Analytical and Numerical Instability Analysis of Corroded and Temperature-Varying Thin-Wall Shells

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Abstract. Thin-wall shells (steel plates, steel cylindrical shells, steel spherical shells, etc.) are widely used in many engineering fields such as construction, machinery, chemical industry, navigation, and aviation because of their light weight and high strength. Their failure modes under static pressure or impact dynamic load are mostly buckling instability, and the failure is very sudden, often causing structural failure or even catastrophic accidents without obvious symptoms. In this framework, the significance of this paper is that it considers the influence of external environment corrosion on steel shells' bearing capacity using plate and shell classical stability theory, and investigates the stable bearing capacity of thin-wall steel shells in view of corrosion impact. By this approach, a theoretical method for the time-varying stable bearing capacity of plate and shell thin-walled steel members under the simultaneous action of corrosion and temperature changes is obtained, providing a useful theory for complex engineering practices such as corrosion and temperature changes, including fire actions.

Keywords: Corrosion, Variable temperature, Fire, Thin-walled shell, Stable bearing capacity.

1 Introduction

Thin shell structures (thin plates, thin cylindrical shells, thin spherical shells, etc.) often work in environments with corrosion and temperature changes. According to relevant research, most of their failure problems are controlled by stability. For their stable (unstable) behavior at this time, classical thin-walled shell theories, e.g., small or large deflection theories, can no longer be used directly. Gutman [1-3] proposed a series of calculation methods for cylindrical shells and spherical shells, mainly in corrosive environments. When dealing with temperature, he does not consider the temperature stress, but only the influence of temperature on the corrosion rate. Lacidogna also mentioned the problem of spherical shell stability after corrosion in literature [4]. However, it mostly focuses on the estimation of the stable bearing capacity based on the non-destructive testing data of the shell, and does not discuss the temperature stress in depth.

Pronina in the literature [5,6] took the spherical shell as an example, considered the influence of corrosion and temperature stress, and gave its stable bearing capacity calculation equation, and obtained a segmented analytical solution. However, as for the corrosion kinetic equation it uses comes from Gutman's metal immersion corrosion experiments and has not been verified to be applicable to the type of air corrosion. Also, during the derivation, it doesn't mention constraints (e.g.: elastic constraints). Although, this has little effect on spherical shells, which are usually considered free constraints. But once it is applied to cylindrical shells or other flat shells, the deficiency will appear. Because cylindrical shells or plate shells often have different levels of restraint at the ends. More importantly, in practice, temperature also changes the physical properties of the shell, such as elastic modulus, yield strength, and possibly even its mode of failure (e.g., stability control or strength control). And those things should be discussed. However, research on this type of content has yet to be found.

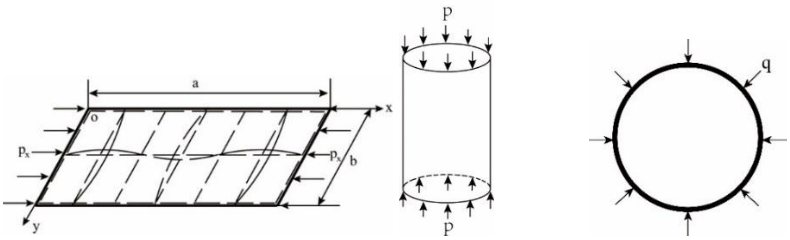
Among the factors that cause temperature changes, such as: temperature difference between day and night, seasonal temperature difference and fire, fire is more harmful. Therefore, as a special kind of variable temperature environment, its influence on the stability of corroded thin shells should also be discussed. In fact, no relevant research has been found so far.

Therefore, in this paper it is believed that it is necessary to use a better corrosion dynamic model, consider temperature stress, and different constraints to study the stability of shells.

2 Problem description

2.1 Objectives of the research

Our purpose is to study the calculation method of the stable bearing capacity of common thin shells (plates, cylindrical shells and spherical shells) in corrosion and variable temperature environments in the elastic range.



(a) (b) (c)

Fig. 1. Thin-wall shells: (a) plates, (b) cylindrical shells, (c) spherical shells

2.2 Stability theory under known ideal environment

In this paper to carry out the research we start from the known small deflection and small deflection stability theory of part-thin shells [7], which is explained by the following equations.

2.2.1 Plate

Small deflection buckling:

$$\sigma_{x,cr}(t) = \frac{k\pi^2 E\cdot h^2(t)}{12(1-\mu^2)b^2}, \quad (1)$$

$$p_{x,cr}(t) = \sigma_{x,cr}(t)\cdot h(t) = \frac{k\pi^2 E\cdot h^3(t)}{12(1-\mu^2)b^2}. \quad (2)$$

Large deflection buckling:

$$\sigma_{xa} = \frac{4\pi^2 D}{hb^2} + \frac{E\pi^2 f^2}{8b^2} = \sigma_{cr} + \frac{E\pi^2 f^2}{8b^2}. \quad (3)$$

2.2.2 Cylindrical shell.

$$\begin{cases} \sigma_{cr}(t) = \begin{cases} \frac{E}{\sqrt{3(1-\mu^2)}}\frac{h(t)}{r}, \text{small deflection buckling,} \\ \frac{1}{3}\frac{E}{\sqrt{3(1-\mu^2)}}\frac{h(t)}{r}, \text{large deflection buckling,} \end{cases} \\ p_{cr}(t) = \sigma_{cr}(t)\cdot h(t), \\ N_{cr}(t) = \sigma_{cr}(t)\cdot 2\pi r\cdot h(t). \end{cases} \quad (4)$$

2.2.3 Spherical shell

$$\sigma_{cr}(t) = \begin{cases} \frac{1}{\sqrt{3(1-\mu^2)}}\frac{Eh(t)}{R}, \text{small deflection buckling,} \\ \frac{1}{4}\frac{1}{\sqrt{3(1-\mu^2)}}\frac{Eh(t)}{R}, \text{large deflection buckling.} \end{cases} \quad (5)$$

$$q_{cr}(t) = \frac{2h(t)}{R}\sigma_{cr}(t) = \begin{cases} \frac{2E}{\sqrt{3(1-\mu^2)}}\frac{h(t)^2}{R^2}, \text{ small deflection buckling,} \\ \frac{E}{2\sqrt{3(1-\mu^2)}}\frac{h(t)^2}{R^2}, \text{large deflection buckling.} \end{cases} \quad (6)$$

2.3 Other required unknown conditions or governing equations

2.3.1 A more general and improved kinetic corrosion model.

We propose an improved empirical model, considering the following equation:

$$\frac{dh}{dt} = Ant^{n-1}\, exp\,\frac{V\sigma}{RT}. \quad (7)$$

2.3.2 Material properties of structural steel at varying temperatures

The elastic modulus expression of steel at high temperature is the following:

$$E(T_s) = \begin{cases} \frac{7T_s-4780}{6T_s-4760} E_s, \ 20°C \le T_s < 600°C, \\ \frac{1000-T_s}{6T_s-2800} E_s, 600°C \le T_s < 1000°C. \end{cases} \quad (8)$$

where, T_s is the temperature of steel; E_{Ts} is the elastic modulus of steel at high temperature (N/mm^2); E_s is the elastic modulus of steel at normal temperature, it can be taken as 206 GPa [9] according to the literature [10].

2.3.3 Relationship between temperature stress and constraints.

Under arbitrary elastic constraints, the temperature stress equation of plates and cylindrical shells are the following.

Plate:

$$\sigma_{cr}(t_c, T_s) = \frac{k\pi^2 \cdot h^2(t_c) \cdot E(T_s)}{12(1-\mu^2)b^2} - k_r \alpha_T (T_s - T_{s0}) E(T_s). \quad (9a)$$

Cylindrical shell:

$$\sigma_{cr}(t_c, T_s) = \begin{cases} \frac{h(t_c)}{R} \frac{E(T_s)}{\sqrt{3(1-\mu^2)}} - k_r \alpha_T (T_s - T_{s0}) E(T_s) \\ \frac{1}{3} \cdot \frac{h(t_c)}{R} \frac{E(T_s)}{\sqrt{3(1-\mu^2)}} - k_r \alpha_T (T_s - T_{s0}) E(T_s) \end{cases} . \quad (9b)$$

The spherical shell is considered free and unconstrained, so its temperature stress is:

$$\sigma_{cr}(t_c, T_s) = \begin{cases} \frac{h(t_c)}{r} \frac{E(T_s)}{\sqrt{3(1-\mu^2)}}, \\ \frac{1}{4} \frac{h(t)}{r} \frac{E(T_s)}{\sqrt{3(1-\mu^2)}}. \end{cases} \quad (9c)$$

2.3.4 Governing equation of fire temperature curve.

Fire temperature curve [13] shows:

$$T_s = (\sqrt{0.044 + 5.0 \times 10^{-5} \alpha \frac{F_i}{V}} - 0.2)t + T_{s0} \quad T_s \le 700^o\text{C}. \quad (10)$$

In the equation, t is fire duration (s); T_{s0} -initial temperature of steel member before fire, which can be taken as 20°C; F_i -fired surface area per unit length of steel member with fire protection (m ^{2}); V-unit length of steel member volume (m ^{3}); α -comprehensive heat transfer coefficient [W/ (m ^{2} °C)].

3 Stability calculation method for thin shells in corrosion environment

The stability conditions are deduced in this section according to Eqs. (1) - (10).

3.1 Plate

The stress of the plate can be expressed as,

$$\sigma(t) = p_0/h(t). \tag{11}$$

Substituting Eq. (11) into (7) gets:

$$\frac{dh}{dt} = -Ant^{n-1} \exp\frac{V\sigma}{RT} = -Ant^{n-1} \exp\frac{Vp_0}{RTh}. \tag{12}$$

We integrate it in the range of h_0 - h, t_0- t to get:

$$t = [-\frac{1}{A}\int_{h_0}^{h} \exp(-\frac{Vp_0}{RTh})\, dh]^{\frac{1}{n}}. \tag{13}$$

In the equation $t = f(h) = [-1/A \int_{h_0}^{h} \exp(-\frac{Vp_0}{RTh})\, dh]^{\frac{1}{n}}$, f expresses the corresponding rule between variable *h* and variable *t*.
Since the inverse operation exists, $h = f^{-1}(t)$, substituting it into Eq. (9a) we obtain:

$$\begin{cases} t = f(h) = [-\frac{1}{A}\int_{h_0}^{h} \exp(-\frac{Vp_0}{RTh})\, dh]^{\frac{1}{n}}, \\ h = f^{-1}(t), \\ \sigma_{x,cr}(t) = \frac{k\pi^2 E}{12(1-\mu^2)} \frac{[f^{-1}(t)]^2}{b^2}, \\ p_{x,cr}(t) = \sigma_{x,cr}(t) \cdot h(t) = \frac{k\pi^2 E}{12(1-\mu^2)} \frac{[f^{-1}(t)]^3}{b^2}. \end{cases} \tag{14}$$

3.2 Cylindrical shells

The stress of the cylindrical shell is expressed as:

$$\sigma(t) = \frac{N}{2\pi r \cdot h(t)}. \tag{15}$$

Considering Eq. (7), we have:

$$\frac{dh}{dt} = -Ant^{n-1} \exp\frac{V\sigma}{RT} = -Ant^{n-1} \exp\frac{VN}{2\pi rhRT}. \tag{16}$$

Integrating it over h_0 to *h* and *0* to *t* gives:

$$\int_{h_0}^{h} -\frac{1}{A} \exp(-\frac{VN}{2\pi rhRT})\, dh = \int_0^t nt^{n-1}\, dt. \tag{17}$$

After integral operation, we get:

$$t = [-\frac{1}{A}\int_{h_0}^{h} exp(-\frac{VN}{2\pi rhRT})\, dh]^{\frac{1}{n}}. \quad (18)$$

Let $t = f(h) = [-1/A\int_{h_0}^{h} exp(-\frac{VN}{2\pi rhRT})\, dh]^{\frac{1}{n}}$, where f express the correspondence rule between variable h and variable t.
Substituting $h = f^{-1}(t)$ into above equations we obtain:

$$\begin{cases} t = f(h) = [-\frac{1}{A}\int_{h_0}^{h} exp(-\frac{VN}{2\pi rhRT})\, dh]^{\frac{1}{n}}, \\ h = f^{-1}(t), \\ \sigma_{cr}(t) = \begin{cases} \frac{E}{\sqrt{3(1-\mu^2)}}\frac{f^{-1}(t)}{r}, \text{small deflection buckling,} \\ \frac{1}{3}\cdot\frac{E}{\sqrt{3(1-\mu^2)}}\frac{f^{-1}(t)}{r}, \text{large deflection buckling,} \end{cases} \\ p_{cr}(t) = h(t)\cdot\sigma_{cr}(t), \\ N_{cr}(t) = 2\pi r\cdot h(t)\cdot\sigma_{cr}(t). \end{cases} \quad (19)$$

Among them, $f^{-1}$is the inverse of function *f*.

3.3 Spherical shell

The stress of the spherical shell is expressed as:

$$\sigma(t) = \frac{qr}{2h(t)}. \quad (20)$$

Considering Eq. (7), we have:

$$\frac{dh}{dt} = -Ant^{n-1}\, exp\frac{V\sigma}{RT} = -Ant^{n-1}\, exp\frac{Vqr}{2hRT}, \quad (21)$$

integrating over h_0 to *h* and t_0 to *t*:

$$\int_{h_0}^{h} -\frac{1}{A} exp(-\frac{Vqr}{2hRT})\, dh = \int_{0}^{t} nt^{n-1}\, dt, \quad (22)$$

we find:

$$t = [-\frac{1}{A}\int_{h_0}^{h} exp(-\frac{Vqr}{2hRT})\, dh]^{\frac{1}{n}}. \quad (23)$$

In the equation $t = f(h) = [-1/A\int_{h_0}^{h} exp(-\frac{Vqr}{2hRT})\, dh]^{\frac{1}{n}}$,f expresses the corresponding rule between variable h and variable t.
Substituting $h = f^{-1}(t)$ it into above equations we obtain:

$$\begin{cases} t = f(h) = [-\frac{1}{A}\int_{h_0}^{h} exp(-\frac{Vqr}{2hRT})\, dh]^{\frac{1}{n}}, \\ h = f^{-1}(t), \\ \sigma_{cr}(t) = \begin{cases} \frac{E}{\sqrt{3(1-\mu^2)}}\frac{f^{-1}(t)}{r}, \text{small deflection buckling,} \\ \frac{1}{4}\frac{E}{\sqrt{3(1-\mu^2)}}\frac{f^{-1}(t)}{r}, \text{large deflection buckling,} \end{cases} \\ q_{cr}(t) = \frac{2h(t)\sigma_{cr}(t)}{r}. \end{cases} \quad (24)$$

4 Calculation method of stable bearing capacity of thin shell under corrosion temperature change

This section is based on the equations deduced in the Section 3, and then considers the influence of variable temperature (stress) (Eq.(9a), Eq.(9b), Eq.(9c)) to continue the derivation.
The relevant results are the following.

4.1 Plate

The critical stress expression when the plate loaded edge is subject to arbitrary elastic constraints is:

$$\begin{cases} \frac{dh}{dt_c} = Ant_c{}^{n-1}\, exp\frac{V\sigma}{RT_g}, \\ E(T_s) = \frac{7T_s-4780}{6T_s-4760}E_s\,, 20°C \le T_s < 600°C, \\ \sigma_{cr}(t_c, T_s) = \frac{k\pi^2 \cdot h^2(t_c)\cdot E(T_s)}{12(1-\mu^2)b^2} - k_r\alpha_T(T_s - T_{s0})E(T_s), \\ p_{cr}(t_c, T_s) = \sigma_{cr}(t_c, T_s)\cdot h(t_c). \end{cases} \quad (25)$$

4.2 Cylindrical shell

When the two loaded sides of a cylindrical shell are subjected to arbitrary elastic constraints, its critical stress can be expressed as:

$$\begin{cases} \frac{dh}{dt_c} = Ant_c{}^{n-1} exp\frac{V\sigma}{RT_g}, \\ E(T_s) = \frac{7T_s - 4780}{6T_s - 4760} E_s \text{ , } 20°C \le T_s < 600\text{°C}, \\ \sigma_{cr}(t_c, T_s) = \begin{cases} \frac{h(t_c)}{R}\frac{E(T_s)}{\sqrt{3(1-\mu^2)}} - k_r\alpha_T(T_s - T_{s0})E(T_s), \text{small deflection buckling} \\ \frac{1}{3}\cdot\frac{h(t_c)}{R}\frac{E(T_s)}{\sqrt{3(1-\mu^2)}} - k_r\alpha_T(T_s - T_{s0})E(T_s), \text{large deflection buckling} \end{cases} \\ p_{cr}(t_c, T_s) = h(t_c)\cdot\sigma_{cr}(t_c, T_s), \\ N_{cr}(t_c, T_s) = 2\pi r\cdot h(t_c)\cdot\sigma_{cr}(t_c, T_s). \end{cases} \tag{26}$$

4.3 Spherical shell

The spherical shell is considered as a free structure in this paper. Therefore, its critical stress can be expressed as:

$$\begin{cases} \frac{dh}{dt_c} = Ant_c{}^{n-1} exp\frac{V\sigma}{RT_g}, \\ E(T_s) = \frac{7T_s - 4780}{6T_s - 4760} E_s \text{ , } 20°C \le T_s < 600\text{°C}, \\ \sigma_{cr}(t_c, T_s) = \begin{cases} \frac{h(t_c)}{r}\frac{E(T_s)}{\sqrt{3(1-\mu^2)}}, \text{small deflection buckling,} \\ \frac{1}{4}\frac{h(t)}{r}\frac{E(T_s)}{\sqrt{3(1-\mu^2)}}, \text{large deflection buckling,} \end{cases} \\ q_{cr}(t_c, T_s) = \frac{2\cdot h(t_c)\cdot\sigma_{cr}(t_c, T_s)}{r}. \end{cases} \tag{27}$$

5 Calculation method of stable bearing capacity of corroded thin shells under variable temperature in fire

This section considers the fire control curve, Eq. (16), on the basis of the equations already derived in Section 4, while ignoring the change of wall thickness h during the fire, that is, $h(t_c) = h(t_0)$ (because usually the fire time is extremely short relative to the corrosion period), and the derivation is carried out.

We easily obtain its result as shown below.

5.1 Plate

$$\begin{cases} T_s = (\sqrt{0.044 + 5.0 \times 10^{-5} \alpha \frac{F_i}{V}} - 0.2) t_s + T_{s0}, T_s \leq 700^o C, \\ E(T_s) = \frac{7T_s - 4780}{6T_s - 4760} E_s, 20°C \leq T_s < 600°C, \\ \sigma_{cr}(t_c, T_s) = \frac{k\pi^2 \cdot h^2(t_c) \cdot E(T_s)}{12(1-\mu^2)b^2} - k_r \alpha_T (T_s - T_{s0}) E(T_s), 20°C \leq T_s < 600°C. \end{cases} \tag{28}$$

Among them, the third equation can also be written as:

$$\sigma_{cr}(t_c, t_s) = \frac{k\pi^2 \cdot h^2(t_c) \cdot E(t_s)}{12(1-\mu^2)b^2} - k_r \alpha_T (T_s - T_{s0}) E(t_s). \tag{29}$$

5.2 Cylindrical shell

The calculation method for the stable bearing capacity of a thin cylindrical shell under variable temperature in fire is as follows:

$$\begin{cases} T_s = (\sqrt{0.044 + 5.0 \times 10^{-5} \alpha \frac{F_i}{V}} - 0.2) t + T_{s0}, T_s \leq 700^o \mathrm{C}, \\ E(T_s) = \frac{7T_s - 4780}{6T_s - 4760} E_s, 20°C \leq T_s < 600°\mathrm{C}, \\ \sigma_{cr}(t_c, T_s) = \frac{1}{3} \cdot \frac{h(t_c)}{R} \frac{E(T_s)}{\sqrt{3(1-\mu^2)}} - k_r \alpha_T (T_s - T_{s0}) E(T_s), 20°C \leq T_s < 600°\mathrm{C}. \end{cases} \tag{30}$$

The third equation can also be written as:

$$\sigma_{cr}(t_c, t_s) = \frac{1}{3} \cdot \frac{h(t_c)}{R} \frac{E(t_s)}{\sqrt{3(1-\mu^2)}} - k_r \alpha_T (T_s - T_{s0}) E(t_s). \tag{31}$$

5.3 Spherical shell.

The calculation method of the stable bearing capacity of a thin spherical shell under fire temperature changes is as follows:

$$\begin{cases} T_s = (\sqrt{0.044 + 5.0 \times 10^{-5} \alpha \frac{F_i}{V}} - 0.2) t_s + T_{s0}, \quad T_s \leq 700^o C \\ E(T_s) = \frac{7T_s - 4780}{6T_s - 4760} E_s, \ 20°C \leq T_s < 600°\mathrm{C}, \\ \sigma_{cr}(t_c, T_s) = \frac{1}{4} \frac{h(t_c)}{r} \frac{E(T_s)}{\sqrt{3(1-\mu^2)}}, \ 20°C \leq T_s < 600°\mathrm{C}. \end{cases} \tag{32}$$

The third equation can also be written as:

$$\sigma_{cr}(t_c, t_s) = \frac{1}{4} \frac{h(t_c)}{r} \frac{E(t_s)}{\sqrt{3(1-\mu^2)}}. \tag{33}$$

6 Conclusions

In this paper, a more general corrosion kinetic model is first proposed. Then, based on the model, starting from the basic stability theory, the calculation method of the stable bearing capacity of the thin shell under different corrosion temperature is deduced by combining the temperature stress and the change of steel physical properties under different constraints. Finally, considering the special variable temperature-fire control curve, the calculation method of the stable bearing capacity of the fire variable temperature corroded thin shell is deduced.

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