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Abstract: Recently, Enhanced Gaussian Noise (EGN) model of few-mode fiber nonlinearity has been proposed improving GN model accuracy. However, EGN is considerably complex. We propose a closed-form EGN model and provide substantial simulations indicating its accuracy.

1. Introduction

The Gaussian Noise (GN) model of Few-Mode Fiber (FMF) nonlinear effects has been provided in [1] as a practical tool for estimating Nonlinear Interference Noise (NLIN) power in uncompensated FMF links. The GN model shows some errors in predicting NLIN power mostly due to the signal Gaussianity assumption [2]. Recently, GN model was extended in [3] and a complete Enhanced GN (EGN) model has been derived considering all NLIN components that improves the NLIN power prediction accuracy. Note that in this EGN model formulation, the carrier phase estimator is implicit in the model [3]. EGN model is computationally complex, which can make its extensive practical use difficult [2]. This paper proposes a closed-form EGN model from the integral-form EGN model presented in [3] and fully verifies it on a theoretical basis. Such approximation is effective in its present form especially for real-time quality of transmission evaluation.

2. Closed-form EGN model

Considering a rectangular Nyquist Wavelength Division Multiplexing (WDM) comb, and flat NLIN Power Spectral Density (PSD), the NLIN variance can be expressed as the multiplication of NLIN PSD and the bandwidth [4]. The PSD of pth mode and nth channel can be formulated by EGN model as in [2]

$$G^p_{EGN}(f_n) = G^p_{GN}(f_n) + G^p_{FON}(f_n) + G^p_{HON}(f_n), \tag{1}$$

where G_{GN}, G_{FON}, and G_{HON} are second, forth, and higher order noise contributions, respectively. Since Self Channel Interference (SCI) and Cross Channel Interference (XCI) are the dominant NLIN terms in dense WDM systems with large symbol rate, we only consider these terms. In other words, Multi Channel Interference (MCI) is not assumed in NLIN formulation [5]. Therefore, and by considering [3], the PSD of pth mode and nth channel can be expressed as

$$\begin{aligned} G^p_{EGN}(f_n) = \sum_{q=1}^{D} \Bigg[& 3\Bigg(\underbrace{\kappa_1^{(n,q)^2}\kappa_1^{(n,p)} P^2_{n,q}P_{n,p}E(n,p,n,q)}_{GN,SCI} + \underbrace{2\sum_{n'\neq n}\kappa_1^{(n',q)^2}\kappa_1^{(n,p)}P^2_{n',q}P_{n,p}E(n,p,n',q)}_{GN,XCI}\Bigg) + \Bigg(\underbrace{\kappa_2^{(n,q)}\Big(\kappa_1^{(n,p)}5P^2_{n,q}P_{n,p}E(n,p,n,q) + \kappa_1^{(n,q)}P_{n,q}P_{n,p}P_{n,q}E(n,p,n,q)\Big)}_{FON,SCI} \\ & \underbrace{+2\sum_{n'\neq n}\kappa_2^{(n',q)}\Big(\kappa_1^{(n,p)}5P^2_{n',q}P_{n,p}E(n,p,n',q) + \kappa_1^{(n,q)}P_{n',q}P_{n',p}P_{n,q}E(n,p,n',q)\Big)}_{FON,XCI}\Bigg) + \Bigg(\underbrace{\kappa_3^{(n,q)}P^2_{n,q}P_{n,p}E(n,p,n,q)}_{HON,SCI} + \underbrace{2\sum_{n'\neq n}\kappa_3^{(n',q)}P^2_{n',q}P_{n',p}E(n,p,n',q)}_{HON,XCI}\Bigg)\Bigg], \end{aligned} \tag{2}$$

where D is number of modes, $P_{n,p}$ is the power of the nth channel of pth mode, $\kappa_1^{(n,p)} = \mu_2^{(n,p)}$, $\kappa_2^{(n,p)} = \mu_4^{(n,p)} - 2\mu_2^{(n,p)^2}$, and $\kappa_3^{(n,p)} = \mu_6^{(n,p)} - 4\mu_4^{(n,p)}\mu_2^{(n,p)} + 12\mu_2^{(n,p)^3}$, where $\mu_2^{(n,p)}$, $\mu_4^{(n,p)}$, and $\mu_6^{(n,p)}$ denote the second, fourth, and sixth order moment of the constellation of nth channel of pth mode, respectively. Moreover, the $E(n,p,n',q)$ term is equal to

$$E(n,p,n',q) = \frac{\tilde{\gamma}^2_{pq}}{4}\iint \left| \frac{1-e^{-\alpha_p L_s e^{j|\beta_q(f_1+f_2-f_n)-\beta_p(f_1)+\beta_p(f_n)-\beta_q(f_2)|L_s}}}{\alpha_p - j|\beta_q(f_1+f_2-f_n)-\beta_p(f_1)+\beta_p(f_n)-\beta_q(f_2)|} \frac{1-e^{j|\beta_q(f_1+f_2-f_n)-\beta_p(f_1)+\beta_p(f_n)-\beta_q(f_2)|N_sL_s}}{1-e^{j|\beta_q(f_1+f_2-f_n)-\beta_p(f_1)+\beta_p(f_n)-\beta_q(f_2)|L_s}} \right|^2 df_1 df_2, \tag{3}$$

where α_p shows the attenuation, L_s is span length, N_s is number of spans, $\beta_p(f)$ is the propagation constant of pth mode, $\tilde{\gamma}_{pp} = \frac{8}{9}\gamma f_{pp}$, $\tilde{\gamma}_{pq} = \frac{4}{3}\gamma f_{pq}$ with γ being the Kerr nonlinearity coefficient, and $f_{pq} = \frac{A_{eff}}{I_p I_q}\iint F_p^2 F_q^2 dxdy$ representing the nonlinear coupling coefficient between the modes p and q where $F_p(x,y)$ is the spatial profile of pth mode, $I_p = \iint F_p^2(x,y)dxdy$, A_{eff} shows the effective area of the fundamental mode [6]. $f_1 \in [f_n - B_{ch,n}/2, f_n + B_{ch,n}/2]$ and $f_2 \in [f_{n'} - B_{ch,n'}/2, f_{n'} + B_{ch,n'}/2]$ with $B_{ch,n}$ and $B_{ch,n'}$ showing the bandwidths of the nth and n'th channels, respectively. We use the incoherency approximation, i.e.,

$(1-e^{j|\beta_q(f_1+f_2-f_n)-\beta_p(f_1)+\beta_p(f_n)-\beta_q(f_2)|N_sL_s})/(1-e^{j|\beta_q(f_1+f_2-f_n)-\beta_p(f_1)+\beta_p(f_n)-\beta_q(f_2)|L_s}) \to N_s$ [5]. Moreover, we can approximate the numerator term $|1-e^{-\alpha_pL_s}e^{j|\beta_q(f_1+f_2-f_n)-\beta_p(f_1)+\beta_p(f_n)-\beta_q(f_2)|L_s}|^2 \approx (1-e^{-\alpha_pL_s})^2$ [4]. Therefore, using $\beta_p(f)=2\pi\beta_{1_p}f+2\pi^2\beta_{2_p}f^2$, (3) can be approximated by

$$E(n,p,n',q) \approx N_s\frac{\bar{\gamma}_{pq}^2}{4}\iint L_{eff}^2/\left[1+L_{eff,a}^2\left(2\pi(\beta_{1_q}-\beta_{1_p})(f_2-f_n)-4\pi^2\beta_{2_q}(f_1-f_n)(f_2-f_n)\right)^2\right]df_1df_2, \tag{4}$$

where β_{1_p} is the inverse group velocity parameter of pth mode, and $\beta_{2_p}=|D_p|c/(2\pi\nu^2)$, D_p is the dispersion coefficient of pth mode, ν is the center frequency of the WDM comb, c is the light speed, $L_{eff}=(1-e^{-\alpha_pL_s})/\alpha_p$, and $L_{eff,a}=1/\alpha_p$. By noting that $\int\rho/(1+\rho^2x^2)dx=tan^{-1}(\rho x)$, $Ti_2(\rho x)=\int_0^x tan^{-1}(\rho y)/(\rho y)dy$ [7], and $Ti_2(x)\approx\pi/2ln(x/2+\sqrt{(x/2)^2+1})$ [7], (4) can be expressed as

$$E(n,p,n',q) \approx N_s\frac{\bar{\gamma}_{pq}^2}{4}\frac{L_{eff}^2}{4\pi\beta_{2_q}L_{eff,a}}\left[ln\left(\pi^2\beta_{2_q}L_{eff,a}B_{ch,n}\left(\frac{\beta_{1_q}-\beta_{1_p}}{2\pi\beta_{2_q}}+f_n-f_{n'}+\frac{B_{ch,n'}}{2}\right)+\sqrt{\left(\pi^2\beta_{2_q}L_{eff,a}B_{ch,n}\left(\frac{\beta_{1_q}-\beta_{1_p}}{2\pi\beta_{2_q}}+f_n-f_{n'}+\frac{B_{ch,n'}}{2}\right)\right)^2+1}\right)+\right.$$
$$\left.ln\left(\pi^2\beta_{2_q}L_{eff,a}B_{ch,n}\left(f_{n'}-\frac{\beta_{1_q}-\beta_{1_p}}{2\pi\beta_{2_q}}-f_n+\frac{B_{ch,n'}}{2}\right)+\sqrt{\left(\pi^2\beta_{2_q}L_{eff,a}B_{ch,n}\left(f_{n'}-\frac{\beta_{1_q}-\beta_{1_p}}{2\pi\beta_{2_q}}-f_n+\frac{B_{ch,n'}}{2}\right)\right)^2+1}\right)\right]. \tag{5}$$

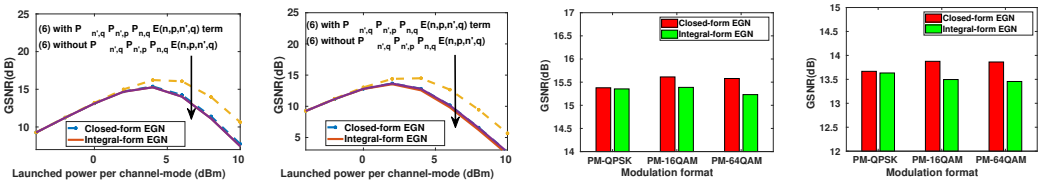
By substituting the obtained $E(n,p,n',q)$ approximation in (2), the PSD of the nonlinear noise of the nth channel of the pth mode can be obtained. However, we show in in simulation results that this formulation underestimates the NLIN PSD. The FON terms have negative coefficient and removing them leads to NLIN PSD overestimation. We depict in our experiments that neglecting the FON term, $P_{n',q}P_{n',p}P_{n,q}E(n,p,n',q)$, can properly compensate this underestimation. Therefore, the PSD of pth mode and nth channel can be formulated by

$$G_{EGN}^p(f_n)=\sum_{q=1}^{D}\left[3\left(\kappa_1^{(n,q)^2}\kappa_1^{(n,p)}P_{n,q}^2P_{n,p}E(n,p,n,q)+2\sum_{n'\neq n}\kappa_1^{(n',q)^2}\kappa_1^{(n,p)}P_{n',q}^2P_{n,p}E(n,p,n',q)\right)+\left(\kappa_2^{(n,q)}\kappa_1^{(n,p)}5P_{n,q}^2P_{n,p}E(n,p,n,q)+\right.\right.$$
$$\left.\left.2\sum_{n'\neq n}\kappa_2^{(n',q)}\kappa_1^{(n,p)}5P_{n',q}^2P_{n,p}E(n,p,n',q)\right)+\left(\kappa_3^{(n,q)}P_{n,q}^2P_{n,p}E(n,p,n,q)+2\sum_{n'\neq n}\kappa_3^{(n',q)}P_{n',q}^2P_{n',p}E(n,p,n',q)\right)\right]. \tag{6}$$

3. Simulation results

In this section, the accuracy of proposed closed-form EGN model versus integral-form full EGN model presented in [3] is substantially investigated. The Generalized Signal to Noise Ratio (GSNR) is considered which contains the effect of NLIN and amplified spontaneous emission. The following FMF link parameters are used; the modulation format is PM-QPSK with the symbol rate 64 *GBaud* and the channel spacing 75 *GHz*. 10 spans are considered each with 100 *km* length. 3 modes, each with 66 channels with the center frequency 1550 *nm* are used. The values for nonlinear coupling coefficient, dispersion coefficient, inverse group velocity, and attenuation are taken from [6]. At each span, an erbium doped fiber amplifier with 5 *dB* noise figure compensates FMF attenuation. All Figs plot GSNR of central channel of LP01 or LP11a/b mode. The link-state is full-load with the same modulation format for different channels and modes.

Figs. 1a and 1b investigate the effect of considering $P_{n',q}P_{n',p}P_{n,q}E(n,p,n',q)$ term in (6) for LP01 and LP11a/b modes, respectively. As seen, considering (6) with $P_{n',q}P_{n',p}P_{n,q}E(n,p,n',q)$ term underestimates the NLIN power, and removing this term results in better accuracy. The accuracy of closed-form EGN model for different modulation formats is indicated by GSNR plots in Figs. 1c and 1d where the optimum launched power is deployed for different channels and modes. The GSNR difference of closed-form and integral-form EGN model is always below 0.4 *dB*. Fig. 2 plots GSNR versus launched power per channel-mode for closed-form and integral-form EGN



(a) (b) (c) (d)

Fig. 1: GSNR versus launched power per channel-mode for closed-form and integral-form EGN model, considering (6) with/without $P_{n',q}P_{n',p}P_{n,q}E(n,p,n',q)$ term for a) LP01 and b) LP11a/b modes; and GSNR of the optimum launched power per channel-mode versus modulation format for closed-form and integral-form EGN model, for c) LP01 and d) LP11a/b modes.

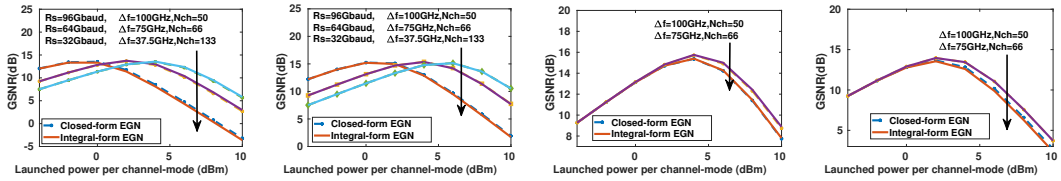


Fig. 2: GSNR versus launched power per channel-mode for closed-form and integral-form EGN model, considering different baudrates-channel spacing for a) LP01 and b) LP11a/b modes; and considering different channel spacing for c) LP01 and d) LP11a/b modes.

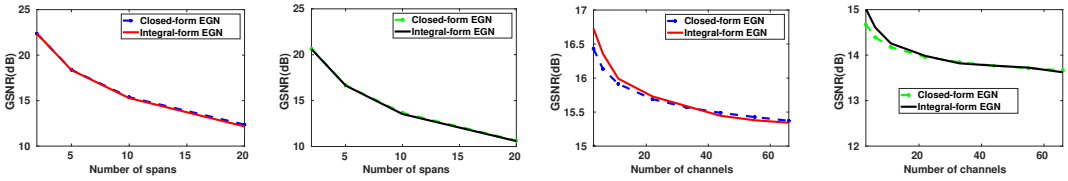


Fig. 3: GSNR versus number of spans, for closed-form and integral-form EGN model, considering the optimum launched power per channel-mode for a) LP01 and b) LP11a/b modes; and GSNR versus number of channels, for closed-form and integral-form EGN model, considering the optimum launched power per channel-mode considering full-load link-state for c) LP01 and d) LP11a/b modes.

model, considering different baudrates-channel spacing (Figs. 2a and 2b) and different channel spacing (Figs. 2c and 2d). It is indicated that closed-form EGN model is quite effective while considering different baudrates and channel spacing in both LP01 and LP11a/b modes. Figs. 3a and 3b show good convergence of closed-form EGN model towards the integral-form EGN model curve considering different number of spans for both LP01 and LP11a/b modes. Figs. 3c and 3d indicate that closed-form EGN model is accurate for sufficiently large number of channels for both LP01 and LP11a/b modes.

4. Conclusion

In this paper, we presented a closed-form EGN model for FMF systems. Substantial simulations indicated the proposed formulation to be accurate for sufficiently large number of channels. The GSNR difference compared with integral-form EGN model is always below 0.4 *dB*.

Acknowledgement

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