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Abstract of the doctoral dissertation

The non-symmetric coupling of virtual and boundary elements for wave propagation problems in unbounded domains

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We consider wave propagation problems described by partial differential equations and defined in unbounded domains. We aim to approximate their solution using the coupling of a domain method with a boundary integral one. Specifically, we use and analyze the non-symmetric coupling of the Virtual Element Method (VEM) with the Boundary Element Method (BEM).

The key aspect of the coupling we consider relies on using the well-known single and double layer integral operators. This makes it computationally efficient compared to other couplings, although, the stability of this scheme is in general still an open problem. Our goal is to advance the state of the art by ensuring stability for nonlinear operators where it was previously not guaranteed.

Overall, the non-symmetric coupling with the virtual element method represents a simple and interesting approach for solving engineering problems in unbounded domains.

Since VEM turns out to be appealing to easily handle high-order approximations on generic polygonal and curvilinear mesh elements, we also propose an approach to discretize the integral operators involved in the time-dependent wave equation using high-order convolution quadrature techniques for the time discretization. Precisely, we consider and analyze Runge-Kutta convolution quadrature based on the Gauss-Legendre quadrature nodes. Among the stable convolution quadrature techniques based on ordinary differential equation solvers, this method reveals to be, in certain situations, the one with the highest convergence order in terms of stages involved.

Additionally, we deal with the analysis of the interior Navier equation reformulated, by means of a Helmholtz-Hodge decomposition, as a couple scalar Helmholtz equations. Some insights on the corresponding exterior problem are also provided.

To validate the proposed approaches, we include several numerical tests along with implementation guidelines.

The results obtained in this thesis are contained in the scientific papers written in collaboration with L. Banjai, L. Desiderio, S. Falletta, and L. Scuderi, published or submitted to international peer-reviewed journals.