Models for Geometry: thoughts for an interdisciplinary dissemination

Laura Carlevaris, Graziano Mario Valenti

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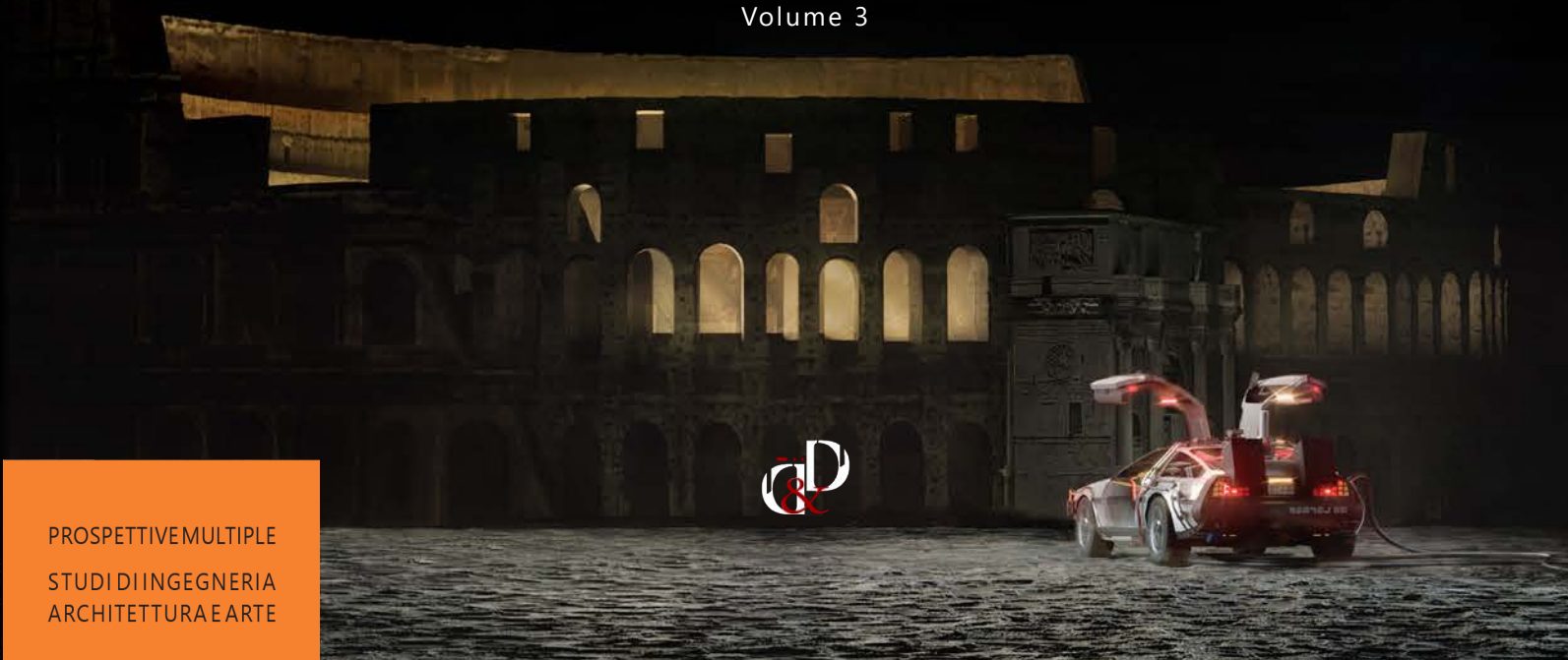
Digital & Documentation

Reading and Communicating Cultural Heritage

Volume 3



PROSPETTIVE MULTIPLE

STUDI DI INGEGNERIA
ARCHITETTURA E ARTE

Laura Carlevaris, Graziano Mario Valenti

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Reading and Communicating Cultural Heritage

Volume 3

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The volume consists of a collection of contributions from the seminar *Digital & Documentation. Reading and Communicating Cultural Heritage*, realised on online platform on December 4th, 2020. The event, organized by Department of History, Representation and Restoration of Architecture, Sapienza University of Rome, promotes the themes of digital modeling and virtual environments applied to the documentation of the tangible, intangible and natural Cultural Heritage. The event has provided the contribution of external experts who are engaged in the management and conservation of the most important Italian cultural assets.
The scientific responsible for the organization of the event is Prof. Graziano Mario Valenti, Sapienza University of Rome.

This publication is made with the contribution of DSDRA, Department of History, Representation and Restoration of Architecture of Sapienza University of Rome.

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Department of History, Representation
and Restoration of Architecture
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The event *Digital & Documentation - 2020* has seen the participation of professors, researchers, scholars and private Institutions

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His scientific interests deal with historical architectural drawing and survey, critic analysis of the connections between drawing, images and texts in historical architectural librarian production/ press, interactions between architectural shapes, design and Geometry. UID adherent fellow.

07 MODELS FOR GEOMETRY: THOUGHTS FOR AN INTERDISCIPLINARY DISSEMINATION

MARTINO PAVIGNANO

Abstract

The contribution proposes a series of methodological reflections aimed at the knowledge and enhancement of the models of mathematical surfaces preserved in the Collections of the University of Torino, as expressions of cultural assumptions that allow us to understand Geometry as a language and connection tool between Architecture and Mathematics. These artifacts, over time, have made their geometric assumptions tangible so that, to date, a critical reinterpretation is proposed aimed at their reinterpretation, including digital, as the first design step of their renewed prototyping.

Il contributo propone una serie di riflessioni metodologiche tese alla conoscenza e alla valorizzazione dei modelli di oggetti matematici conservati presso le Collezioni dell'Università di Torino, quali espressioni di presupposti culturali che permettono di intendere la Geometria quale linguaggio e strumento di connessione tra Architettura e Matematica. Tali artefatti, nel tempo, hanno reso tangibili i propri presupposti geometrici tanto che, ad oggi, se ne propone una rilettura critica finalizzata alla loro reinterpretazione, anche digitale, quale primo *step* progettuale della loro rinnovata prototipazione.

Foreword

The contribution proposes a thought on a methodology for the valorisation of physical models for the study of geometric surfaces belonging to museum heritage ff the University of Torino. They are preserved at the "G. Peano" Library (Dept. of Mathematics) and their interest arises stronger even due to the needs highlighted in the historical period that started on the first months of 2020. This thought affects both the accessibility of this heritage, as well as its dissemination to a heterogeneous unspecialized public, also in response to Third Mission actions of the two Universities involved.
The interest in these artifacts arises within the MAG. IA interdisciplinary research project, based on the effective collaboration between the disciplines of Architecture and Mathematics, in the respective declinations of Drawing (Icar/17) and Geometry (Mat/03). It focuses on the recognition and critical analysis of Geometry as a shared language, which can also be used as a tool for the enhancement of heritage (architectural and otherwise).
In this sense, it is important to remember that Guarino Guarini, architect, and mathematician, upheld the fundamental importance of Geometry for Architecture [Guarini 1737, p. 3], to be considered the result of a process of interpretation of mental models geometrically analysed and defined [Leonardis 2016, pp. 93-94], or expression of forms of representations based on the geometric foundations of Descriptive Geometry [Bianchini 2008, pp. 27-28].
In these terms, the MAG.IA project refers to the fundamentals and specific declinations of Geometry as possible interpretative expressions [Migliari 2009; Gay 2000; Giaquinto 2007; Friedman 2018] of two different ways of analysing, describing, and representing the world.

State of the art

Once defined the specific cultural and disciplinary context, it is necessary to promptly highlight the value attributed to the idea of tangible model in its primary essence as physical artefact, built starting from a project. To this end, in the interdisciplinary spirit of the project, a reinterpretation on two levels is proposed: both in the role of tangible artifact, created to facilitate the study and visualization of abstract concepts, in the purely mathematical declination [Giacardi 2015], as well as in the role of medium for the dissemination/ enhancement of cultural heritage, through its reduction 'to scale' [Cumino et al. 2021]. In this sense, the most recent thoughts developed within the research project led to consider the physical model as the result of a figurative path of eidetic nature: sometimes it is speculative, but it is always recognizable as an artifact of great value (not only historical) for both Architecture and Mathematics [Cumino et al. 2020]. In the first case (fig. 1), Massimo Scolari analysed the different values the physical model gained, referring to the process of morphological transformation that saw its meaning shifted from that of a votive icon to that of a validation tool for the project, closely linked to the libido edificandi. Nevertheless, A. Smith specified its essence as a fundamental "machine" for "mediating between divine chaos and humanity's cosmos" [Smith 2004, p. 122]. Likewise, for the mathematical context (fig. 2), Marcs D. Giaquinto confirmed the epistemolog ical role assumed by the physical model in the field of Visual Thinking for Mathematics [Giaquinto 2007], clarifying its function of material projection. Moreover, Michael Friedman highlighted the role of the physical model for the study of Geometry in Klein's view "to understand the relations valid for them as evident consequences of the principles of spatial intuition" [Friedman 2018, p. 123].

Fig. 1 - Architecture and models: from symbolic to analytical function. Theatrical scene, I cent. DC; Fortress of Zara, plan relief, 1566; Domenico Cresti, Michelangelo presenting his model of San Pietro to the Pius IV, 1618-1619; Antoni Gaudì's study strings and weights model, XIX cent. first quarter [Scolari 2005; Smith 2004].

Fig. 2 - Mathematics and models: Fabre de Lagrange's string surface models, 1872 (Royal Society 2012).

Object of study

The collection of models of mathematical surfaces of the Peano Library (fig. 3) was born at the behest of the mathematician Enrico d'Ovidio between 1880 and 1881 [Giacardi 2015, p. 22; see also Giacardi 2004], following the remarkable success received from such artifacts at the academic level following the early works of Felix Klein and Alexander von Brill [Seidl 2018], that gained the favor of the scientific community also thanks to the functional inclusion in specific technical Catalogs [Pavignano et al. 2020., pp. 3661-3662]. The collection accounts about seventy models made with different techniques: brass and silk threads, plaster, glass, cardboard. The same techniques correspond to different possibilities of interpreting the artifacts, since the wire models highlight ruled surfaces, the plaster ones, as well as the glass sets, emphasize the overall nature of surfaces, while the cardboard models represent them using their notable sections. Some mathematical objects are therefore present in two or more types, as discussed later.

This heritage is currently recognized as an expression of that scientific/mathematical culture of a positivist nature which has its roots in Monge's successors' studies on Descriptive Geometry [Moon, Abel 2016; Gay 2000].

Since the end of the 20th century, the scientific community became aware of the importance of this heritage, starting to investigate their value of material expression of mathematical culture of the time [Palladino, Palladino 2001; Ferrarese 2004]. Meanwhile, following the success of digital tools, i.e. ray tracing software [see Sdegno 2004], collections of virtual alter egos were created. In the case of the UniTo models, numerous digital artifacts were created with PovRay (fig. 4) and published online by Professor Ferrarese (early 2000s and still accessible).

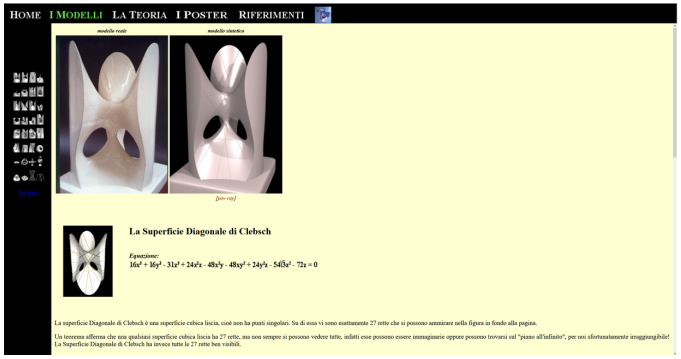
Regardless of the possible digital interpretations, the UniTO heritage is currently not easily accessible. In fact, this collection is kept in the archives of the Peano Library (fig. 5), thus being available only to authorized scholars. Moreover, a general public may encounter difficulties in analysing its in-depth use (or not only aesthetic) due to the requirements for specific knowledge.

The models were created with the primary purpose of visualizing abstract objects, studying their peculiarities, but, even if they have already been recognized as a heritage, they often have been interpreted as Wunderkammer objects, even though they were subjects of those "didactic theaters" which were an integral part of technical and polytechnic culture [Gay 2000, pp. 43-47].

This research, however, aims at placing the main geometric values of these objects in the foreground, since they are configured both as the result of a process of analysis of mathematical entities, and as the result of a real process of communication and visualization of the same. Furthermore, they can become the basis for renewed types of communication of Geometry, between Architecture and Mathematics. since it is known that this kind of heritage, once was a harbinger of interesting ideas for the renewal of architectural form, within the modernist architecture and not only [see Vierling-Classen 2010; Farinella, Baglioni 2018; Greco et al. 2018]. Thus, it is also important to summarize the main features that allow a possible definition of recognized interdisciplinary heritage, between the two disciplines.

In this regard, we take exemplify the practices of enhancement/communication already developed in the Italian and international context, where important collections the are variously exhibited online. I.e. Universities of Pavia, Naples Federico II, Padua; University of Coimbra; Institut Henri-Poincaré, Museum of the University of Tübingen; University of Arizona; Smithsonian Institution and Univestity of Illinois (fig. 6) provide different accesses to their specific heritage.

In all these cases, it is possible to recognize a great attention for the mathematical interpretation of these



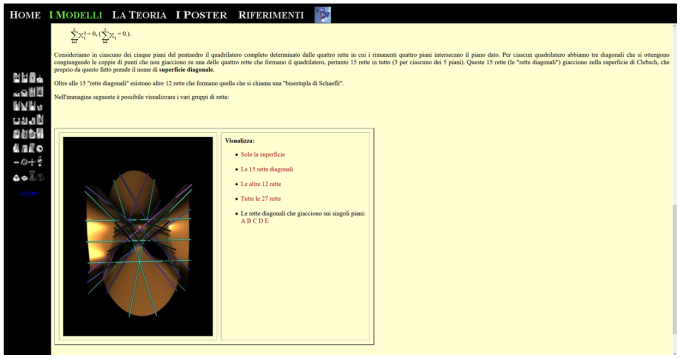


Fig. 3. Selezione di modelli della collezione UniTo (M. Pavignano 2020).

Fig. 4. Digital collection of models of the Biblioteca Peano.

artifacts, mostly thanks to the use of formulas to represent the geometric realities of models, with specific interests for the need to 'shaping' an abstract idea, while defining the contours regarding the production of aids for the teaching of mathematics for mathematicians. In this sense, there is little interest in the 'structural' substance of these products, which can be defined as the results of semi-artisanal processes that clearly avoided the need for numerical control of the representation, thus declined in the form of symbolic elaboration of shapes.

In this sense, it is important to underline that these products are closer to called unique pieces [see Bortot 2019].

In this direction, the project raises questions relating to the dimension of the enhancement of the concept of shared heritage and the documentation of the cultural values of which these artefacts are bearers.

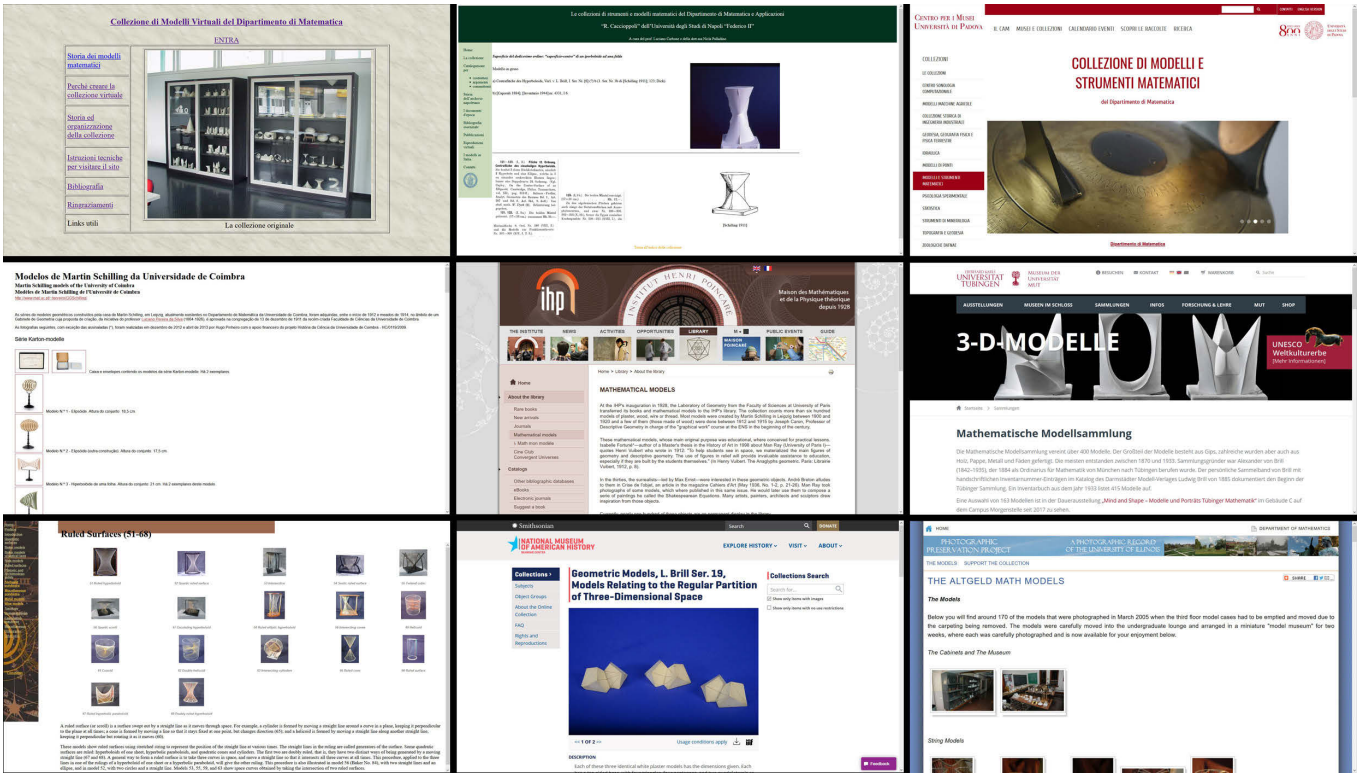


Fig. 5. Display cabinets for models' presentation at the Biblioteca Peano (M. Pavignano 2020).

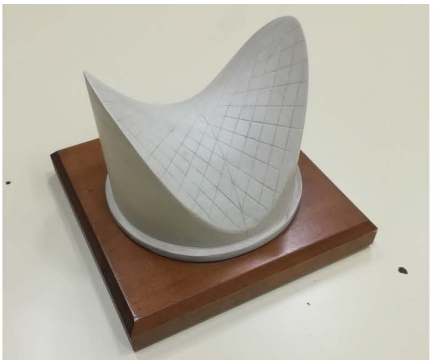
Fig. 6. Examples of collections of models on the web.

Methodological process

We then structured the methodological process by identifying a model that was represented in the greatest number of types. This model is the hyperbolic paraboloid (fig. 7), a surface of great interest for both disciplines [see Inzerillo, Di Paola 2012]. Obviously, the three models do not represent the exact same surface, nevertheless they propose three different interpretations. The string model highlights the nature of the ruled surface, in addition to the notable parabolic and hyperbolic sections; the plaster model, whose surface is engraved with portions of the (infinite) generatrixes, emphasizes the continuity of the surface by offering as a valid, even if not explicit, synthetic representation. The third model shows vertical parabolic sections, integrated by the

traces of one horizontal hyperbolic section proposes an extremely limited reinterpretation, since the viewer must fulfil the virtual reconstruction of the surface by himself. On the other hand, it is important to emphasize that the third solution presents itself as the simplest to be reproduced.
This analysis highlights how the different models communicate the same geometries in a different way. The degree of approximation with which they were made does highlights the need of a proper survey, at least for the main purpose set out here. In fact, the feasibility for a subsequent development of the project is not excluded. These three models highlight a fundamental aspect of their conception, in fact we express the choices made for the material discretization of the represented surface which, in the intent of the proposed study, can support the definition of renewed critical interpretations of these prototyping processes.
Another possible implementation is the construction of digital artifacts modelled with Dynamic Geometry Software, which make possible the coexistence of the visual representation of the object and its clearly mathematical analytical description (fig. 8), defining an interactive and easily implementable tool.

The same surface can digitally represented with CAD software (fig. 9). These models become tools for direct exploration the object and for prototyping it into a tangible artefact. We investigated the possibilities of laser cut and 3D print. In both cases, we started with the definition of the digital model and its subsequent elaboration for the creation of models for vertical and horizontal sectional planes in vegetable cardboard, or for the creation of solid models in PLA (fig. 10).
These artifacts represent a reinterpretation of models of the Peano collection and highlight their geometric characteristics by integrating the original weaknesses by means of an interaction made possible by our design choices. The models, in fact, have been created to facilitate mutual interpenetration, by proposing complementary physical representations.
Models were used during the events linked to the European Researchers' Night 2020, thus becoming mediated by the digital medium and setting up a series of short documentaries and subsequently made available on social network platforms (YouTube).
The primary purpose of the events was to make high school students participate in the communicative potential of prototyped models, starting from their geometric genesis as ideal prosecution of the functions performed by historicized artifacts.

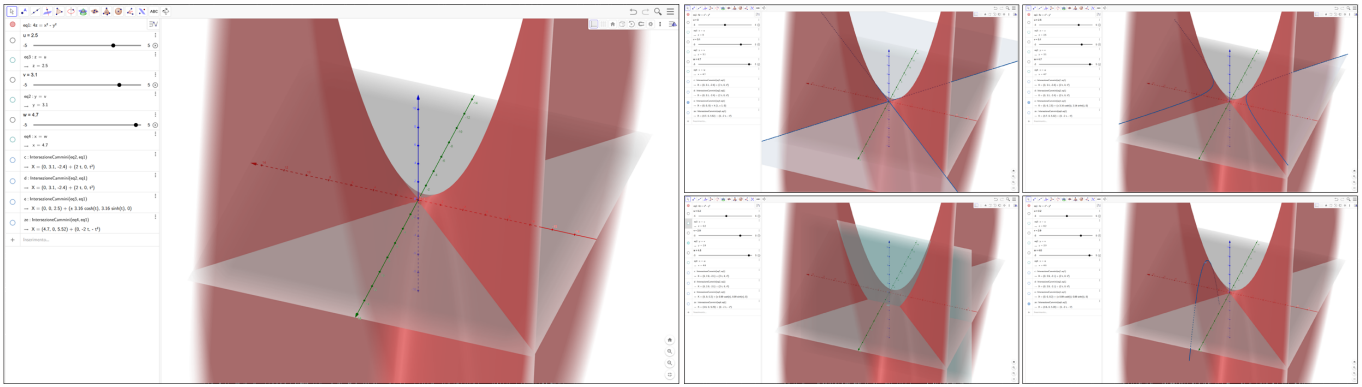
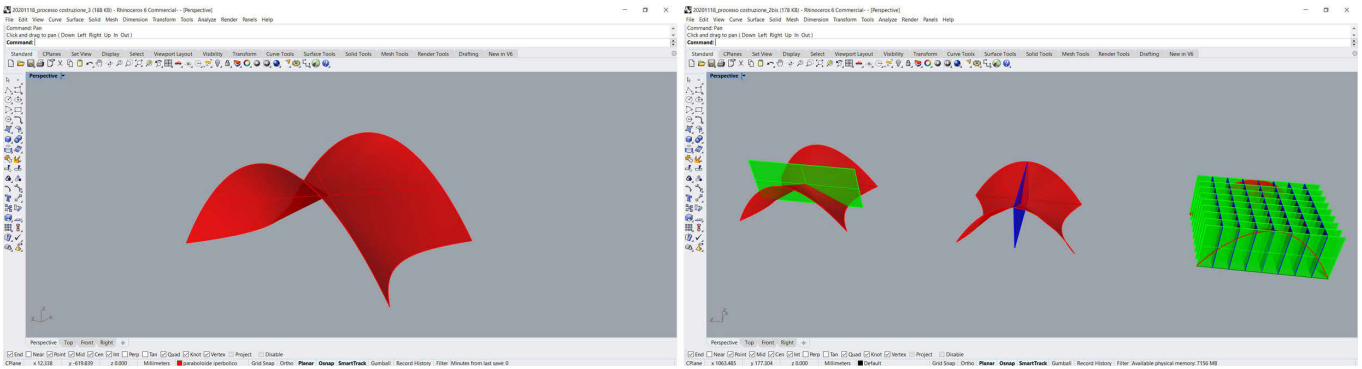
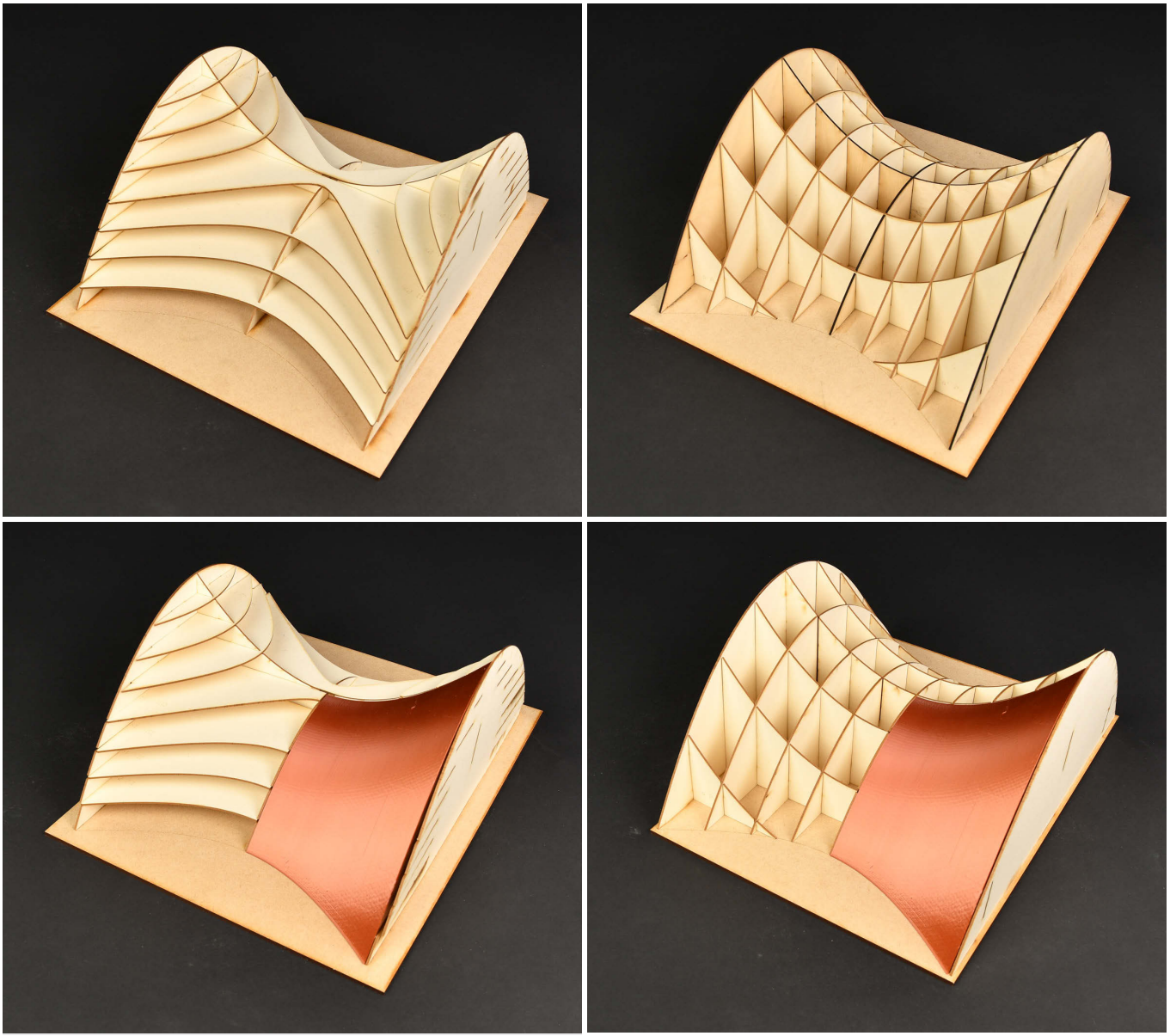



Fig. 7 - The results of the intervisibility analysis at territorial scale from several simultaneous points of view.

Fig. 8 - DGS model describing a hyperbolic paraboloid and its sections (courtesy of C. Cumino).

Fig. 9 - Digital models for communication and prototyping of new models (elab. M. Pavignano).





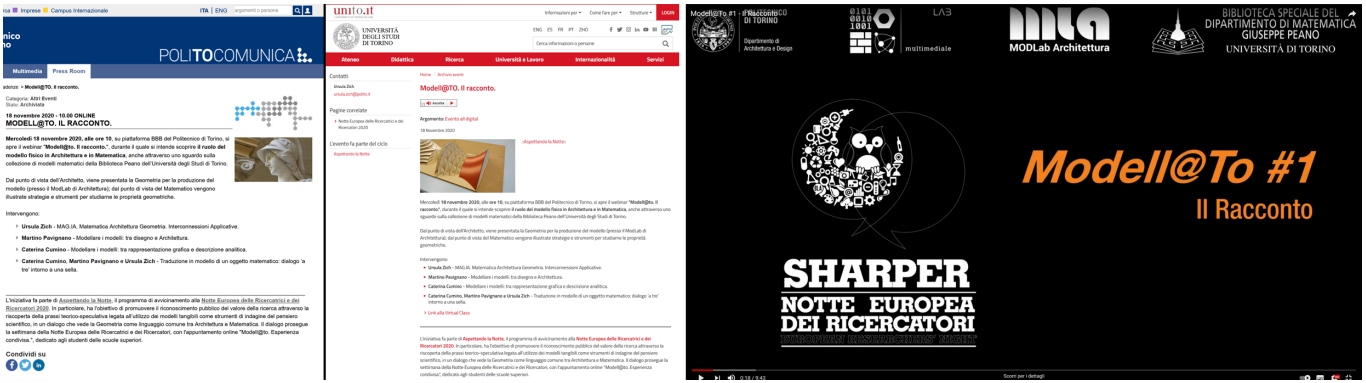


Fig. 10 - Laser cut and 3D print models and their interaction (elab. M. Pavignano).

Fig. 11 - Different possibilities of enhancing models' knowledge and communication: European Researchers' Night 2020 (MAG.IA team).

Discussion and (open) conclusion

The experience discussed clearly aims at defining new supports for the interdisciplinary dissemination (referring to the last two years of high school and first year of university) between Architecture and Mathematics, to support a mutual dialogue.
Therefore, it is clear how this methodology allows an interconnection between the physical model of heritage value, its updated reinterpretation in the light of the possibilities offered by rapid prototyping techniques and their mediation through digital tools. Furthermore, paraphrasing what Graziano Mario Valenti asserted [Valenti 2019, p. 47], it is again highlighted how Geometry, by means of Drawing as a tool of ideation and generation, can manifest itself through physical and digital models. In this sense, through the hybridization of languages and communication methods, it becomes possible to propose opportunities for the active enhancement of a shared heritage, as the bearer of specific geometric characteristics useful both for the definition of its mathematical properties, as well as possible suggestions for understanding of complex architectural postulates.

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Notes

1. *Mathematics Architecture Geometry. Interconnections and Applications*. 2018-on going.

2. <www.toonz.com/personal/todesco/superfici_algebriche/index.html> (last accessed August 18 2021).

3. <www.unito.it/eventi/modellto-il-racconto> (last accessed August 18 2021).

4. <www.youtube.com/watch?v=zxiHU1dqJb0&t=76s> (last accessed August 18 2021). In the initial intentions of the project, it was envisaged the possibility of using the models during a series of guided visits to the Peao Library, however this was not possible due to evident limitations linked to the contingent situation.

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