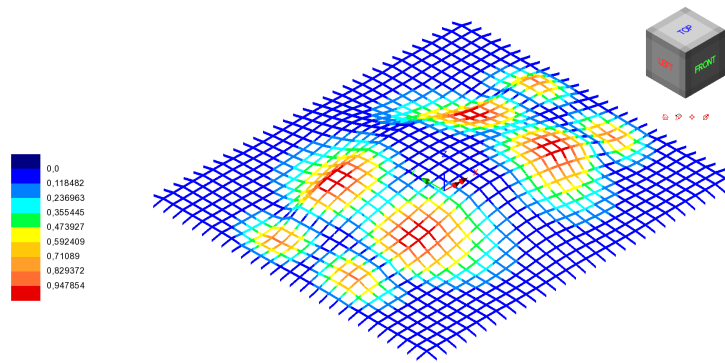
386 extraction of the eigenvalue  $\lambda$  from the problem  $\det(K - \lambda K_G) = 0$ , where  $K$   
387 and  $K_G$  are the global and geometric stiffness matrices respectively. Lanczos  
388 solution method has been used and the error norm resulted in lower than  
389  $1E - 11$  for all the computed modes.

390 The first buckling mode of the two geometries is illustrated in Figure 13  
391 and 14, respectively for the symmetrically and the asymmetrically distributed  
392 loads. The eigenshapes illustrated are the linear buckling deformations with  
393 the lowest energy. Consequently, they are utilized as a reference in investi-  
394 gating the impact of the imperfections on structural behaviour.

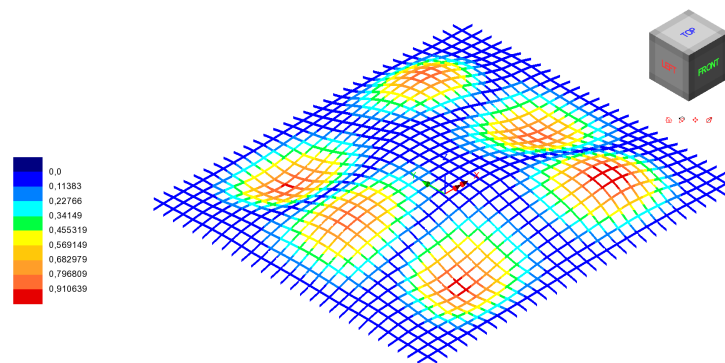
395 Finally, Table 2 presents the linear buckling eigenvalues for both struc-  
396 tural configurations, considering three different slenderness ratios. The eigen-  
397 values represent the critical load multipliers that lead to structural instabil-  
398 ity. As a result, the table reports the relative difference in load multiplier  
399 between the studied geometries.

400 The differences, denoted as  $\Delta$ , are computed as percentages. This cal-  
401 culation is performed according to Equation 8, where  $\omega_{MRA}$  represents the  
402 buckling load multipliers associated with the MRA structure, and  $\omega_{i-MRA}$   
403 corresponds to those related to the i-MRA structure.

$$\Delta[\%] = 100 \left( 1 - \frac{\omega_{MRA}}{\omega_{i-MRA}} \right) \quad (8)$$

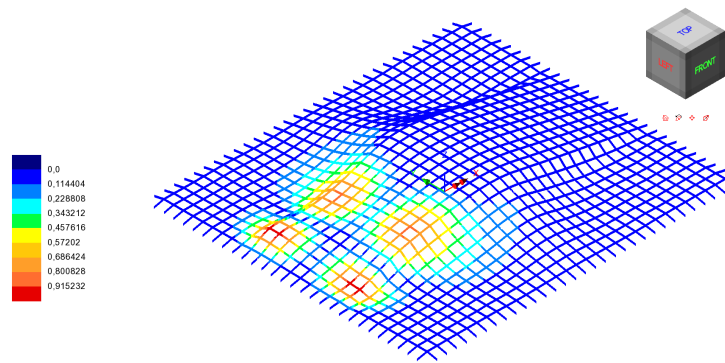


(a) MRA

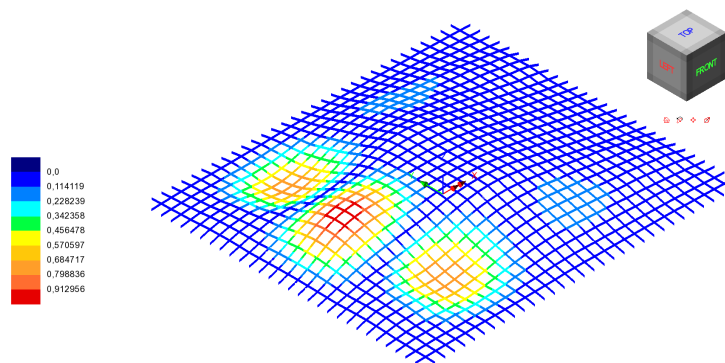


(b) i-MRAd

Figure 13: Mode 1 eigen-shapes obtained with linear buckling analysis on the symmetrical loaded structures.



(a) MRA



(b) i-MRA

Figure 14: Mode 1 eigen-shapes obtained with linear buckling analysis on the asymmetrical loaded structures.

$\lambda$	Mode	MRA		i-MRA		Difference %	
		Sym	Asym	Sym	Asym	Sym	Asym
<b>50</b>	<b>1</b>	1935.7	3041.6	1204.1	1412.8	37.8%	53.6%
	<b>2</b>	1973.3	3454.8	1305.2	1502.8	33.9%	56.5%
	<b>3</b>	2194.0	3795.0	1351.1	1664.2	38.4%	56.1%
<b>100</b>	<b>1</b>	94.8	156.0	72.7	91.1	23.3%	41.6%
	<b>2</b>	97.8	174.3	77.3	94.4	21.0%	45.8%
	<b>3</b>	104.5	192.4	78.3	100.2	25.1%	47.9%
<b>150</b>	<b>1</b>	17.4	27.6	14.0	18.6	19.5%	32.6%
	<b>2</b>	18	30.6	14.2	18.9	21.1%	38.2%
	<b>3</b>	18.9	33.1	14.4	19.8	23.8%	40.2%

Table 2: Dakar Mosque linear buckling eigenvalues (load multipliers) considering symmetric and asymmetric load condition.

#### 404 4.2. GNIAs

405 To investigate the post-buckling behaviour of the structure, an incremen-  
406 tal load Geometrically Nonlinear Analysis with Imperfections (GNIA) has  
407 been conducted. The co-rotational formulation [55] has been employed to  
408 account for geometrical non-linearity. Body strains have been derived at  
409 each step from the rigid body motion, considering both the element and the  
410 force vector rotation. Step by step, therefore the geometric stiffness matrix  
411 has been computed taking into account the elements rotations and deforma-  
412 tions, and consequently the equilibrium paths have been outlined.

413 Imperfections have been modeled using the first linear buckling mode  
414 associated with each loading condition. The eigenshapes have been scaled to  
415 have a maximum deviation from the original geometry as defined in 4.

416 The GNIA has been performed to determine the loading path considering  
417 both the symmetrical and the asymmetrical loading patterns. The analyses  
418 have been then repeated in order to asses the effects of geometrical imper-  
419 fections and changes in the structural element slenderness.

420 The initial study was conducted by varying the slenderness of the struc-  
421 tural components while maintaining constant the imperfection. The displacement-  
422 load paths of the gridshells with a maximum geometric imperfection of 1 cm  
423 are presented in Figure 15. In the graphs, the sum of the structural element  
424 reactions  $F_z$  to the applied loads  $Q$  is shown as a function of the displacement  
425 of the central structural node  $\delta$ . Results are reported for both geometries,  
426 considering two distinct loading conditions and three different element slen-  
427 derness  $\lambda$ . Furthermore, to emphasize the relationship between the linear  
428 buckling and the nonlinear load path, the linear buckling load is depicted as

429 a dashed horizontal line.

430 In Table 3, the results of the analyses are reported in terms of Euler load  
431  $Q_{CR}$ , elastic buckling load  $Q_{EB}$  and Yielding load  $Q_y$ . These loads represent  
432 critical situations in terms of structural integrity. For this reason in the Table  
433 is also reported the design load  $Q_d$  defined as  $\min\{Q_{CR}, Q_{EB}, Q_y\}$ .

434 In the table, a comparison is made between the two structures, consid-  
435 ering the relative difference for each critical loading situation. Specifically,  
436 the differences, denoted as  $\Delta_Q$ , are reported as percentages and calculated  
437 as per Equation 9, where  $Q$  represents  $Q_{CR}$ ,  $Q_{EB}$ ,  $Q_y$ , or  $Q_d$ , depending on  
438 the specific situation under consideration.

$$\Delta_Q[\%] = 100 \left( 1 - \frac{Q_{MRA}}{Q_{i-MRA}} \right) \quad (9)$$

439 It is worth noting that the numbers in the table represent vertical loads  
440 that are deemed positive when acting in the direction of gravity acceleration.

441 It should be highlighted that the yielding load  $Q_y$  is the lower critical load  
442 for the MRA case, considering all the possible slenderness values. Thus, the  
443 stress distribution considering the minimum between the Euler load  $Q_{CR}$  and  
444 the elastic buckling load  $Q_{EB}$  is computed. The stress distribution related  
445 to the structures with a slenderness  $\lambda = 100$  is reported in Figures 16 and  
446 17.

447 Moreover, the displacements to this loading condition with a slenderness  
448  $\lambda = 100$  is reported in Figures 18 and 19. The displacement configuration is  
449 key to understand the structural behaviour in the critical loading conditions.

450 Finally, the displacement-load paths have been extracted for increasing  
451 magnitude of imperfections. The first linear buckling eigen-shape has been

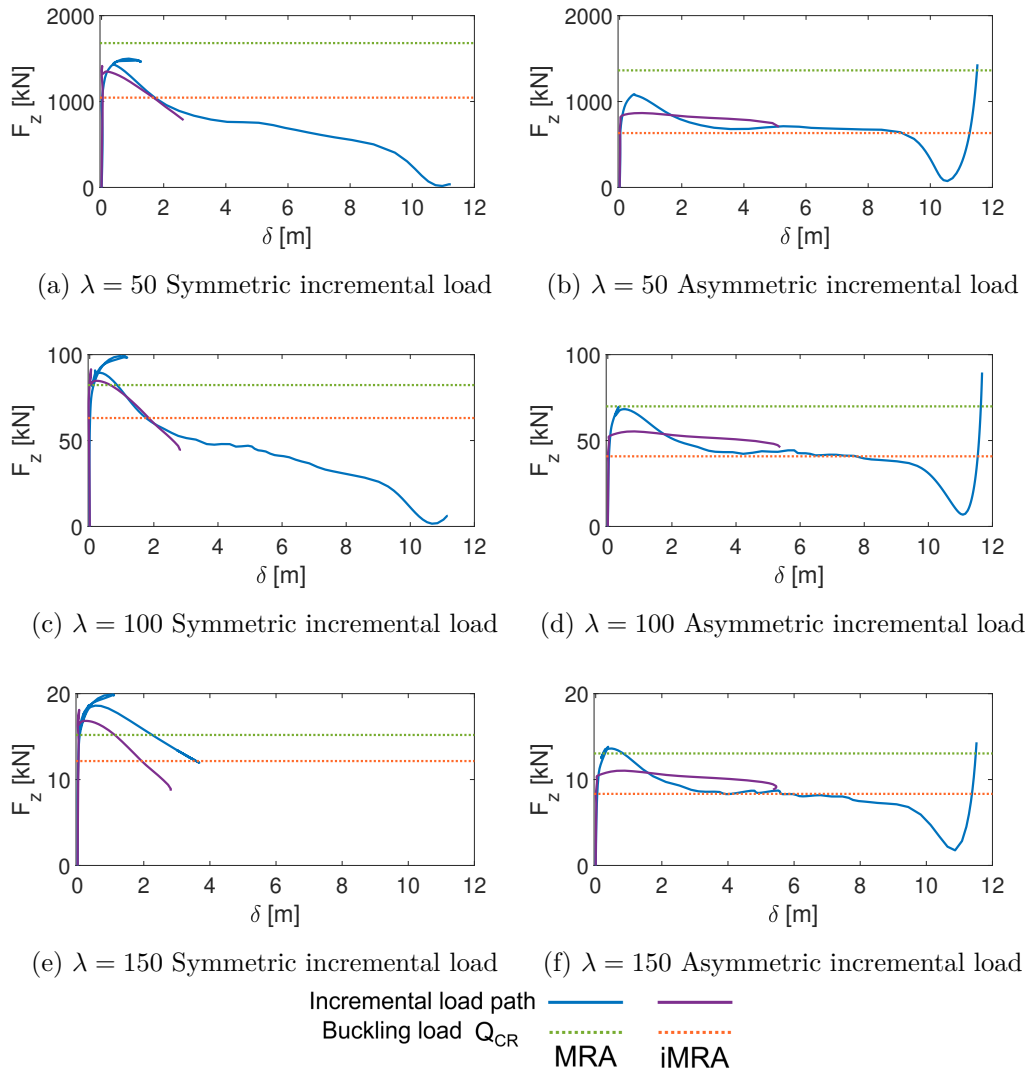
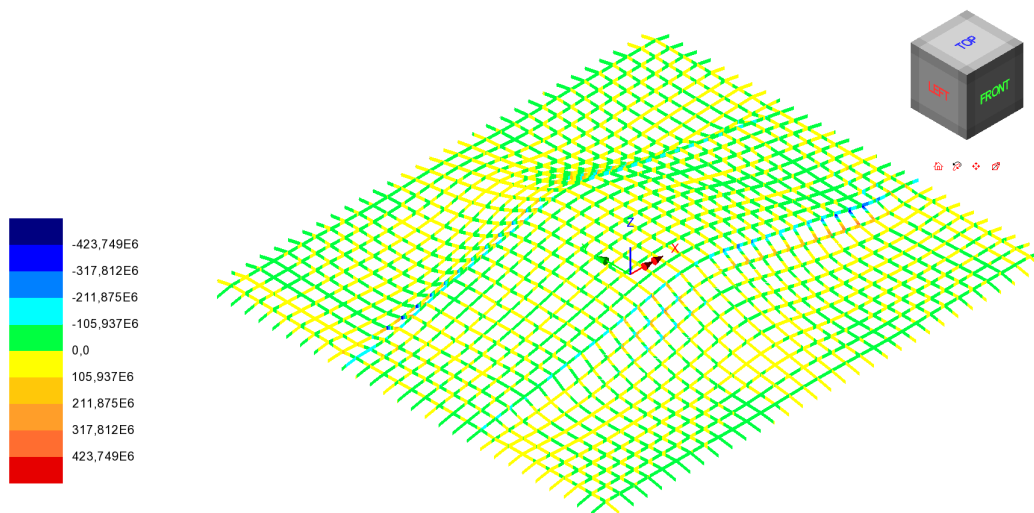


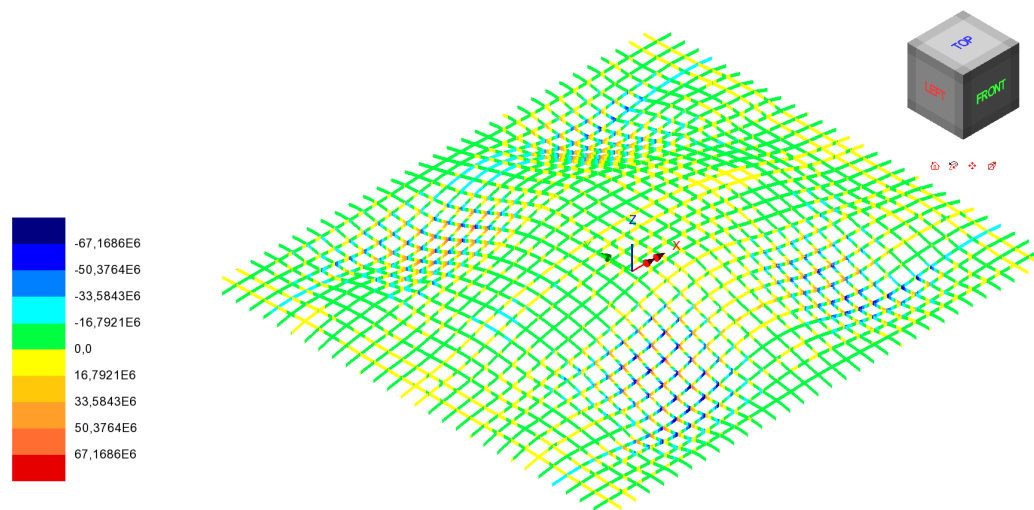
Figure 15: Displacement-load path of the central node of the Dakar mosque gridshell considering an incremental load geometric nonlinear analysis.

	$\lambda$	MRA		iMRA		Difference [%]	
		Sym	Asym	Symm	Asym	Sym	Asym
$Q_{CR}$	<b>50</b>	1680.1	1362.6	1045.2	632.9	37.8%	53.6%
$Q_{EB}$		1428.2	1084.3	1349.0	866.0	5.5%	20.1%
$Q_y$		<b>607.6</b>	<b>343.2</b>	<b>925.3</b>	<b>483.8</b>	-52.3%	-41.0%
$Q_{CR}$	<b>100</b>	82.3	69.9	<b>63.1</b>	<b>40.8</b>	23.3%	41.6%
$Q_{EB}$		90.9	68.2	84.8	55.3	6.7%	18.9%
$Q_y$		<b>75.0</b>	<b>43.2</b>	83.2	51.3	-11.0%	-18.7%
$Q_{CR}$	<b>150</b>	<b>15.2</b>	13.0	<b>12.2</b>	<b>8.3</b>	20.0%	36.1%
$Q_{EB}$		18.6	13.6	16.8	11.0	9.3%	19.1%
$Q_y$		<b>15.2</b>	<b>10.8</b>	16.5	10.1	-8.5%	6.6%
$Q_d$	<b>50</b>	607.6	343.2	925.3	483.8	-52.3%	-41.0%
	<b>100</b>	75.0	43.2	63.1	40.8	15.9%	5.6%
	<b>150</b>	15.2	10.7968	12.2	8.3	20.1%	22.8%

Table 3: Critical instability loads in [N] considering the structural element slenderness  $\lambda = [50, 100, 150]$  and both symmetrical and asymmetrical load patterns.

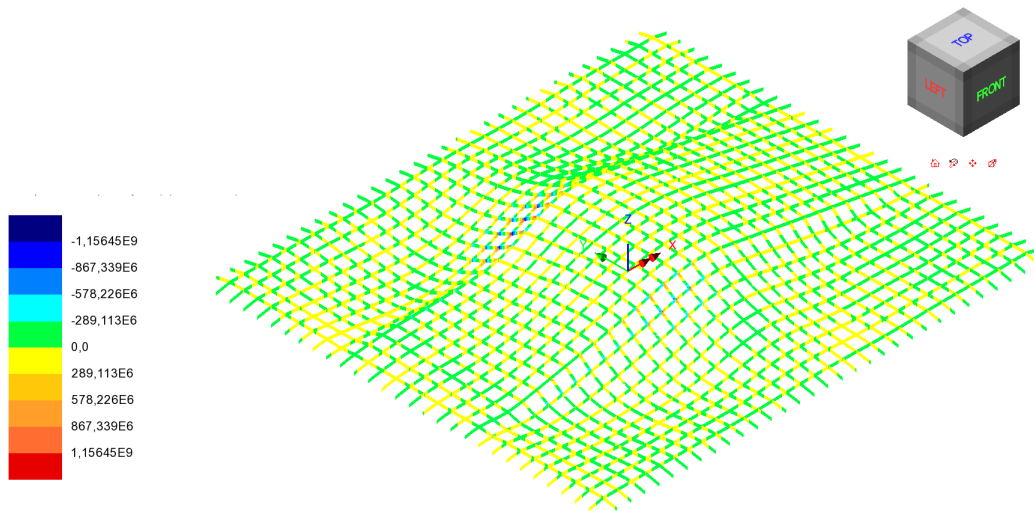


(a) MRA:  $|\sigma_z| = 424MPa$

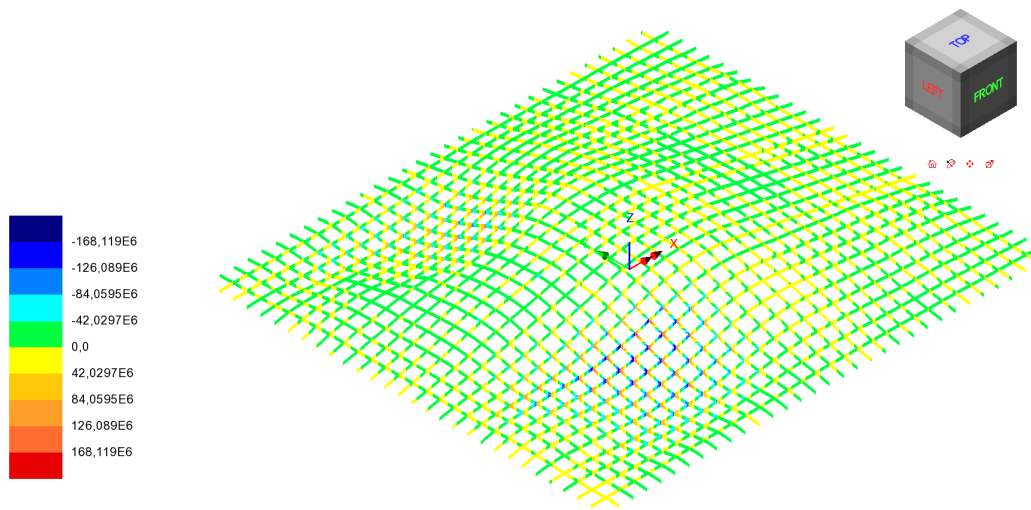


(b) i-MRA:  $|\sigma_z| = 67MPa$

Figure 16: Stress distribution [MPa] related to the minimum critical load between Euler load  $Q_{CR}$  and the elastic buckling load  $Q_{EB}$  in the symmetrical load configuration.

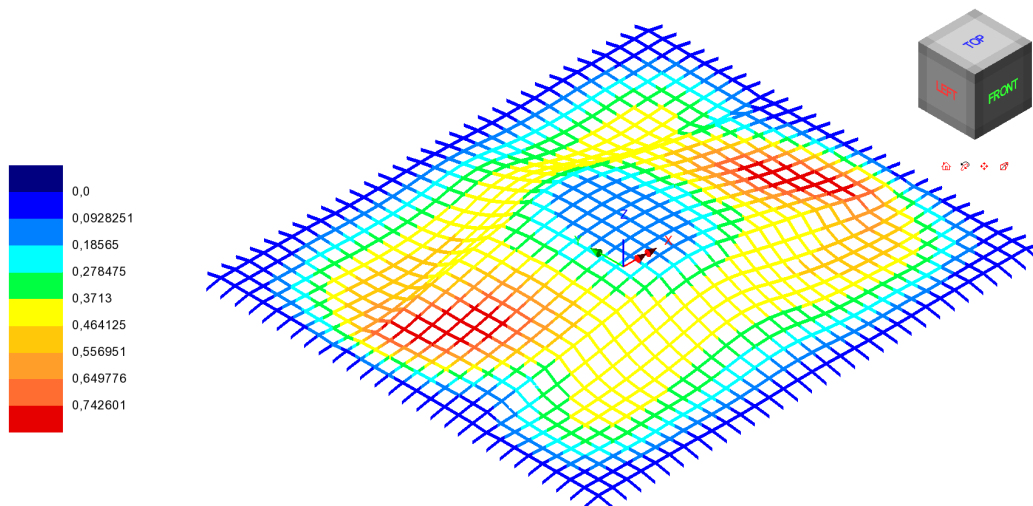


(a) MRA:  $|\sigma_z| = 1157MPa$

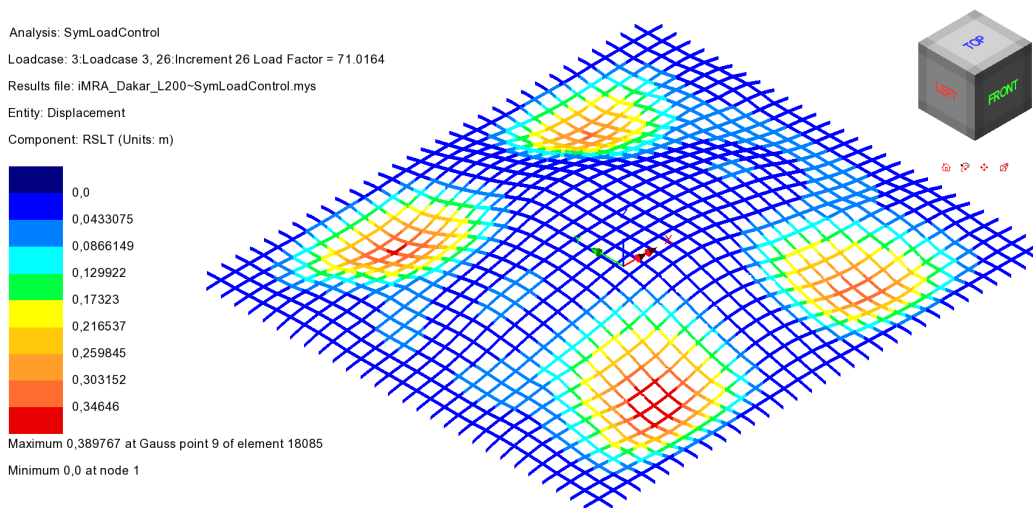


(b) i-MRA:  $|\sigma_z| = 168MPa$

Figure 17: Stress distribution [MPa] related to the minimum critical load between Euler load  $Q_{CR}$  and the elastic buckling load  $Q_{EB}$  in the asymmetrical load configuration.

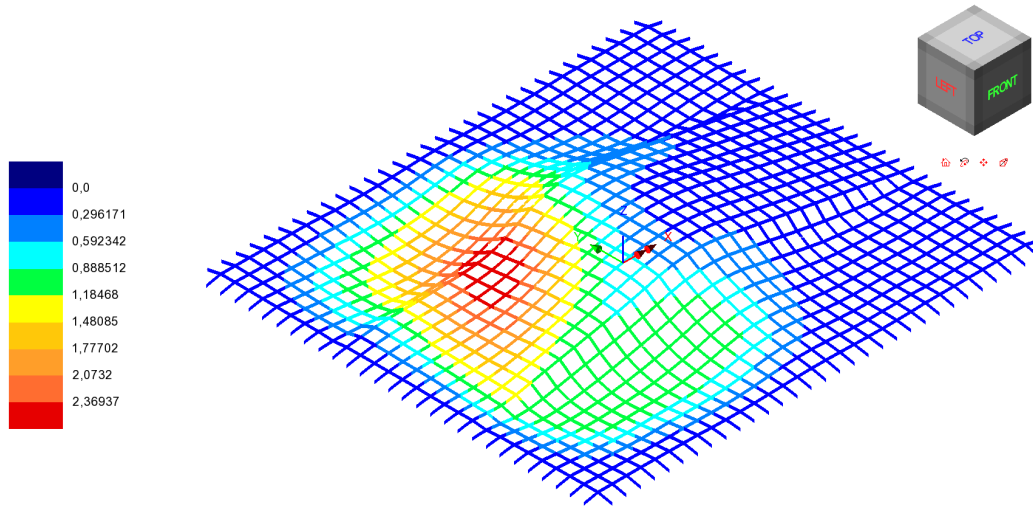


(a) MRA

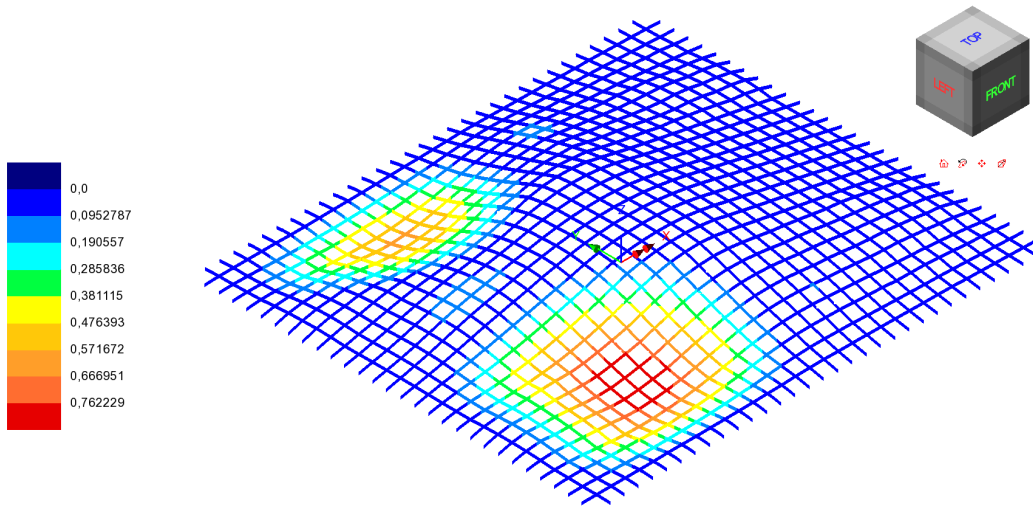


(b) i-MRA

Figure 18: Displacement distribution [m] related to the minimum critical load between Euler load  $Q_{CR}$  and the elastic buckling load  $Q_{EB}$  in the symmetrical load configuration.



(a) MRA



(b) i-MRA

Figure 19: Displacement distribution [m] related to the minimum critical load between Euler load  $Q_{CR}$  and the elastic buckling load  $Q_{EB}$  in the asymmetrical load configuration.

452 used to define the geometrical imperfections. The results for maximum de-  
453 viations of  $1\text{cm}$ ,  $L/1000 = 6\text{cm}$  and  $L/500 = 12.5\text{cm}$  are shown in Figure  
454 20. In this case, the slenderness of the structural elements has been kept  
455 constant and equal to  $\lambda = 100$ .

456 It should be highlighted that the monotonously increasing branch shown  
457 in Figure 20(a) represents an unstable equilibrium path. It is essential to  
458 note that this equilibrium path can only be realized in a numerical solu-  
459 tion. In practice, the behaviour of the structure will always adhere to the  
460 stable equilibrium path. Consequently, during numerical simulations, after  
461 attempting to follow the unstable path, the system will return to find equi-  
462 librium configurations associated with the stable path.

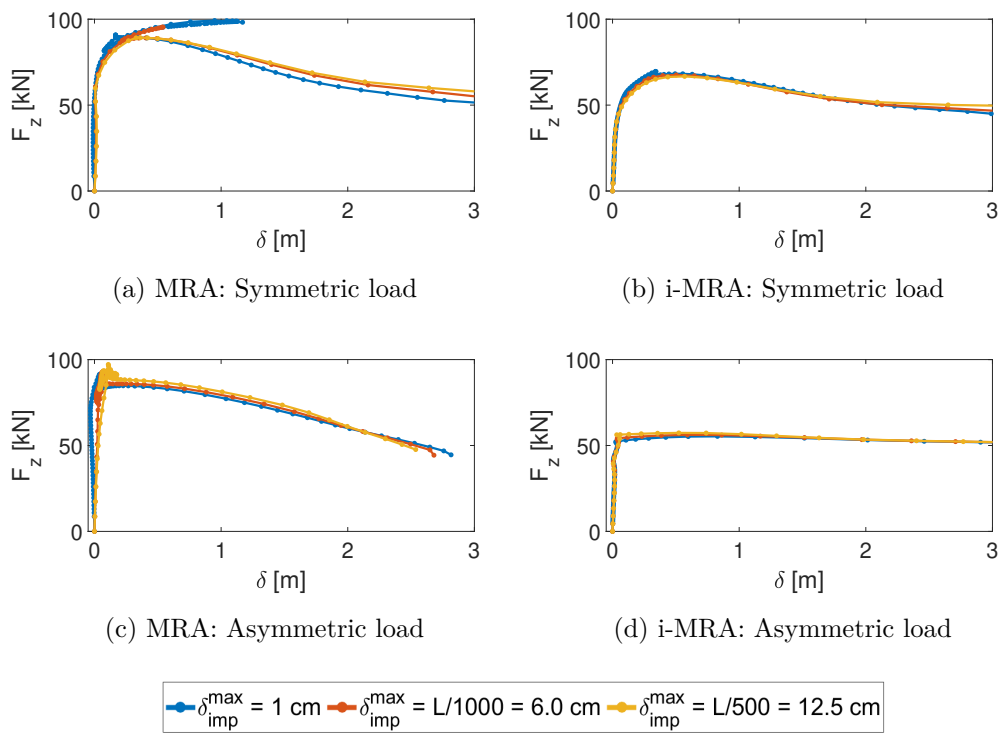


Figure 20: GNIA considering three magnitudes of imperfections on the Dakar mosque roof with  $\lambda = 100$ .

463 *4.3. GMNIAs*

464 In Section 4.2, the GNIA findings demonstrates that the yielding load  $Q_y$   
465 and instability loads ( $Q_{CR}$  and  $Q_{ED}$ ) have the same order of magnitude. The  
466 structural elements in some regions of the structure may reach the yielding  
467 limit before the development of the instability phenomenon.

468 It is not possible to neglect the material non-linearity while investigating  
469 the interaction between yielding and instability. Indeed, the yielding of some  
470 structural components may alter the displacement field that is considered  
471 during the GNIA.

472 The elasto-plastic steel S355 constitutive law is added into the FEM to  
473 evaluate the effect of material non-linearity. An incremental load geomet-  
474 rically and material nonlinear with imperfection analysis (GMNIA) is per-  
475 formed on the structures with  $\lambda = 100$ . Initially, the structures with a  
476 maximum deflection equal to  $1cm$  are investigated. The analysis is repeated  
477 for both the geometries and load configurations. In Figure 21, the GMNIA  
478 results are compared to those obtained with the GNIA. A dashed red line  
479 represents the  $F_z$  value at which the yielding limit is exceeded. It is there-  
480 fore possible to investigate how the structural behavior varies after passing  
481 yielding limit.

482 Finally, the analysis has been repeated for different magnitudes of imper-  
483 fection to study the effects due to the combination of material non-linearity  
484 and geometric deviations. The resulting load paths are reported in Figure  
485 22.

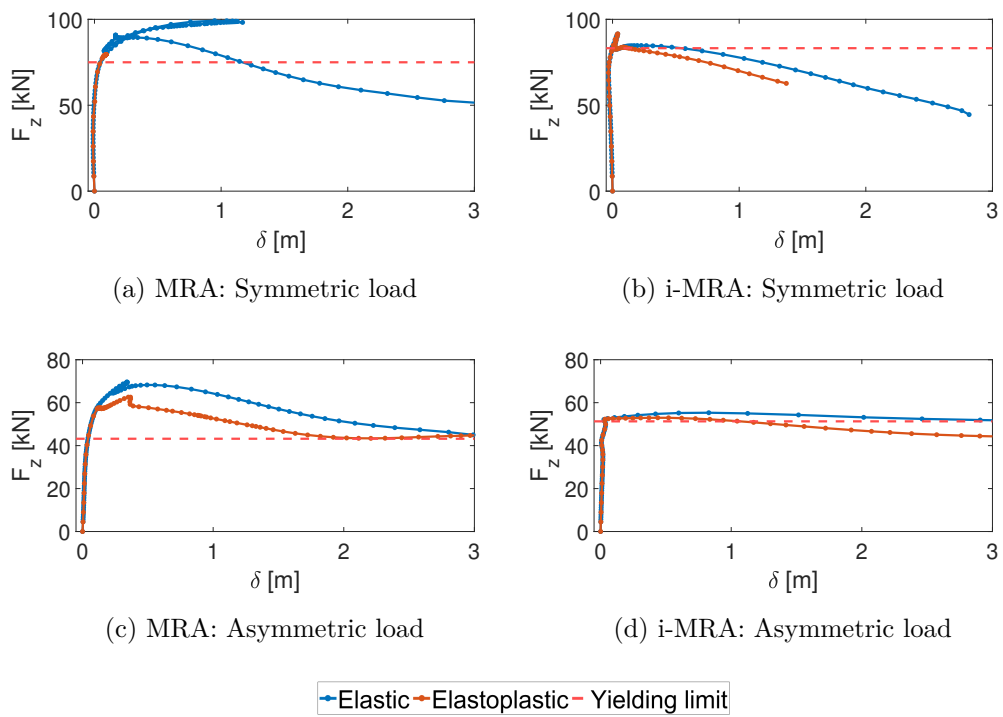


Figure 21: Comparison between GMNIAs with elastic and elasto-plastic material,  $\lambda = 100$ .

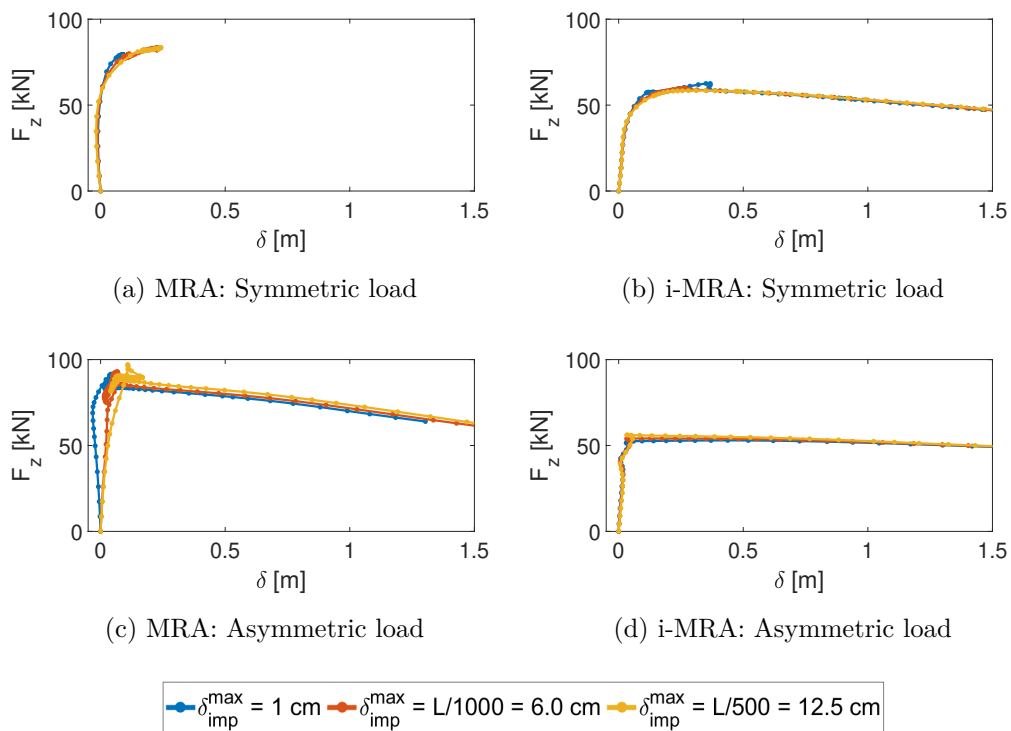


Figure 22: GMNIA considering elasto-plastic material and three magnitudes of imperfections on the Dakar mosque roof with  $\lambda = 100$ .

## 486 5. Discussion

487 The previous sections illustrate the results of the analyses carried out on  
488 the roof of the Dakar mosque. These studies investigated the influence of  
489 geometry, structural elements slenderness, material non-linearity, and imper-  
490 fections on structural behaviour. The findings gathered from the analyzed  
491 data are detailed in the following.

### 492 5.1. Effect of geometry: MRA vs. iMRA

493 The geometries generated using the two form-finding methodologies differ  
494 significantly in terms of response with respect to the instability phenomena.

495 The linear buckling analysis produces eigenshapes that have distinct de-  
496 formations patterns. As a result, the linear instability behaviour in the two  
497 geometries is qualitatively different, as seen in Figure ?? and 14. The MRA  
498 case shows typical linear buckling of lowered arches, with the main defor-  
499 mations around the haunches of the central arch. This holds for both the  
500 studied load patterns. Instead, the buckling deformation in the i-MRA shape  
501 engages a greater number of structural members. Its behavior is similar to  
502 that seen in flat shell stability, or form resistance structures in general.

503 The eigenvalues provided in Table 2 reveal that the MRA structure per-  
504 forms better overall in terms of linear buckling resistance. Indeed, the critical  
505 loads  $Q_{CR}$  are greater than those displayed by the i-MRA geometry for all  
506 the investigated scenarios. As introduced, this result can be invalidated by  
507 nonlinear analyses; when this happens, it generally implies for such cases the  
508 presence of catastrophic postbuckling behavior. However, the results of lin-  
509 ear buckling analyses are essential to carry out the GNIA. In details, they

510 have been used to define imperfection patterns, load patterns and factors.

511 In Section 4.2, the results of the GNIA for both the geometric configura-  
512 tions have been reported. The analyses allows for a meaningful comparison  
513 in terms of post-buckling behaviour. In the symmetrical load situation, the  
514 behaviour of the two geometries is remarkably similar, as evident in Figure  
515 15. Even for the critical load  $Q_{CR}$  is considerably different, the symmetrical  
516 equilibrium load path are similar. The differences in terms of elastic buckling  
517 load  $Q_{EB}$  are smaller than 10%, as reported in Table 3. The postbuckling  
518 behaviour induced by the symmetric load pattern can classified as benign,  
519 since  $Q_{EB} > Q_{CR}$  with the exception of the MRA case with  $\lambda = 150$ . This  
520 means that the equilibrium paths can be assimilated to those represented in  
521 Figure 1b).

522 It is worth noting that the differences in postbuckling behaviour become  
523 more pronounced when investigating the asymmetric loading case. In gen-  
524 eral, the critical loads are smaller when the load is distributed asymmetri-  
525 cally. Consequently, the asymmetrical loading condition results to be the  
526 critical one. The equilibrium paths in Figure 15 describe a very different  
527 exhibited behaviour by the two geometries.  $Q_{EB} > Q_{CR}$  in the i-MRA struc-  
528 ture indicates a benign postbuckling. In the MRA shape, however,  $Q_{EB}$  is  
529 generally smaller than  $Q_{CR}$ . The structures exhibit catastrophic postbuck-  
530 ling behaviour. As a result, the equilibrium paths can be associated to the  
531 one represented in Figure 1a).

532 Figure 23 provides insights of the structure under asymmetrical loading  
533 conditions with  $\lambda = 100$ . The graph illustrates the evolution of displacements  
534 and stresses during the incremental load GNIA.

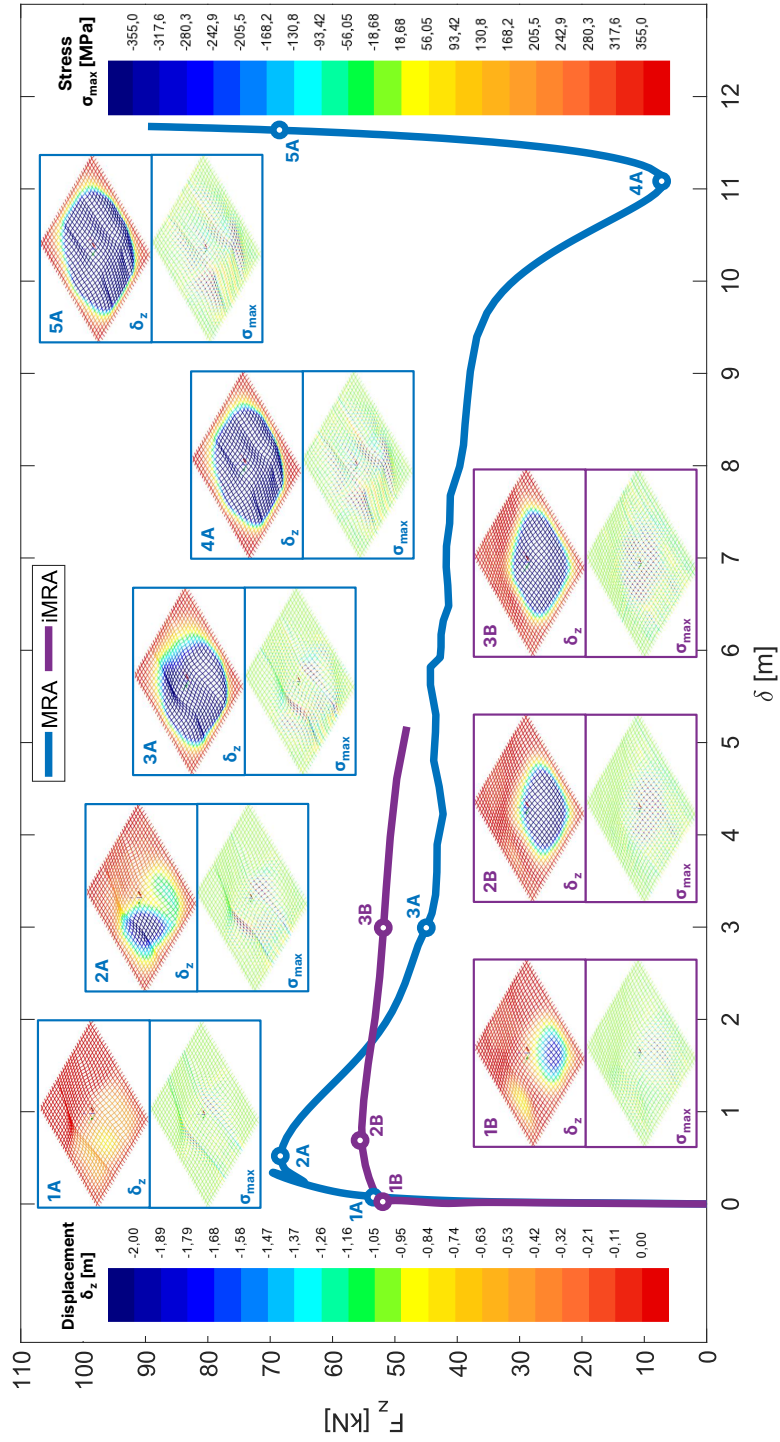


Figure 23: Postbuckling behaviour of the Dakar Mosque

535 In the initial phase, displacement increases approximately linearly as the  
536 load increases in both of the studied structures. Snap-through occurs in  
537 the MRA case with a sudden transition from peak configuration 2A to final  
538 configuration 5A. The collapse is catastrophic with a complete overturning  
539 of the roof.

540 Despite a reduced peak load  $Q_{EB}$ , the i-MRA configuration exhibits be-  
541 nign post-buckling performance. In the post-peak phase, the structure ex-  
542 hibits a plateau. The partial collapse experienced in the initial stages grad-  
543 ually extends. There is a smooth transition between a partial and global  
544 collapse.

545 The load-displacement graph in Figure 23 can be used to trace the col-  
546 lapse mechanism. The stress graphs illustrate that there is a larger concentra-  
547 tion of stresses in the MRA case. As a result, the source of this concentration  
548 has been investigated, and the maximum axial force  $N_{max}$  and bending mo-  
549 ment  $M_{max}$  have been computed for each load increment. Figures 24, 25, and  
550 26 illustrate the values of  $N_{max}$  and  $M_{max}$  as the load increases. The Figures  
551 refer to structures with slenderness  $\lambda$  50, 100 and 150, respectively. In any  
552 case, it is clear that the maximum axial force and bending moment are always  
553 greater in the case of MRA than those on the i-MRA structure. This con-  
554 centration implies that yielding will occur in specific structural nodes before  
555 the critical buckling load is attained.

556 In order to investigate the interaction between yielding and instability  
557 phenomena, the elasto-plastic constitutive law has been introduced in the  
558 model. In Figure 21 the elastic model is compared with the GMNIAs results.

559 Implementation of the elasto-plastic constitutive law to the MRA model

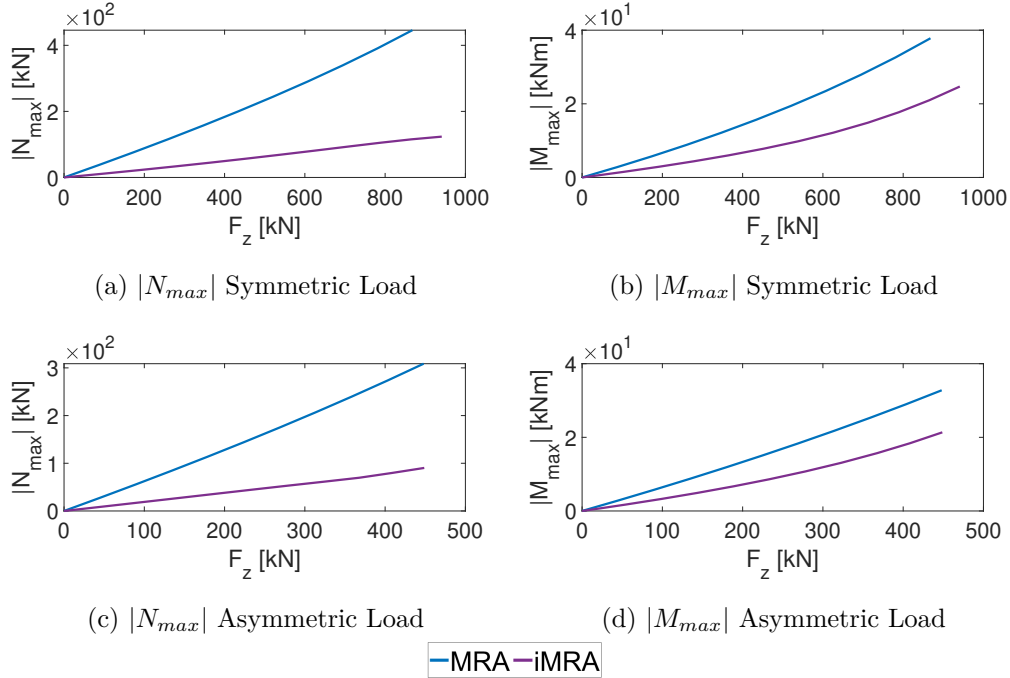


Figure 24: Maximum absolute axial force and bending moment in the structural elements with  $\lambda = 50$  considering to increase the load to perform the GNIA.

560 reduces the linear buckling load, then results in  $Q_{EB} < Q_{EP}$ . In the case  
 561 of i-MRA, yielding is observed in proximity to the elastic peak. As a re-  
 562 sult, the elastic buckling load  $Q_{EB}$  is roughly equivalent to the elastic-plastic  
 563 buckling load  $Q_{EP}$ . In either case, a reduction in stiffness is observed in the  
 564 postbuckling tails.

### 565 5.2. Effect of member slenderness

566 Structural members' slenderness effects have been investigated by analysing  
 567 structures with three different  $\lambda$ . In particular, Table 2 reports the eigenval-  
 568 ues for the linear buckling analysis of structures characterised by  $\lambda$  equal to  
 569 50, 100 and 150. The difference between the two structural configurations is

570 minimized when  $\lambda$  increases, as seen in the Table. Local instability phenom-  
571 ena become more prevalent than global ones when slenderness rises. As a  
572 result, the effects of the geometric differences between the two configurations  
573 tend to be less significant.

574 Considering the GNIA, In Figure 15 the same qualitative behaviour is  
575 observed for all the slenderness ratios. The major differences are related to  
576 the yielding load  $Q_y$ . In Table 3, the design load results to be the yielding  
577 one in the cases with thicker structural elements. For the i-MRA case, as the  
578 slenderness increases the instability becomes the most critical phenomenon.

579 At the same time, yielding seems to be the main challenge for all the  
580 slenderness ratios, if we consider the MRA geometry. This is a consequence  
581 of the stress concentration observed in Figures 16 and 17.

### 582 5.3. Effect of material yielding

583 The GNIA results show that the yielding load  $Q_y$  is the most critical in  
584 some of the investigated structural configurations. In particular, in the MRA  
585 case, it is observed that the  $Q_y < Q_{EB}$  for all the slenderness ratios. In order  
586 to understand the effects related to material non-linearity, the elasto-plastic  
587 constitutive law has been included in GMNIAs. The results of the GMNIAs  
588 are shown in Figure 21, demonstrating that the yielding of the structural  
589 nodes can alter the structural behaviour.

590 In the MRA case is evident that after the yielding the global structural  
591 stiffness decreases. As a result, it is observed that  $Q_{EP} < Q_{EB}$ .

592 Considering the MRA geometry, the material non-linearity effects are  
593 exploited only on the postbuckling load paths. The stiffness decreases only  
594 after the peak and  $Q_{EP} \approx Q_{EB}$ .

595 The yielding of some structural nodes reduces the peak load in compar-  
596 ison to the MRA situation. In contrast, there is no significant effect on the  
597 behaviour of the i-MRA structure. Although in GNIAs the peak load of the  
598 MRA case was much higher, the concentration of stresses produced by such  
599 geometry results in the yielding of structural nodes prior to the establish-  
600 ment of buckling mechanisms. As a result of introducing an elasto-plastic  
601 constitutive law, the disparity between the two peak loads relative to the two  
602 analyzed geometries is greatly reduced due to local plastic flows.

#### 603 *5.4. Imperfection Sensitivity*

604 The structure defined by  $\lambda = 100$  was examined by taking into account  
605 different levels of deviation from the basic geometry. The geometric imper-  
606 fections were introduced by scaling the deformed relative to the first linear  
607 buckling mode. The resulting geometry was scaled so that the maximum  
608 distance between the deformed and undeformed configurations was 1 cm,  
609  $L/1000 = 6.0cm$ , and  $L/500 = 12.5cm$ .

610 A geometrically nonlinear analysis with imperfection was conducted on  
611 the deformed configurations. The results of this analysis are shown in Figure  
612 20.

613 The graphs indicate the presence of geometric deviations causes minor  
614 changes to the equilibrium paths. Deviations from the initial geometry ap-  
615 pear to increase the peak load. This phenomena has been widely documented  
616 in the literature [56].

617 Finally, the analyses have been repeated after introducing material non-  
618 linearity of the material; results are reported in Figure 22. Also in this case  
619 a slight improvement in the peak load is observed as the geometric deviation

620 increases.

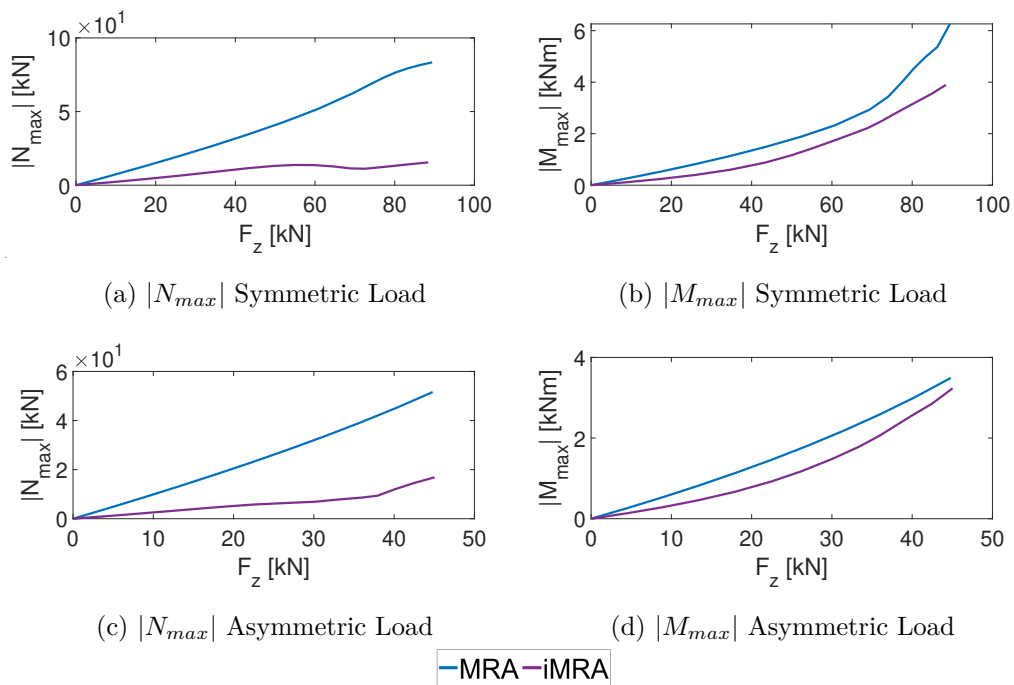


Figure 25: Maximum absolute axial force and bending moment in the structural elements with  $\lambda = 100$  considering to increase the load to perform the GNIA.

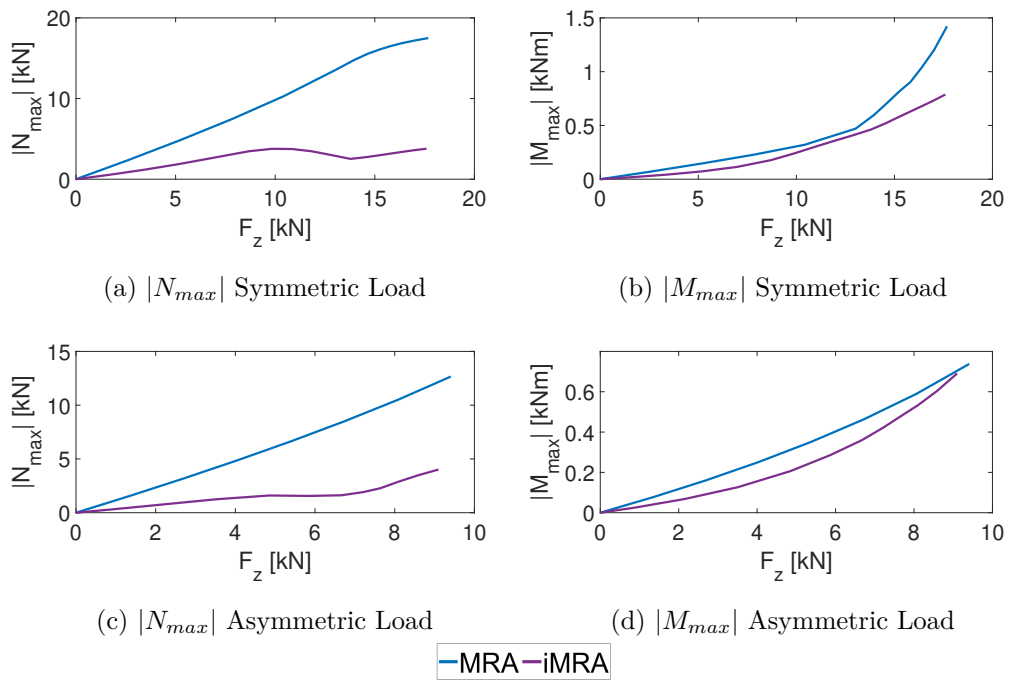


Figure 26: Maximum absolute axial force  $|N_{max}|$  and bending moment  $|M_{max}|$  in the structural elements with  $\lambda = 150$  considering to increase the load to perform the GNIA.

## 621 **6. Conclusions**

622 In this paper, a comparative analysis of two form-finding methods with  
623 respect to structural stability is provided. The case study is the gridshell  
624 that constitutes the roof of the mosque in Dakar, Senegal. The Multibody  
625 Rope Approach (MRA) and its enhanced variant, the i-MRA, were used  
626 to generate two different structure geometries. A preliminary analysis was  
627 performed to establish the differences between the two configurations in the  
628 static and dynamic fields as a result of permanent loads. The two structural  
629 geometries are then investigated through a linear eigenvalue buckling analysis  
630 under two different loading conditions. Three distinct slenderness ratios  $\lambda$  of  
631 the structural components are considered in the analyses.

632 The findings of these investigations are used to define the shape of the  
633 geometric deviations from the starting geometry. The rescaled first linear  
634 buckling mode define the geometry of structural imperfections. An incre-  
635 mental load geometric nonlinear analysis with imperfection (GNIA) is then  
636 performed under these conditions.

637 The outcomes define the equilibrium paths of the investigated structural  
638 configurations.

639 Ultimately, the impacts of material nonlinearity are considered by intro-  
640 ducing an elastoplastic constitutive law. Subsequently, the structures are  
641 investigated by Geometrically incremental load and Material Nonlinear Im-  
642 perfection Analyses (GMNIAs).

643 Different structural behaviours are exhibited by the two geometries. In  
644 particular, The MRA case shows the typical behaviour of lowering arches.  
645 Both static and instability analyses indicate this behaviour. This structure

646 shows a structural hierarchy where the central arch works as the main sup-  
647 porting structure.

648 Conversely, the i-MRA case behaviour resembles a shell structure. In this  
649 configuration, no obvious structural hierarchies are observed. The outcome  
650 is a more even distribution of stress across the structure.

651 For the two investigated geometries, two distinct structural behaviours  
652 are identified by the equilibrium paths resulting from the GNIA. The MRA  
653 case is characterized by a higher elastic buckling load  $Q_{EB}$ . However, for  
654 the MRA geometry  $Q_{EB}/Q_{CR} < 1$ , which is an indicator of catastrophic  
655 postbuckling. On the contrary, in the i-MRA case, postbuckling can be  
656 defined as benign being  $Q_{EB}/Q_{CR} > 1$ . Therefore, even if the peak load  
657  $Q_{EB}$  is lower, the i-MRA example exhibits a qualitatively better postbuckling  
658 structural behaviour.

659 The ratios between the elastic buckling load  $Q_{EB}$  and the Euler critical  
660 load  $Q_{CR}$  for the different structural configurations are displayed in Figure  
661 27. It is evident that this ratio is always higher in the i-MRA scenario than  
662 it is in the MRA case.

663 Finally, the GNMIA results demonstrated a reduction in the peak load  
664 for the MRA case, obtaining  $Q_{EB} > Q_{EP}$ . This reduction is due to the stress  
665 concentration induced by the MRA geometry. As a result, the yield strength  
666 limit is exceeded before the buckling loads are reached.

667 This phenomenon is not observed in the i-MRA case. The lowering of  
668  $Q_{EB}$  resulting from the elastoplastic model introduction is evident in Figure  
669 27 and is limited to the MRA scenario. As a result, the elasto-plastic buckling  
670 loads  $Q_{EP}$  related to the two geometries are comparable.

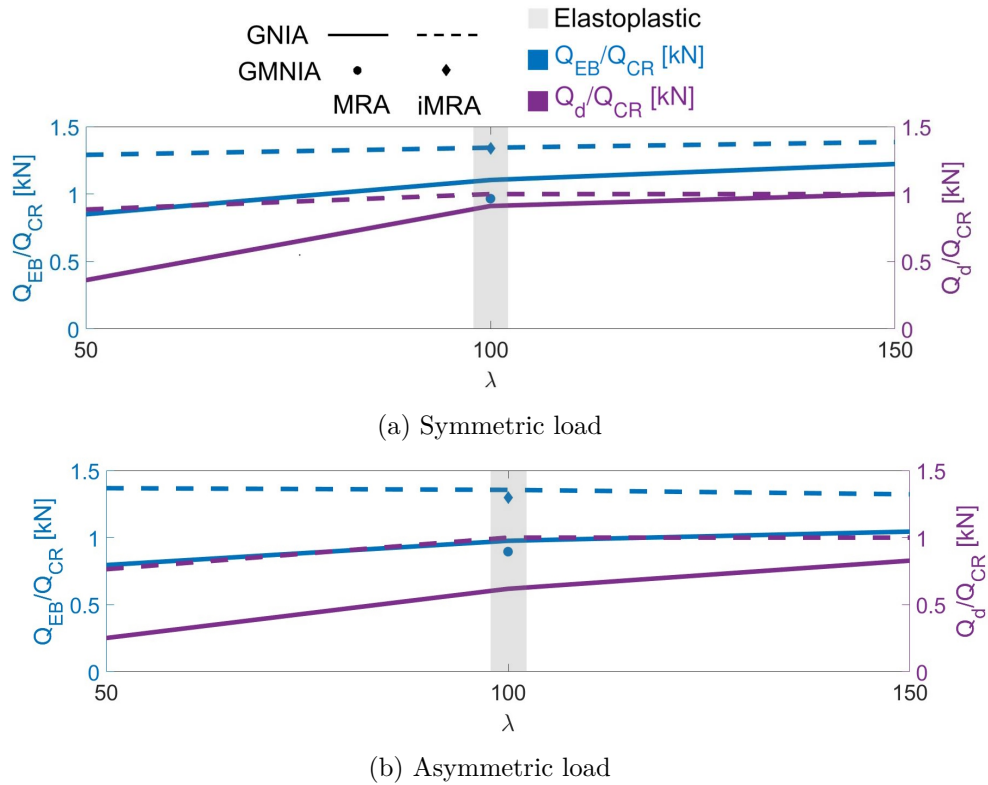


Figure 27: Ratios between the elastic buckling load  $Q_{EB}$  and the Euler critical load  $Q_{CR}$  compared with ratios between design load  $Q_d$  and the Euler critical load  $Q_{CR}$ , for the different structural configurations.

671 In conclusion, the i-MRA technique enables us to define a structural ge-  
 672 ometry that is advantageous from a construction perspective. The i-MRA  
 673 form exhibits superior stress distribution in relation to static loads. Further-  
 674 more, a qualitatively better postbuckling behaviour is observed compared to  
 675 the MRA case. Finally, if the non-linearity of the materials is considered,  
 676 the peak loads are comparable in the two studied geometries.

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