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***Abstract*—Data-driven rational fitting algorithms are the methods of choice for generating behavioral circuit models for complex multiport components. These approaches generate the circuit model by minimizing the deviation of one of its network functions (e.g., its scattering matrix) from the reference response. In this paper, we show that this commonly-employed fitting condition, even when met with high accuracy, is not sufficient to guarantee the reliability of the macromodel when it is used as building block in larger electrical interconnections. We address this issue by suitably modifying the cost function that drives the rational fitting process, and we outline how to modify the Vector Fitting (VF) iteration accordingly. The effectiveness of the resulting scheme is tested over a relevant Power Integrity application.**

I. Introduction

Nowadays, macromodels are widely exploited in commercial electronic design automation tools to efficiently explain the behavior of complex multiport components. When dealing with system-level simulations, the ultimate goal of a macromodel is to accurately predict how the underlying multiport will interact with other sub-networks in a larger electrical interconnection, for the sake of design verification and optimization. Rational fitting algorithms [1], [2] are the standard approach for generating behavioral models of Linear and Time-Invariant (LTI) systems in a data-driven setting. These approaches represent the macromodel as a small-size LTI circuit, by matching one of its network functions (most commonly the scattering, impedance or admittance matrix) with the corresponding one of the underlying multiport.

In this contribution, we study and address one relevant but often overlooked issue affecting macromodels generated pursuing this strategy, namely the sensitivity of their accuracy to the actual loading condition occurring in a system-level simulation [3]–[6]. We show that the feedback interactions between the macromodel and its terminations can lead to significant magnification of the (even small) residual error of the rational fitting process, thus making the macromodel unable to reproduce accurately the port behavior of the reference structure in realistic scenarios. To overcome this problem, we propose to enrich the rational fitting conditions with additional accuracy constraints, that explicitly force the model to reproduce the input-output behavior of the reference structure when it is interconnected with an arbitrary set of admissible LTI networks. We show how to compute numerically these constraints based on a frequency domain characterization of the admissible loads; then, we outline how to incorporate them in the Vector Fitting (VF) iteration, in order to generate macromodels that remain accurate under multiple loading conditions. The effectiveness of the proposed strategy is experimentally verified over a realistic Power Distribution Network (PDN) modeling problem.

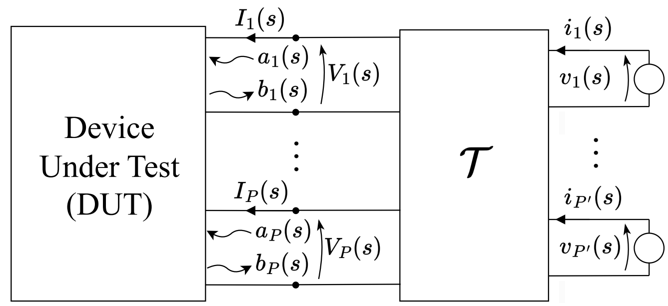

Fig. 1. A P-port representing the structure under modeling inserted in a larger electrical network. The device to be modeled (DUT) is loaded with a generic termination network $\mathcal{T}$. Hollow circles represent either ideal current or voltage sources.

II. Behavioral Models for System-Level Analyses

We consider an arbitrarily complex electromagnetic structure, denoted in the following as Device Under Test (DUT), accessible through a set of P well-defined ports. We assume that the behavior of the electrical variables at the DUT ports can be fully characterized in terms of one of its network functions. Without loss of generality, we assume that the DUT is characterized in terms of its scattering matrix $\breve{\mathbf{S}}(s) \in \mathbb{C}^{P\times P}$, highlighting that considering different network functions (e.g. impedance or admittance matrices) does not impair the validity of our derivations.

The objective of macromodeling is to synthesize a small-size LTI network, with scattering matrix $\mathbf{S}(s) \in \mathbb{C}^{P\times P}$, able to efficiently predict the port behavior of the DUT when it is loaded with other sub-networks within a larger electrical system. To formalize this concept, consider the generic electrical network template depicted in Fig. 1, where the DUT is interconnected with an arbitrary LTI termination network $\mathcal{T}$, having P ports connected to the DUT and P' ports closed on ideal voltage or current sources, that we gather in a single vector of excitations $\boldsymbol{w}$. A closed form expression for the behavior of the DUT in this interconnection can be obtained in the Laplace domain, for instance in terms of power waves at its ports. Defining with $\boldsymbol{a}^{\mathcal{T}}(s)$ and $\boldsymbol{b}^{\mathcal{T}}(s)$ the vectors of power waves that are incident into and reflected from $\mathcal{T}$ at its internal ports, due to the superposition principle it holds

$$\boldsymbol{b}^{\mathcal{T}}(s) = \mathbf{\Gamma}(s)\boldsymbol{a}^{\mathcal{T}}(s) + \mathbf{P}(s)\boldsymbol{w}(s), \tag{1}$$

where s is the Laplace variable and $\mathbf{\Gamma}$, $\mathbf{P}$ are the transfer functions from $\boldsymbol{a}^{\mathcal{T}}$ to $\boldsymbol{b}^{\mathcal{T}}$ and from $\boldsymbol{w}$ to $\boldsymbol{b}^{\mathcal{T}}$, respectively. Moreover, defining the vector of the waves impinging on the DUT $\breve{\boldsymbol{a}}(s) = [a_1(s), \dots a_P(s)]$, and that of scattered waves $\breve{\boldsymbol{b}}(s) = [b_1(s), \dots b_P(s)]$, the electrical interconnection imposes the following feedback constraints

$$\breve{\boldsymbol{b}}(s) = \breve{\mathbf{S}}(s)\,\breve{\boldsymbol{a}}(s), \quad \boldsymbol{a}^{\mathcal{T}}(s) = \breve{\boldsymbol{b}}(s), \quad \boldsymbol{b}^{\mathcal{T}}(s) = \breve{\boldsymbol{a}}(s), \tag{2}$$

where the symbol $\breve{}$ is used to indicate that the corresponding quantity pertains to the *true* DUT rather than its macromodel. Coupling these constraints with (1), we obtain the expression for the DUT port variables

$$\begin{aligned}\breve{\boldsymbol{a}}(s) &= (\mathbf{I} - \mathbf{\Gamma}(s)\breve{\mathbf{S}}(s))^{-1}\mathbf{P}(s)\boldsymbol{w}(s) \triangleq \breve{\mathbf{M}}^a(s)\boldsymbol{w}(s),\\ \breve{\boldsymbol{b}}(s) &= \breve{\mathbf{S}}(s)(\mathbf{I} - \mathbf{\Gamma}(s)\breve{\mathbf{S}}(s))^{-1}\mathbf{P}(s)\boldsymbol{w}(s) \triangleq \breve{\mathbf{M}}^b(s)\boldsymbol{w}(s),\end{aligned} \tag{3}$$

where $\mathbf{I}$ is the identity matrix. The pair of vectors $(\breve{\boldsymbol{a}}(s), \breve{\boldsymbol{b}}(s))$ represents the port behavior of the DUT when it is interconnected with $\mathcal{T}$ and excited by $\boldsymbol{w}$.

When a macromodel with scattering matrix $\mathbf{S}(s)$ takes the place of the DUT in the interconnection, the power waves $\boldsymbol{a}(s)$ and $\boldsymbol{b}(s)$ at its ports are analogously defined as

$$\begin{aligned}\boldsymbol{a}(s) &= (\mathbf{I} - \mathbf{\Gamma}(s)\mathbf{S}(s))^{-1}\mathbf{P}(s)\boldsymbol{w}(s) \triangleq \mathbf{M}^a(s)\boldsymbol{w}(s),\\ \boldsymbol{b}(s) &= \mathbf{S}(s)(\mathbf{I} - \mathbf{\Gamma}(s)\mathbf{S}(s))^{-1}\mathbf{P}(s)\boldsymbol{w}(s) \triangleq \mathbf{M}^b(s)\boldsymbol{w}(s).\end{aligned} \tag{4}$$

Thus, a macromodel can be used to seamlessly replace the DUT if, for each admissible termination $\mathcal{T}$ and excitation signals $\boldsymbol{w}$, it holds that

$$\boldsymbol{a}(s) \approx \breve{\boldsymbol{a}}(s), \quad \boldsymbol{b}(s) \approx \breve{\boldsymbol{b}}(s). \tag{5}$$

III. Robust Macromodeling via Augmented Vector Fitting

Data-driven macromodeling approaches, such as VF, assume that a closed form expression for DUT scattering matrix is not available, but that its value can be numerically retrieved at discrete frequency configurations

$$\breve{\mathbf{S}}_k = \breve{\mathbf{S}}(\mathrm{j}\omega_k), \quad k = 1, ..., K \tag{6}$$

by means of real or virtual measurements, computed in the latter case via high-fidelity circuit or electromagnetic simulations. Based on these data, the model is generated via rational fitting, which identifies a model with rational scattering matrix $\mathbf{S}(s)$

$$\mathbf{S}(s) = \sum_{i=1}^{\nu} \frac{\mathbf{R}_i}{s - p_i} + \mathbf{R}_0 \tag{7}$$

by optimizing the unknown model poles $\{p_i\}$ and residues $\{\mathbf{R}_0, \mathbf{R}_i\}$ so that $\mathbf{S}(s)$ matches the reference response up to a small user-defined target error $\delta\mathbf{S}_k$

$$\mathbf{S}_k = \mathbf{S}(s)|_{s=\mathrm{j}\omega_k} = \breve{\mathbf{S}}_k + \delta\mathbf{S}_k, \quad \delta\mathbf{S}_k \approx 0,\ k = 1, ..., K. \tag{8}$$

Although macromodels derived with this approach are reliable in general, it is important to observe that the fulfillment of (8) does not formally guarantee that the macromodel will also fulfill (5) when interconnected with a network $\mathcal{T}$. In fact, evaluating for instance $\boldsymbol{a}(s)$ in (4) at a frequency point ω_k, according to (8) and (3) we obtain

$$\begin{aligned}\boldsymbol{a}(\mathrm{j}\omega_k) &= (\mathbf{I} - \mathbf{\Gamma}(\mathrm{j}\omega_k)(\breve{\mathbf{S}}_k + \delta\mathbf{S}_k))^{-1}\mathbf{P}(\mathrm{j}\omega_k)\boldsymbol{w}(\mathrm{j}\omega_k) =\\ &= (\breve{\mathbf{M}}^a(\mathrm{j}\omega_k) + \delta\mathbf{M}_k^a)\boldsymbol{w}(\mathrm{j}\omega_k) = \breve{\boldsymbol{a}}(\mathrm{j}\omega_k) + \delta\mathbf{M}_k^a\boldsymbol{w}(\mathrm{j}\omega_k),\end{aligned}$$

where the error matrix $\delta\mathbf{M}_k^a$ appears due to the presence of the unavoidable error term $\delta\mathbf{S}_k$ affecting the model. Since the matrix transformation relating $\breve{\mathbf{S}}_k$ to $\breve{\mathbf{M}}_k^a$ involves a matrix inversion operation, the entries of $\delta\mathbf{M}_k^a$ may be large even when the perturbation error $\delta\mathbf{S}_k$ is kept under control, depending on the characteristics of the network $\mathcal{T}$. This error amplification phenomenon, known as macromodel sensitivity, has been already studied in a number of previous works, see e.g. [4], [6] for a more thorough analysis. If not properly addressed, this sensitivity can severely compromise the macromodel reliability, as will be experimentally shown in Sec. IV.

In order to address this issue, we propose to augment the fitting conditions that drive the macromodel generation, by explicitly enforcing the correct approximation of the DUT port behavior for several different termination schemes. Let us define a set of M admissible termination/excitation conditions

$$\mathcal{T}^{(m)} : \left\{\mathbf{P}^{(m)}(s), \mathbf{\Gamma}^{(m)}(s), \boldsymbol{w}^{(m)}(s)\right\}, \quad m = 1, \dots, M,$$

assuming that the value of the involved quantities is known at the DUT sampling points ω_k. Then, using (3), we can compute the DUT port behaviors for each frequency and termination condition

$$(\breve{\boldsymbol{a}}^{(m)}(\mathrm{j}\omega_k), \breve{\boldsymbol{b}}^{(m)}(\mathrm{j}\omega_k)) \triangleq (\breve{\boldsymbol{a}}_k^{(m)}, \breve{\boldsymbol{b}}_k^{(m)}) \qquad \forall\, k, m$$

and augment the standard macromodeling accuracy requirements (8) with the additional constraints

$$\mathbf{S}_k\breve{\boldsymbol{a}}_k^{(m)} \approx \breve{\boldsymbol{b}}_k^{(m)}, \qquad \forall\, k, m. \tag{9}$$

These constraints force the model to behave as similarly as possible to the DUT when it is interconnected with the networks $\mathcal{T}^{(m)}$, and, in principle, can be used to guide any macromodeling algorithm based on rational fitting. In this work, we incorporate them in the VF scheme, as follows.

In compliance with standard VF formulations (see e.g. [1]), we rewrite the rational model structure (7) in a barycentric form using a set of ν auxiliary fixed basis poles $\{q_i\}_{i=1}^{\nu}$ and two rational functions $\mathbf{N}(s) \in \mathbb{C}^{P\times P}$ and $d(s) \in \mathbb{C}$

$$\mathbf{S}(s) = \frac{\mathbf{N}(s)}{d(s)} = \frac{\mathbf{N}_0 + \sum_{i=1}^{\nu} \frac{\mathbf{N}_i}{s-q_i}}{d_0 + \sum_{i=1}^{\nu} \frac{d_i}{s-q_i}}. \tag{10}$$

Then, we solve for the VF unknowns $\{\mathbf{N}_0, \mathbf{N}_i, d_0, d_i\}$ by enforcing $\forall k, m$ the linearized approximation conditions

$$\mathbf{N}(\mathrm{j}\omega_k) \approx \breve{\mathbf{S}}_k\, d(\mathrm{j}\omega_k), \quad \mathbf{N}(\mathrm{j}\omega_k)\breve{\boldsymbol{a}}_k^{(m)} \approx \breve{\boldsymbol{b}}_k^{(m)} d(\mathrm{j}\omega_k) \tag{11}$$

in a least-squares sense [6]. Notice that (11) includes a linearized version of the proposed constraints (9). The modified VF iteration repeatedly solves (11), each time updating the set of basis poles $\{q_i\}$ with the zeros of the denominator function $d(s)$ found at the previous iteration, as in standard VF. The iteration stops when either the basis poles stabilize or when the least-squares approximation error hits a user defined tolerance. See [6] for a complete outline of the modeling algorithm and for a comprehensive analysis of its computational complexity.

IV. Numerical Results

The proposed method was applied to a PDN modeling problem. The structure of interest comes from a real design (courtesy Intel Corp.), originally introduced in [5]. The PDN is accessible from the outer environment at $P = 18$ electrical ports. Its scattering matrix is measured in correspondence of $K = 602$ sample points in the range $[0, 3]$ GHz. For this structure, we built a model of order $\nu = 12$ applying the proposed approach. Here, macromodel accuracy is optimized with respect to a termination network $\mathcal{T}_1$ comprising series RC branches connected to chip-side ports 2-15, together with a small resistance representing the voltage regulator at port 1. Ports 16-18 are left open. Network $\mathcal{T}_1$ represents the intended use of this macromodel in a target simulation. To further assess the robustness of the method, an additional termination network $\mathcal{T}_2$ made up of voltage sources with series $R_s = 0.1\,\Omega$ resistances was included. This corresponds to optimizing a 50-Ω scattering parameters model to be accurate when terminated by $R_0 = 0.1\,\Omega$ resistances. Overall, $\mathcal{T}_1$ and $\mathcal{T}_2$ give $M = 33$ constraints of the form (9) to be included in the VF iteration.

The top panel of Fig. 2 shows the agreement between the scattering parameters of the proposed model and a standard VF model: both match the reference response. However, when the standard VF model is used to predict the output impedance of the PDN loaded with the network $\mathcal{T}_1$, significant deviations are observed (middle panel). Instead, the proposed approach effectively increases the robustness of the macromodel accuracy with respect to the prescribed termination, providing more accurate results. Finally, the bottom panel of Fig. 2 shows that having optimized the proposed macromodel with respect to the low impedance termination $\mathcal{T}_2$ guarantees remarkably accurate predictions of the admittance matrix of the bare PDN. The same accuracy is not guaranteed by the standard VF model, which suffers from a large error sensitivity under this change of representation. In conclusion, using the proposed fitting scheme, the resulting macromodel matches correctly the S, Y, and reterminated Z parameters of the PDN.

V. Conclusions

In this contribution, we have shown how macromodels that fulfill standard accuracy requirements can still fail at predicting the underlying multiport behavior at the system level. To address this problem, we augmented the VF iteration with additional frequency domain fitting conditions, effectively increasing the robustness of the resulting macromodel with respect to an arbitrary set of admissible loading conditions.

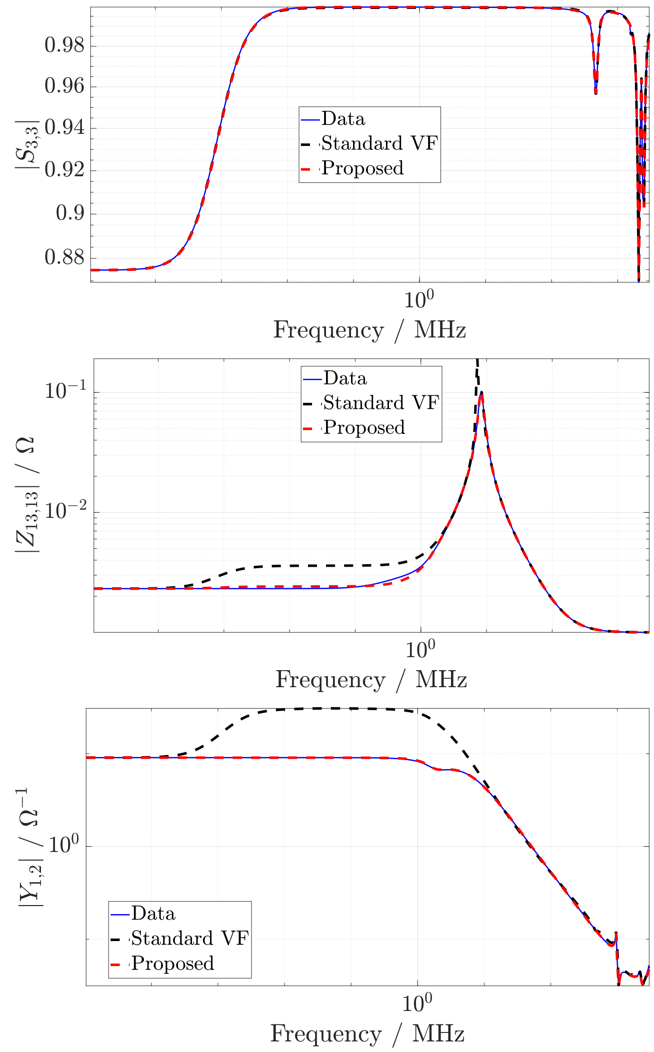

Fig. 2. Top: a PDN scattering response compared to the proposed and a standard VF model. Middle: prediction of the PDN impedance when terminated by $\mathcal{T}_1$. Bottom: prediction of one PDN admittance response.

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