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A non-intrusive algorithm for parameterized model order reduction of LTI systems with guaranteed dissipativity

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We present a non-intrusive approach for generating LTI parameterized reduced-order models (PROMs) that are dissipative by construction. The approach is based on a convex formulation and enforcement of the dissipativity constraints, thus avoiding potentially unreliable heuristic post-processing schemes. Let $\boldsymbol{\vartheta} \in \Theta \subset \mathbb{R}^d$ be a vector of external parameters and s the Laplace variable. We assume that the frequency response of the underlying system $\breve{H}(s, \boldsymbol{\vartheta}) \in \mathbb{C}^{P \times P}$ is available at discrete frequency $s_k = j\omega_k$, $k = 1 \ldots K$ and parameter samples $\boldsymbol{\vartheta}_m$, $m = 1 \ldots M$ through some measurement or first-principle solver. The PROM is generated by enforcing a fitting condition $\mathsf{H}(j\omega_k, \boldsymbol{\vartheta}_m) \approx \breve{H}(j\omega_k, \boldsymbol{\vartheta}_m)$, assuming the following parameterized rational model structure

$$\mathsf{H}(s, \boldsymbol{\vartheta}) = \frac{\mathsf{N}(s, \boldsymbol{\vartheta})}{\mathsf{D}(s, \boldsymbol{\vartheta})} = \frac{\sum_{i=0}^{\bar{n}} \sum_{\boldsymbol{\ell} \in \mathcal{I}_{\bar{\boldsymbol{\ell}}}} R_{i,\boldsymbol{\ell}} \, b_{\boldsymbol{\ell}}^{\bar{\boldsymbol{\ell}}}(\boldsymbol{\vartheta}) \, \varphi_i(s)}{\sum_{i=0}^{\bar{n}} \sum_{\boldsymbol{\ell} \in \mathcal{I}_{\bar{\boldsymbol{\ell}}}} r_{i,\boldsymbol{\ell}} \, b_{\boldsymbol{\ell}}^{\bar{\boldsymbol{\ell}}}(\boldsymbol{\vartheta}) \, \varphi_i(s)} \tag{1}$$

where $R_{i,\boldsymbol{\ell}}$, $r_{i,\boldsymbol{\ell}}$ are unknown coefficients, and $\varphi_i(s) = (s - q_i)^{-1}$ with $\mathrm{Re}\{q_i\} < 0$ are partial fraction frequency-dependent basis functions, as in Vector Fitting schemes [2]. Parameterization is induced by multivariate Bernstein polynomials $b_{\boldsymbol{\ell}}^{\bar{\boldsymbol{\ell}}}(\boldsymbol{\vartheta})$ with degree $\bar{\boldsymbol{\ell}} = (\bar{\ell}_1, \ldots, \bar{\ell}_d)$, where $\mathcal{I}_{\bar{\boldsymbol{\ell}}}$ denotes the set of admissible multi-indices. Due to the presence of unknowns at both numerator and denominator in (1), determination of model coefficients is performed through an iteratively re-weighted linearized least squares process known as PSK iteration [3].

Both asymptotic stability and dissipativity are enforced uniformly $\forall \boldsymbol{\vartheta} \in \Theta$ by exploiting the uniform positivity and partition of unity properties of Bernstein polynomials. On one hand, stability is implied by the denominator $\mathsf{D}(s, \boldsymbol{\vartheta})$ being a uniformly Positive Real function, whereas dissipativity is implied by $\mathsf{H}(s, \boldsymbol{\vartheta})$ being a uniformly Bounded Real function, assuming a scattering representation. Both conditions are formulated algebraically through the appropriate Kalman-Yakubovich-Popov lemmas, which lead to parameter-dependent Linear Matrix Inequality (LMI) conditions, to be enforced uniformly $\forall \boldsymbol{\vartheta} \in \Theta$. All terms in these LMIs are then represented as truncated Bernstein polynomial expansions, including also the energy storage function. The result is a finite set of independent LMIs in the model coefficients, which are enforced concurrently with the model-data fitting. Exploiting the degree elevation property of Bernstein polynomials is shown to effectively reduce the conservativity of the LMI discretization, leading to certified uniformly stable and dissipative models, with controlled accuracy. Technical derivations and proofs are available in [1]. Application examples in the area of electronic CAD will be presented.

References

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