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Gates' bidding model allows anticipating the probability of submitting the lowest bid in a future auction. Despite its relative simplicity, this classical model has been shown to outperform many other bidding models in real auction settings. However, Gates' model is accurate if, and only if, bidders' bid probability distributions are from the proportional hazards family. Unfortunately, checking this assumption in practice is difficult ex-ante (before the auction takes place) due to limited access to similar previous auctions' information. In this paper we propose an approach to quantitatively measure the tenability of the proportional hazards assumption in real auction settings. By resorting to Monte Carlo simulation, we develop a method to measure the consistency of the pairwise probability matrix that stores the probabilities of every bidder individually underbidding each other. Application of our method will allow bidding decision-makers to assess Gates' forecasts reliability for upcoming auctions.

Keywords: Gates; bidding; consistency; statistics; simulation

MEDICIÓN DE LA HIPÓTESIS DE RIESGOS PROPORCIONALES EN EL MODELO DE LICITACIÓN DE GATES

El modelo de licitación de Gates permite anticipar la probabilidad de presentar la oferta económica más baja en una futura licitación. A pesar de su relativa simplicidad, este modelo clásico ha demostrado ser superior a muchos otros modelos de licitación en subastas reales. Sin embargo, el modelo de Gates es preciso si, y solo si, las distribuciones de probabilidad de las ofertas de los licitadores pertenecen a la familia de riesgos proporcionales. Desafortunadamente, comprobar esta hipótesis en la práctica es difícil a priori (antes de que la licitación tenga lugar) debido al acceso limitado que se suele tener a información de licitaciones previas. En esta comunicación proponemos un método para medir cuantitativamente la defendibilidad de la hipótesis de riesgos proporcionales en situaciones de licitación reales. Por medio de simulaciones de Monte Carlo, desarrollamos un método para medir la consistencia de la matriz de comparaciones pareadas, la cual almacena las probabilidades de que cada licitador presente una oferta económica menor a cada uno de los otros licitadores. La aplicación de este método permitirá a aquellos que toman las decisiones en las licitaciones el evaluar la fiabilidad de las predicciones efectuadas por el modelo de Gates.

Palabras clave: Gates; licitación; consistencia; estadística; simulación

1. Introduction

Marvin Gates published a paper in 1967 which proposed optimal bidding strategies when a bidder was facing one or several competitors (Gates, 1967). Gates' paper also explored other important and current bidding topics such as least-spread strategies [e.g., (Skitmore et al., 2001)] and unbalanced bidding [e.g. (An et al., 2018; Hyari, 2015; Wu and Xu, 2021)].

Gates' all-bidders-known formula lacked mathematical justification, which delayed the recognition of his paper as a seminal one in the bidding domain (Engelbrecht-Wiggans, 1980). This formula is capable calculating the probability of one bidder underbidding a group of competitors whose identities are known. However, in his paper, Gates also indirectly discredited Lawrence Friedman's (1956) bidding model, which triggered an intense debate.

Some researchers attempted to compare the performance of both models with real auction datasets [e.g. (Benjamin, 1972; Benjamin and Meador, 1979)], but their results were inconclusive. Some researchers were in favor of Friedman's model [e.g. (Fuerst, 1976; Morin and Clough, 1969, 1972; Park, 1966)], whereas others were in favor of Gates' [e.g. (Baumgarten, 1970; Dixie, 1974a; b)]. here were also some researchers who claimed that both Friedman and Gates' models could be correct [e.g. (Näykki, 1973; Rosenshine, 1972)], or even both incorrect [e.g. (Stark, 1968)].

The issue that kept arising during this debate was that Gates' model lacked a valid mathematical justification [e.g. (Engelbrecht-Wiggans, 1980)]. Some tried to rectify this situation [e.g. (Benjamin, 1969; Rosenshine, 1972)], even Gates himself (Gates, 1970; 1976b), but they all failed.

As a way of defending the validity of his model, Gates (1976a) published an extensive Monte Carlo-based comparison experiment between his model and Friedman's. In that experiment, Gates managed to prove that his model was far superior to Friedman's in most bidding situations. However, that analysis also attracted significant opposition (Fuerst, 1977). In fact, Gates' Monte Carlo analysis partially misconceived Friedman's model by neglecting bid variability (McCaffer, 1976). Bid variability was dealt with inherently in Gates' model, but it required some additional calculations from Friedman's that Gates (probably unintentionally) omitted (Rothkopf, 1969).

Since Gates' (1979) last attempt to justify the validity of his model, a few more works were published [e.g. (King and Mercer, 1985; 1988; Rothkopf, 1980a; 1980b; Rothkopf and Harstad, 1994)], but none ended the controversy (Crowley, 2000).

This situation lasted until Skitmore et al. (2007) proved that Gates' model is exact (correct) if, and only if, the distributions modelling the bidders' bids belong to the proportional hazards family (PHF). Some examples of proportional hazards distributions are the Exponential and Weibull distributions. In that paper, Skitmore et al. (2007) also provided the long-desired mathematical proof of Gates' *all-bidders known* formula. Skitmore's (2014) analysis of three different auction datasets also found that Gates' model performs better than most bidding models in real contexts.

Hence, Gates' model is accurate as long as the statistical distributions modelling the bidders' bids closely resemble a distribution from the PHF. Checking this assumption is key to assess Gates' forecasts reliability. However, some problems arise when trying to gauge the tenability of the proportional hazards assumption from historical bidding data.

The first problem and, perhaps, most evident, is the frequent shortage of homogeneous historical auctions' data. By homogeneous we refer to the availability of previous auctions which share similar traits with the auction to be forecasted (similar type of project, similar client, similar project size, etc.). Most projects tend to be one-of-a-kind. Hence, it is generally necessary to relax the homogeneity/similarity restriction in order to increase the number of previous auctions with which being able to perform forecasts.

The second problem is the current absence of statistical tests that perform the proportional hazards assumption check on bidders' bids. Filling this void is precisely the aim of this paper. Namely, in this paper we propose an approach to obtain a p-value which measures the probability of the bidders' bids being Exponential (the most common PHF distribution). This particular distribution is chosen both for its convenience (it is one of the simplest distributions mathematically) and also because Gates' all-bidders-known formula is based on a colored-balls in the urn model where the ball draws are perfectly represented by this distribution. Additionally, as the calculation of this p-value by means of an analytical expression seems an unsurmountable task, we will resort to Monte Carlo simulations to calculate the tenability of the PHF assumption given the frequently limited access to a homogeneous auctions dataset.

2. Background: Gates' all-bidders known formula

On page 85 of his 1967 paper, Gates stated that his all-bidders-known formula (Eq. 15 in the original paper) could be "*considered as the mathematical model of a colored balls in the urn problem*". This brief statement gave the floor to many researchers trying to find out which exact urn problem was that, but, as mentioned earlier, none succeeded, not even Gates, until Skitmore et al. (2007).

The scenario in which all competitor bidders' identities are known, is one of the more general cases reported in Gates' 1967 paper. To calculate the probability of bidder *i*'s bid (b_i) winning (P_{iw}) according to Gates, one must apply this formula:

$$P_{iw} = Prob\left(b_i < min\left(b_j\right) \text{ with } j = 2, \dots n\right) = \frac{1}{1+\sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{1-P_{ij}}{P_{ij}}} = \frac{1}{\sum_{j=1}^{n} \frac{1-P_{ij}}{P_{ij}}} \quad (1)$$

Where P_{ij} is the probability of bidder *i* underbidding bidder *j* in an auction with *n* competitors.

The attractiveness of Gates' expression is that, in order to estimate the P_{ij} values, one just needs to perform a simple count of how many times the reference bidder *i* underbid bidder *j* in the past (i.e., submitted a lower bid in previous auctions) and divide it by the total number of times they both competed. That is, no underlying distribution assumption is *apparently* required for Gates' model to be applied. Yet, as mentioned earlier, expression 1 will only be exact if the proportionality assumption holds among bidders. What does this mean, though?

The proportionality assumption is closely related to a transitivity check. This is common to some multicriteria decision making techniques, such as the Analytical Hierarchy Process (AHP) (Saaty, 1994). In AHP, for example, we perform a consistency check over the decision maker's criteria or alternative preferences to ensure their choices are coherent.

In our particular case, if the odds of bidder 1 beating (underbidding) bidder 2 were 4 to 7 (O_{12}=4/7), whereas the odds of bidder 2 beating bidder 3 were 5 to 6 (O_{23}=5/6), then, the odds of bidder 1 beating bidder 3 should necessarily be O_{13}=O_{12}*O_{23}=(4*5)/(7*6)=20/42=10/21. If O_{13}≠10/21, then, bidders' behaviors would not be proportional and Gates' formula would lose accuracy.

The problem is that this kind of proportionality check among bidders' bids is very tricky to perform in most real situations. The major reason is the limited access to previous similar auctions (Ballesteros-Pérez et al., 2015; 2019). This, as data scarcity will produce inaccurate estimates of the P_{ij} values. These rough estimates may not keep any apparent proportionality from the limited encounters we have sampled (even though bidders' underlying bid distribution may actually be proportional).

Gates' model is, in fact, a particularization of Cox' (1972) proportional hazards model. The only differences between both models being that: a) what is time in Cox' model becomes money in Gates' model (the bid values); and b) what are considered covariates in Cox' model become the different competitors in Gates'.

Cox proportional hazards model was first published in 1972, but it was not started to be used by a handful of researchers (mostly statisticians) until 1975 (Breslow, 1975). The conditions that make Cox' model valid in empirical settings are also as hard to test as Gates' in most real situations (Reid, 1994). Yet, to facilitate performing the proportionality check in Gates' model, it is easier to express equation (1) as a function of odds (O_{ij}) instead of probabilities (P_{ij}).

Particularly, knowing that the odds of any bidder *i* underbidding bidder *j* (i.e., O_{ij}) equal:

$$O_{ij} = \frac{P_{ij}}{P_{ji}} = \frac{P_{ij}}{1-P_{ij}} \tag{2}$$

Gates' expression (1) can also be written as a function of odds like:

$$P_{iw} = Prob\big(b_i < min(b_j) \text{ with } j = 2, \dots n\big) = \frac{1}{\sum_{j=1}^{n} \frac{1-P_{ij}}{P_{ij}}} = \frac{1}{\sum_{j=1}^{n} \frac{1}{O_{ij}}} = \frac{1}{\sum_{j=1}^{n} \frac{1}{O_{ji}}} \tag{3}$$

Expression (3) will be the one used moving forward. This, as working with odds makes calculations more straightforward and also closer to the consistency check performed in the pairwise comparisons matrix in the AHP method.

3. Proportionality assumption check

In order to propose an approach to quantitatively measure the tenability of the proportional hazards assumption in a real auctions dataset, we will resort to Monte Carlo simulation. Namely, we will assume that the (limited) sample of auctions we are observing (the set of historical auctions we have managed to gather) is just a random output of many other possible which could have been produced by the same bidders submitting Exponential-distributed bids. Exponential distributions are the simpler PHF distributions that make Gates' model valid.

From now on and for the sake of clarity, we will explain the proposed method to perform the proportionality check along with a case study. These will be the steps followed:

1. Gather a set of (homogeneous) auctions where all relevant bidders participated.
2. Calculate the Odds matrix (O_{ij}) of every bidder underbidding each other.
3. Assimilate each bidder to an Exponential distribution and generate many artificial auctions.
4. Calculate the maximum eigen value (λ_{max}) from the odds matrix of step 2.
5. Calculate the Odds matrix and corresponding λ_{max} value for every artificial set of auctions generated in step 3.
6. Rank the value (in a per-unit scale) of the λ_{max} value generated in step 4 compared to the ones generated in step 5.

The result of step 6 corresponds to the p-value expressing the bidders' level of proportionality (consistency). The closer of this p-value to zero, the higher the chances of bidders' bid distributions coming from a PHF distribution; and the higher the chances of expressions (1) and (3) being accurate. Let's go over these six steps in more detail.

<u>Step 1: Gather a set of (homogeneous) auctions</u>

The first step for a bid decision-maker would be to retrieve a series of auctions as similar as possible to the one to be forecasted. We would look for auctions that took place recently, with a similar scope of works, with similar budgets, location, client, etc. If those similar auctions were scarce, then, it would be necessary to cast a wider net and relax some of the similarity assumptions to gather a minimum number of auctions with which to perform a forecast.

A strict assumption, though, is that the same competitors against whom we want to apply Gates' formula, should have participated. It is not necessary that all competitor bidders have participated in all auctions of our dataset. However, for obvious reasons, the higher their participation rates, the higher the chances of our P_{ij} and O_{ij} values being more representative.

In our example, let's assume we managed to gather 15 auctions in which four (relevant) bidders participated. However, as cautioned above, not all four bidders participated in all of them. These 15 auctions are represented in Figure 1. It is assumed that the bidder *i* submitting the lowest bid eventually won the auction *k*.

Figure 1: The 15-auction dataset with four bidders (from now on, the 'real auctions dataset')

	Bidders' bids ($)→	1	2	3	4
Auction (k) ↓	**1**	75,613.89		76,487.59	
	2	175,401.79	211,015.80	220,114.40	263,641.23
	3	102,811.60	103,593.81		141,425.19
	4	150,395.21	145,576.06	149,984.74	
	5	64,049.44	57,903.34	70,682.64	77,133.78
	6	6,238.53	7,238.87	7,241.67	9,074.34
	7	226,415.88	231,494.95	255,125.25	216,758.74
	8	28,837.87	31,023.39	30,042.45	40,017.69
	9	190,243.50	195,127.45	211,461.38	222,693.82
	10	33,419.21	40,776.28	42,533.72	41,525.55
	11	81,982.02	83,377.94	99,363.01	106,750.86
	12	30,806.52	32,121.32	31,480.90	31,779.74
	13	30,887.85	31,160.08	35,276.13	33,557.57
	14	104,720.52	111,216.28	120,625.52	120,455.12
	15	20,679.50	20,960.41	25,369.39	22,738.93

Step 2: Calculate the Odds matrix (O_{ij})

Now that the set of auctions from Figure 1 are known, it would be easy to perform a count of how many times each bidder *i* (individually) underbid every other bidder *j*. We could also perform the count of how many times each bidder *j* underbid bidder *i*. The ratio of both counts would correspond to the odds of bidder *i* beating bidder *j*, that is O_{ij}. Those ratios would populate the odds matrix shown at the top of Figure 2 (greyed area).

Figure 2: Odds matrix (O_{ij}) calculation from the real auctions' dataset
(O_{ij} = Prob. of *i* underbidding *j* / Prob. of *j* underbidding *i*)

Bidders i↓ j →	1	2	3	4
1	1.00	6.00	13.00	12.00
2	0.17	1.00	5.50	5.50
3	0.08	0.18	1.00	1.40
4	0.08	0.18	0.71	1.00
Prob j winning (Gates' formula) =	**0.75**	**0.14**	**0.05**	**0.05**
Prob j winning (by simulation) =	**0.61**	**0.20**	**0.14**	**0.06**

It is interesting to note that the main diagonal of the odds matrix always has 1s, as every bidder has a 1:1 chance of beating itself. The odds matrix is also reciprocal, that is, $O_{ij}=1/O_{ji}$. This means that the lower triangle values correspond to the inverse values of those hosted in the upper triangle. As can be seen, this odds matrix holds many similarities with the pairwise comparisons matrix used in the AHP method.

Additionally, at the bottom of Figure 2, we have included two probability calculations of every bidder *j* (equivalent to bidder *i* by rows) winning. The calculation performed in the last but one row corresponds to Gates' formula as per expressions (1) or (3). Particularly, Gates' expression (3) merely corresponds to the inverse of the sum of all values of the odds matrix placed in the same column. That is, for example, P_{1w}=0.75=1/(1.00+0.17+0.08+0.08) and P_{2w}=0.14=1/(6.00+1.00+0.18+0.18).

The probabilities of every bidder *j* winning displayed in the bottom row were also obtained by simulation. Namely, in this particular case, the 15 auctions displayed in Figure 1 were generated artificially according to some Lognormal distributions modelling the bidders' behavior. Lognormal distributions are not from the PHF, hence, Gates' probabilities from the upper row should not be very accurate. Yet, depending on the particular choice of the lognormal distributions parameters (i.e., each bidder's lognormal mean and standard deviation), their results may resemble a PHF distribution (for example, when all bidders are represented by Lognormal distributions with very similar means and standard deviations).

Overall, resorting to lognormal distributions modelling the bidders' bids was both representative and convenient. It was representative as it has been studied that most bids distribution in construction auctions closely resemble lognormal distributions (Ballesteros-Pérez & Skitmore, 2017; Baek & Ashuri, 2019; Ballesteros-Pérez et al., 2021). But it was also convenient, as these distributions allowed the authors to easily modify the bidders' behavior at will and generate multiple disparate samples of auction datasets.

Playing with different auction sample sizes, different numbers of bidders and different bidders' behaviors was an essential part of this piece of research. However, those calculations are not reported in this paper for the sake of brevity. Let us just rescue that, the lower the proportionality among bidders' bids, the lower Gates' forecasts accuracy (i.e., the higher the differences between the probability values of the lowest two rows of Figure 2). In the example of Figure 2, a moderate difference of accuracy can be noticed between Gates' probabilities of winning and those calculated by Monte Carlo simulations in the last row (which are considered the most exact approximation of the true probabilities of winning).

<u>Step 3: Generate multiple artificial sets of auctions</u>

If all bidders' bids were proportional, then, they would be closely represented by a distribution of the PHF. The simplest distribution that also keeps hazards constant is the Exponential distribution. In Gates' model, the parameter (λ) of each of these Exponential distributions representing each bidder's bids also coincides with its probabilities of winning (Ballesteros-Pérez et al., 2023).

Hence, we can safely assume that, if a bidder *i*' bids are actually proportional with every other bidder *j*, they could be closely represented by an Exponential distribution whose parameters equals P_{iw}. As noted above, P_{iw} can be calculated for every bidder with expressions (1) or (3). They coincide with the probabilities of each bidder *i* winning the auction, or what is the same, ending in the first position ($P_{(1)j}$).

Following with our example, we assume our bidders {1, 2, 3, 4}'s chances of winning are {0.75, 0.14, 0.05, 0.05}, respectively, as obtained in Figure 2. It is also worth noting that these probabilities of winning cannot be obtained directly from the number of times each bidder won in the 15 auctions from Figure 1. This because these probabilities assume that all four bidders participated all the time and this condition is not fulfilled in auctions 1, 3 and 4.

Hence, once Exponential distributions have been specified for all bidders, we can just generate random bids from each bidder in a similar fashion to the real auctions' shown in Figure 1. This means we generate artificial Exponential-like bids for every bidder *j* (*j*=1…4) and auction *k* (*k*=1…15) but removing (not considering) those in which a particular bidder did not participate. By observing Figure 1 it is easy to see that bidder 2 did not participate in auction 1, bidder 3 did not participate in auction 3, and bidder 4 did not participate in auction 4. An iteration reproducing this same participation pattern is reproduced in Figure 3. This particular set of 15 auctions was calculated 999 more times with Monte Carlo simulations to perform the calculations of the next steps.

Figure 3: One of the 1000 iterations of the artificially-generated 15-auction dataset.

Bidders (i) →	1	2	3	4
Exponential distribution (λ) = $P_{(1)j}$ =	0.75	0.14	0.05	0.05

		Artificially-generated bidders' bids ($)			
	Bidders (i) →	1	2	3	4
Auction (k) ↓	1	0.31		3.35	
	2	1.17	6.73	57.31	67.24
	3	0.01	29.90		36.82
	4	1.61	7.29	28.58	
	5	1.36	2.89	19.21	101.47
	6	0.57	5.23	8.70	15.59
	7	2.91	4.18	55.40	3.16
	8	1.29	15.42	10.06	48.72
	9	2.51	2.52	31.21	40.75
	10	4.70	5.06	18.49	33.14
	11	1.69	6.29	72.36	10.48
	12	0.16	0.07	17.65	49.96
	13	1.47	6.19	11.44	3.47
	14	1.17	5.27	42.88	21.10
	15	1.08	5.71	38.92	30.47

Step 4: Calculate the maximum eigen value (λ_{max}) from the Odds matrix of the real auctions.

From the Odds matrix obtained from the real auctions dataset shown in Figure 1, we can calculate the maximum eigen value as shown in Figure 4. This calculation closely resembles the consistency calculation performed in the AHP method with the pairwise comparisons matrix (Saaty, 1994).

Figure 4: Eigen value (λ_{max}) calculation performed with the real 15-auction set from Figure 1.

	Actual Odds matrix = O_{ij}					Prob i winning = $P_{(1)i}$		$O_{ij} \cdot P_{(1)i}$		$O_{ij} \cdot P_{(1)i} / P_{(1)i}$
Bidders i↓ j →	1	2	3	4		(Gates' formula)				(element by element)
1	1.00	6.00	13.00	12.00		0.75		2.81		3.73
2	0.17	1.00	5.50	5.50		0.14		0.81		5.96
3	0.08	0.18	1.00	1.40	×	0.05	=	0.20	→	4.09
4	0.08	0.18	0.71	1.00		0.05		0.17		3.44
									Avg. = λ_{max} =	**4.31**
									p-value =	0.190

Basically, we multiply the Odds matrix by the probabilities of winning obtained with Gates' expression (3). This allows us to obtain the values labelled as $O_{ij} \cdot P_{(1)i}$ in Figure 4. An element-by-element division of the latter values with Gates' probabilities $P_{(1)i}$ yields the values in the rightmost column. Finally, the average of those values corresponds to the maximum eigen value of our Odds matrix (λ_{max}=4.31 in our example).

This eigen value will generally be equal or higher than the number of bidders *n* (4 in our example). However, the bigger the difference between λ_{max} and *n*, the lower the chances of the bidders' bids being proportional.

Step 5: Calculate the Odds matrices and λ_{max} values for all Monte Carlo iterations.

Following the exact same calculations described in steps 2 and 4, we apply them now to each of the 1000 iterations artificially generated with Monte Carlo simulations emulating Exponential bids (as described in step 3). An analogous calculation to the one we performed in Figure 4 with the real auctions dataset is shown in Figure 5, but in this occasion applied to the 15 artificial auctions generated in Figure 3.

Figure 5: Consistency check (λ_{max} calculation) performed with the artificially-generated random sample of 15-auction data of Figure 3.

Bidders i↓ j→	Simulated Odds matrix = O^*_{ij} 1	2	3	4		Prob i winning = $P^*_{(1)i}$ (Gates' formula)		$O^*_{ij} \cdot P^*_{(1)i}$		$O^*_{ij} \cdot P^*_{(1)i} / P^*_{(1)i}$ (element by element)
1	1.00	13.00	14000001	13000001		0.93		3.84		4.14
2	0.08	1.00	12.00	5.50		0.07		0.14		2.02
3	0.00	0.08	1.00	1.40	×	0.00	=	0.01	→	81788.05
4	0.00	0.18	0.71	1.00		0.00		0.01		165695.95
								Avg. = λ^*_{max} =		**61872.54**

From those repeated calculations for all iterations, we obtain another 1000 λ_{max} values. The difference of the λ_{max} value obtained in Figure 4 and these other 1000 λ_{max} values is that the latter are certain to come from bidders behaving like distributions coming from the PHF.

It is also worth noting that some of the λ_{max} values may yield exceptionally high results (61872>>4 in the example of Figure 5, for instance). This happens because in some of the 1000 iterations, some bidders never won another, hence, their odds would approach infinity. In our example, bidder 1 always underbid bidders 3 and 4 in the 15 auctions of that particular iteration. Hence, O_{13} and O_{14} contain very high figures (14000001 and 13000001, respectively) compared to other cells of the odds matrix. These exceptionally high values attempt to resemble infinity.

Step 6: Obtain the proportionality p-value from the actual vs simulated λ_{max} values.

The last calculation step involves sorting the 1000 λ_{max} values in increasing order (from lowest to highest) and find out in which position the real auctions' λ_{max} value can be found. Dividing that position (ranking) by the total amount of Monte Carlo iterations (1000 in our example) would produce the p-value that represents the chances of our bidders' bids being proportional.

In our example, the λ_{max} value from Figure 4 was 4.31. That particular λ_{max} value fell in the 190th position among the 1000 λ_{max} values obtained as in Figure 5. That p-value corresponded then to 190/1000=0.190.

4. Results and Discussion

The relevant question is: is a p-value of 0.190 too high or too low to accept the proportionality assumption before applying Gates' formula? Well, as usual in statistics, there is no definitive answer.

If we were to 'accept' the hypothesis that all bidders are proportional, then we would seek a p-value<0.05. And if we were to 'reject' the hypothesis that all bidders are proportional, then we would seek a p-value>0.95.

Our p-value of 0.190 falls in between 0.05 and 0.95. Hence, we cannot accept, nor reject the proportionality assumption with a high level of confidence. Yet, it seems bidders are somewhat closer to being proportional (closer to 0) than otherwise (closer to 1).

However, by our p-value not being that close to 0, we can expect some level of inaccuracy when applying Gates' all-bidders-known expression. Namely, if we go back to the two bottom rows of Figure 2, we obtained that Gates' expression (3) results were {0.75, 0.14, 0.05, 0.05} for each of the four bidders, respectively. However, the exact probabilities of each of them winning calculated with Monte Carlo simulation were {0.61, 0.20, 0.14, 0.06}. There certainly are differences between both sets of values. But how big or small those differences are may depend on the bid decision-maker's eyes.

Additionally, it is worth noting that in most real auction settings, the true probabilities of winning will never be known. Hence, counting on some estimates like the ones provided by Gates' expressions can become very helpful.

Therefore, even though a p-value may not always be helpful to accept or reject the proportionality assumption hypothesis among bidders' bids, it does provide richer quantitative information on the tenability of such assumption. Arguably, the latter is preferred.

5. Conclusions

For a prolonged period, there was a heated debate on the superiority of bidding models: Friedman's (1956) or Gates' (1967). The issue was significant, as many classical bidding models shared assumptions and hypotheses with one of these models. Although Friedman's model is mathematically correct, it requires a lot of information and is not suitable for most construction bidding situations as it necessitates the specification of all bidders' bid distributions. In contrast, Gates' all-bidders-known bidding model was considered empirical, but it is accurate only when bidders' bid distributions come from the proportional hazards family (PHF). In fact, Gates' model is a specific version of the Cox (1972) proportional hazards model, which was published later.

Gates' model can be represented by a *colored balls in the urn model*, as Gates himself initially claimed. However, even when bidders' bids may not belong to the PHF, Gates' model is much easier to apply in most construction auction settings and still remains reasonably accurate compared to Friedman's model.

However, there is frequently an insufficient number of previous auctions to infer all bidders' P_{ij} probabilities with a high degree of precision. These P_{ij} values correspond to the probabilities of the reference bidder i underbidding the rest of competitors j individually. This also leads to a severe difficulty in checking whether the probabilities of some bidders underbidding each other are proportional as well. The latter is also relevant as the higher this unproportionality, the higher the chances of Gates' all-bidders-known formula yielding inaccurate results.

In this paper we have proposed a Monte Carlo-based approach to calculate the probability of the bidders' bids being proportional with each other. To do so, we have assumed that bidders' bids resembled Exponential distributions whose parameters coincided with the individual probabilities of each bidder winning when facing all competitors simultaneously in the same auction. However, by resorting to artificially-generated data with Monte Carlo, we have also been able to reproduce auctions in which not all of them competed at the same time (like in real auctions). This yields realistic and representative results.

Application of our method will allow bidding decision-makers to assess Gates' forecasts reliability for upcoming auctions. This is relevant, as Gates' model is one of the most widespread bidding models found in the literature. With our contribution, we ensure that users will be able to discriminate in which situations Gates' model may render unreliable results. In the latter situations, other bidding models might constitute a better alternative.

6. Limitations and future research continuations

In this paper, we have provided evidence on the existence of a positive relationship between the p-value measuring the proportionality found between the values of the Odds matrix and the accuracy of Gates' forecasts. However, it remains unclear what kind of relationship that is exactly (linear, quadratic, etc.) Similarly, it would be interesting to understand how many auctions are needed to reach a reliable calculation of the p-value, perhaps also considering the number of competitors as an additional independent variable. All these aspects are to be developed in a future research journal paper.

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