Grid-Shell Multi-step Structural Optimization with Improved Multi-body Rope Approach and Multi-objective Genetic Algorithm

Original
Grid-Shell Multi-step Structural Optimization with Improved Multi-body Rope Approach and Multi-objective Genetic Algorithm / Melchiorre, J.; Soutiropoulos, S.; Manuello Bertetto, A.; Marano, G. C.; Marmo, F.. - STAMPA. - 437:(2024), pp. 62-72. (2nd Italian Workshop on Shell and Spatial Structures, IWSS 2023 Torino (Ita) 26 June 2023 through 28 June 2023) [10.1007/978-3-031-44328-2_7].

Availability:
This version is available at: 11583/2984105 since: 2024-06-12T21:30:24Z

Publisher:
Springer Science and Business Media Deutschland GmbH

Published
DOI:10.1007/978-3-031-44328-2_7

Grid-shell Multi-step Structural Optimization with Improved Multi-body Rope Approach and Multi-objective Genetic Algorithm

Jonathan Melchiorre[1*][0000−0002−8721−8365], Stefanos Soutiropoulos[1], Amedeo Manuello[1][0000−0003−1474−0176], Giuseppe Carlo Marano[1,2][0000−0002−8748−0312], and Francesco Marmo[3][0000−0002−3456−2867]

[1] Politecnico di Torino, DISEG, Department of Structural, Geotechnical and Building Engineering, Corso Duca Degli Abruzzi, 24, Turin 10128, Italy.
`jonathan.melchiorre@polito.it`
[2] College of Civil Engineering, Fuzhou University, Fuzhou 350108, China.
[3] University of Naples Federico II Department of Structures for Engineering and Architecture.

Abstract. Grid-shell structures are popular in roofing for their aesthetic appeal and structural advantages, enabling the construction of lightweight, large roofs using slender elements. However, their widespread use is limited by the complexities involved in their construction. This paper presents an innovative solution to address the challenges associated with grid-shell construction. The proposed approach combines the enhanced Multibody Rope Approach (i-MRA) form-finding method with metaheuristic optimization algorithms to identify optimal design solutions from both structural and constructional perspectives. To facilitate practical implementation, a parametric code was developed using MATLAB, which was later converted to C# for integration with the parametric design software ”Grasshopper”. A multi-objective optimization problem was formulated to minimize the use of different structural elements, reduce material consumption, account for production waste, and ensure compliance with structural verification requirements. The optimization process yielded a Pareto front solution that enables the conceptual design of grid-shell structures. This approach provides an efficient and innovative solution to the complex construction of grid-shell structures, which could potentially pave the way for their widespread use in the future.

Keywords: Structural optimization · Form-finding · Pareto front · Gridshell · Multi-objective optimization.

1 Introduction

Structural optimization plays a crucial role in achieving efficient and cost-effective designs in various fields of engineering [1–3]. By utilizing optimization techniques [4, 5], engineers can enhance the performance of structures while minimizing material usage and construction costs [6]. In the realm of gridshell structures, which offer unique architectural possibilities, structural optimization becomes even more vital to strike a balance between form, function, ease of construction management and resource utilization [7]. In this paper, a comprehensive approach for the optimization of gridshell structures is presented. The method is specifically focused on the combination of form-finding techniques with meta-heuristic multi-objective optimization methods to identify a range of optimal solutions to guide the designer in the preliminary design phase. The proposed optimization methodology combines the improved Multi-body Rope Approach (i-MRA) method for form-finding of gridshell [8] with a multi-objective optimization genetic algorithm. The i-MRA method enables the determination of structurally optimal gridshell shapes by considering the given loading conditions, while the multi-objective optimization algorithm minimizes material usage while reducing the construction complexity by minimizing the number of types of elements to be handled on the construction site. The entire process and analyses are conducted using a self-made code in MATLAB [9], providing a versatile computational framework for optimization and analysis. To facilitate the practical application of the i-MRA form-finding method, a software component has been developed to integrate it within parametric design software Grasshopper, a tool that runs in Rhinoceros3D environment [10]. This component enhances accessibility by allowing structural designers and architects to utilize the i-MRA method in the design of gridshell structures. The primary objectives of this study are two-fold: first, to simplify the construction management of gridshell structures by reducing the number of different structural components required for their construction; and second, to minimize material consumption through the optimization process. Furthermore, the Cutting Stock Problem is incorporated to optimize the cutting pattern and minimize material waste during the production of structural elements. By achieving these objectives, the proposed approach aims to streamline the construction process, reduce costs, and promote sustainable practices in gridshell design. To demonstrate the effectiveness of the methodology, two case studies are presented. The first case study focuses on a square gridshell subjected to self-weight and various nodal forces. Optimization results are presented in terms of the number of different structural element types and the quantity of material used. The second case study involves the optimization of a more complex gridshell. Results are compared for different loading conditions, considering the number of different element typologies and both net material consumption and waste.

2 The MRA for large Grid Shell: The base assumption and the improved version

The Multibody Rope Approach (MRA) [11, 12] is a cutting-edge method for designing gridshell structures. MRA is specifically tailored for gridshell constructions that utilize free-forms and standardized building elements. By employing a dynamic model of falling bodies in space and time, MRA iteratively calculates the final equilibrium configuration for each node using D'Alembert's principle.

In the MRA the structural elements are modelled as ropes that connect the masses in the nodes. Specifically, the rope element exerts forces on the masses so that they do not move apart beyond the prescribed distance. The aim is to generate a geometry that is both structurally efficient and composed a reduced number of structural element typologies. The final equilibrium configuration of the structure is an inverted representation of the hanging net, resulting in a visually striking design.

In order to achieve an efficient construction of a gridshell, it is important to consider the number of required beam elements and their respective lengths. Thus, it is essential to determine the number of structural elements in the final configuration that possess a length equivalent to the target length l_{rope}. In practical applications, the complexity of gridshell assembly increases proportionally with the number of distinct elements needed for construction. Therefore, it is crucial to classify structural elements into three categories based on their length.

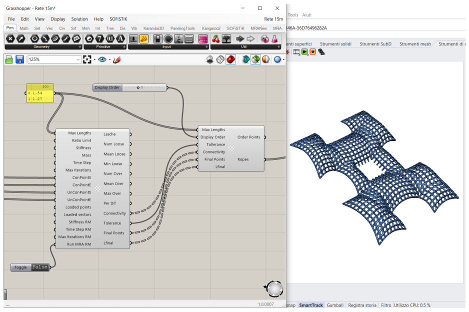


Fig. 1: Grasshopper i-MRA component.

The impovred Multibody Rope Approach (i-MRA) method enhances the MRA approach by incorporating optimization techniques that facilitate the automation of the construction process. There are three main improvements to i-MRA:

- Multiple Orders MRA (MO-MRA) involves grouping together structural elements that share the same length, which reduces the number of distinct structural components required for construction.
- Repulsive Nodes MRA (RN-MRA) applies a repulsive force field to the dynamic model, allowing for minor geometry adjustments to further minimize the number of structural components needed.
- Variable time interval adjusts the time interval used in the dynamical system resolution, which reduces the number of computational iterations required to calculate the final equilibrium configuration, thus decreasing the computational effort involved.

The i-MRA method has been developed with the aim of optimizing the structural geometry of gridshells to streamline the construction process. To make the method easily accessible to designers, an original MATLAB [9] code was developed for its implementation. However, recognizing the need for a more userfriendly platform, the code was translated into a Grasshopper component [10], as depicted in Figure 1.

3 The Optimization Strategy: The Multy-subset Approach

In this paper, the i-MRA method is combined with a multi-objective genetic algorithm (MOGA) using the MATLAB [9] function ”gamultiobj”. The MOGA operates by creating a population of potential solutions and iteratively improving them over generations [13].

The objective functions defined in this work aim to minimize the number of different structural element types required for construction and the quantity of structural material needed to build the gridshell. The use of material is defined also considering the wastes due to cutting the industrially produced logs of 12 m in length. This approach allows for the creation of gridshells that are structurally optimised and efficient in terms of material use and construction management.

Figure 2 illustrates the workflow of the optimization process. In this process, the design variables are the parameters governing the form-finding process. Initially, a basic grid is employed, and the i-MRA is utilized to determine the structurally efficient shape under a given loading condition.

Subsequently, the Cutting Stock Problem (CSP) [14, 15] is employed to calculate the minimum number of 12m-long billets required to obtain all the elements necessary for constructing the structure. This approach is adopted to account for the industrial production process of steel beams, which results in production waste from cutting the billets. The optimal cutting pattern can be determined by employing the algorithm to solve the CSP, minimizing waste during the production of structural elements.

Then, a finite element model is created based on the obtained structural shape, employing CHS-type (Circular Hollow Section) profiles made of S355 steel. Finite element analyses are conducted using a proprietary MATLAB code [16, 17] to determine nodal displacements and stress characteristics acting on individual beams. These results are then used to verify if the generated structure meets the criteria for limit displacement, yield, and Eulerian buckling, as in the equation 1.

$$\delta_{max} < \delta_{lim} \qquad |\sigma_{max}| < f_y \qquad N_{max}^{Compression} < N_{Buckling} \tag{1}$$

Where δ_{max} is the maximum nodal displacement experienced by the structure and δ_{lim} is the limit fixed to be $\frac{1}{250}$ of the maximum structural dimension without external constraints. $|\sigma_{max}|$ is the absolute maximum stress in the structure and should be less than f_y which represents the yielding stress of the employed steel. Finally, $N_{max}^{Compression}$ is the maximum compressive axial force and $N_{Buckling}$ is the critical eulerian buckling axial force.

The cross-section dimensions are increased iteratively until a CHS profile that enables the structure to pass the verifications is attained. This iterative process enables the determination of the cross-sectional area of the structural

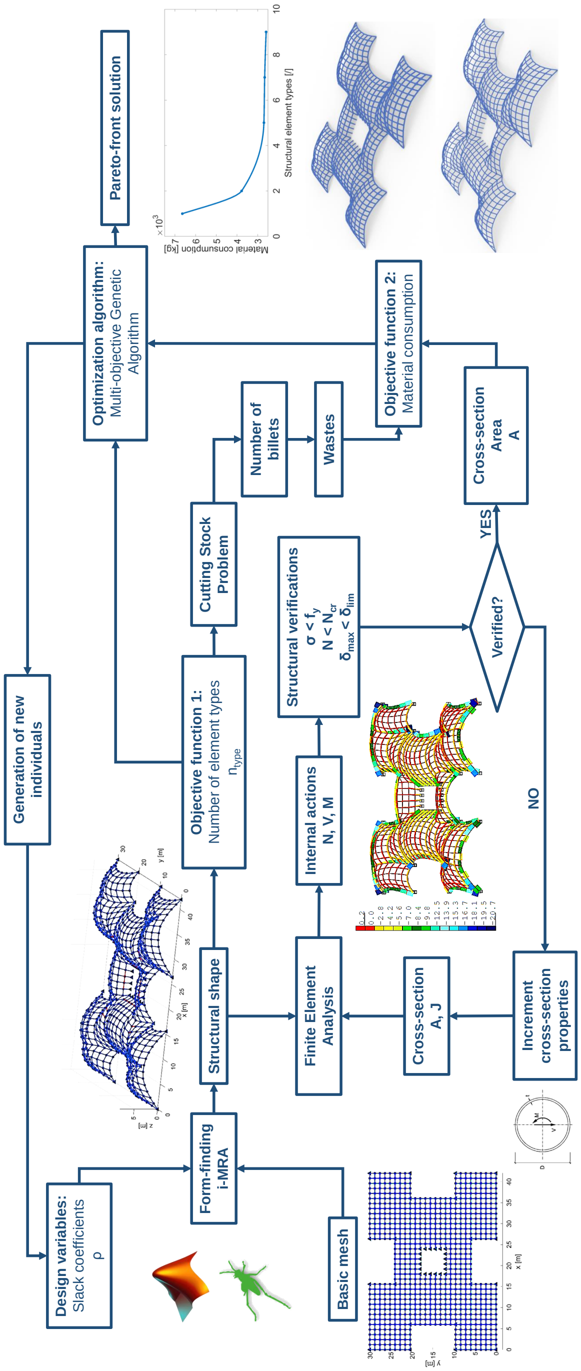


Fig. 2: Optimization procedure workflow.

elements, which, when multiplied by the number of billets resulting from the CSP solution, determines the actual amount of material required for constructing the structure. Consequently, the fitness function evaluated by the MOGA considers two components: the number of different structural element types n_{type} of varying lengths and the amount of material used W. The process is iterated until a set of solutions is obtained, defining a Pareto front where each point represents an optimal structural geometry. This allows the designer to utilize the resulting curve to select the most suitable structure for the specific project, taking into consideration architectural, structural, economic, and management requirements.

4 Methods and testing: Regular and Complex shape Grid shells

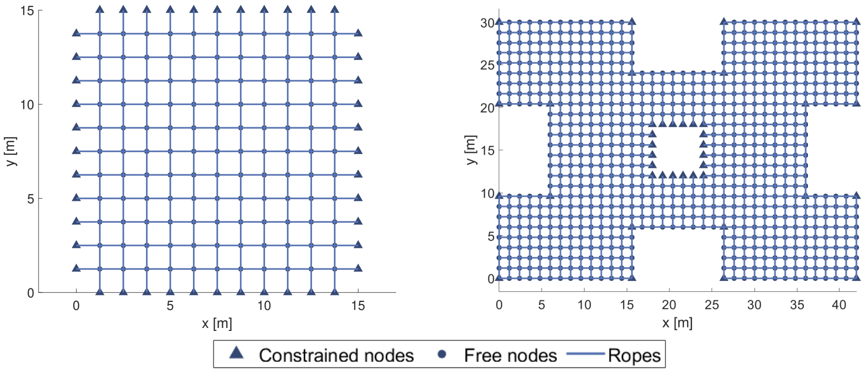
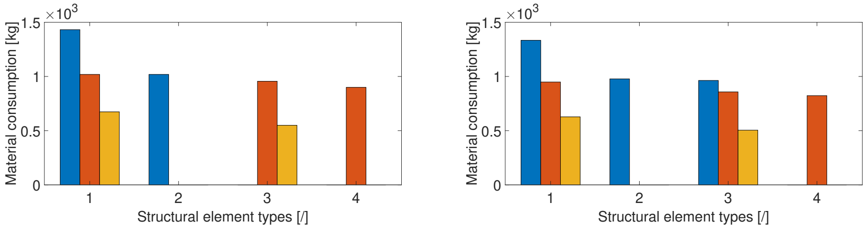


Fig. 3: Base mesh for the case studies

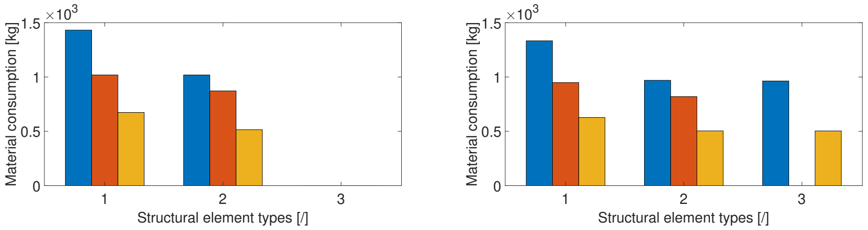
		CSP				**No CSP**			
	n_{type}	**50 kN**	**10 kN**	**5 kN**	**1 kN**	**50 kN**	**10 kN**	**5 kN**	**1 kN**
1 var	**1**	7131	1432	1019	673	6641	1333	949	627
	2	3905	1019	/	/	3773	977	/	/
	3	/	/	956	549	/	963	857	505
	4	3330	/	900	/	/	/	823	/
	5	2683	/	/	/	2698	/	/	/
	7	2659	/	/	/	/	/	/	/
	9	2589	/	/	/	/	/	/	/
2 var	**1**	7131	1432	1019	673	6641	1333	949	627
	2	2552	1019	872	515	2492	970	820	504
	3	/	/	/	/	/	963	0	504

Table 1: First case study optimization results in terms of use of material [kg].

In this section, two examples of the application of the optimization process to form-finding are presented. The first case pertains to a square plan gridshell with a side length of 15m, as reported in Figure 3(a). Figure 4 displays the optimization results for this structure subjected to self-weight and various nodal loads. Specifically, the results depict structures loaded at nodes with respective loads of 1kN, 5kN, and 10kN.



(a) 1 Design variable with CSP (b) 1 Design variable without CSP

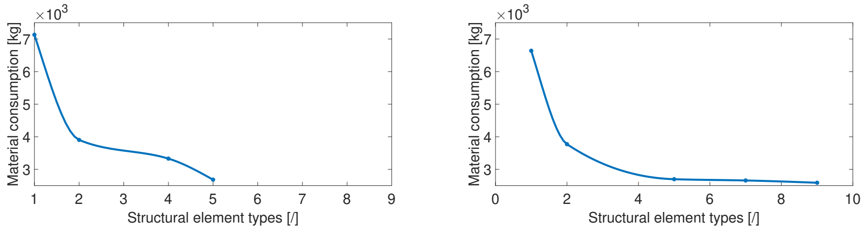


(c) 2 Design variables with CSP (d) 2 Design variables without CSP

🟦10kN 🟥5kN 🟨1kN

Fig. 4: First case study optimization results.

The reported results correspond to optimizations that were conducted considering waste using the CSP (Figures 4(a) and (c)) or without considering waste, as in the cases of Figures 4(b) and (d). Finally, the optimization was performed considering either one (Figures 4(a) and (b)) or two (Figures 4(c) and (d)) design variables represented by the slack coefficients assigned to the ropes for form-finding using the i-MRA method.



(a) CSP (b) No CSP

Fig. 5: 1^{st} case study Pareto front solutions considering a nodal overload of 50kN.

The results are also presented in Table 1, which includes the case with a nodal load of 50kN. It can be observed that the number of different structural element types is inversely proportional to the amount of material required for constructing the structure. Consequently, a range of optimal structural shapes that form a Pareto front is obtained, as depicted in Figure 5.

(a) $n_{type} = 1 \qquad m = 7.13ton$

(b) $n_{type} = 2 \qquad m = 2.55ton$

Fig. 6: Optimized structural shapes for case study 1 considering self-weight and a nodal overload of 50kN.

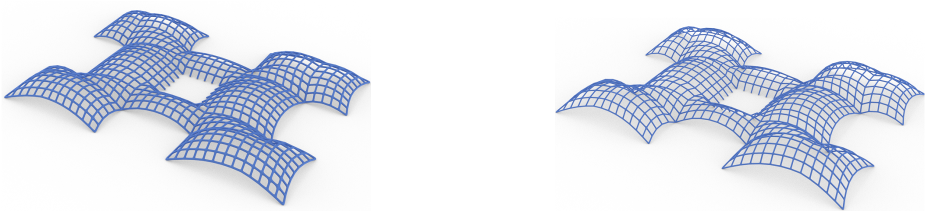
Notably, the lower-profile structural configurations tend to feature thicker structural elements due to the higher involved compression forces. However, they can be constructed using a smaller number of different structural element types. Conversely, the less low-profile configurations consist of structural elements of various types but allow for the utilization of slender elements.

Figure 6 presents the two optimal configurations for the first case study, considering both self-weight and a nodal overload of 50kN.

		CSP				**No CSP**			
	n_{type}	**50 kN**	**10 kN**	**5 kN**	**1 kN**	**50 kN**	**10 kN**	**5 kN**	**1 kN**
1 var	**3**	56388	16797	10183	5399	52029	15498	9396	4903
	4	50390	15747	8662	5399	46865	/	/	/
	5	44670	13009	8149	4633	41934	/	7650	4416
	7	/	12224	6851	4361	/	/	6631	4189
	9	/	11464	/	/	/	/	/	/
	11	/	/	/	/	/	10513	/	/

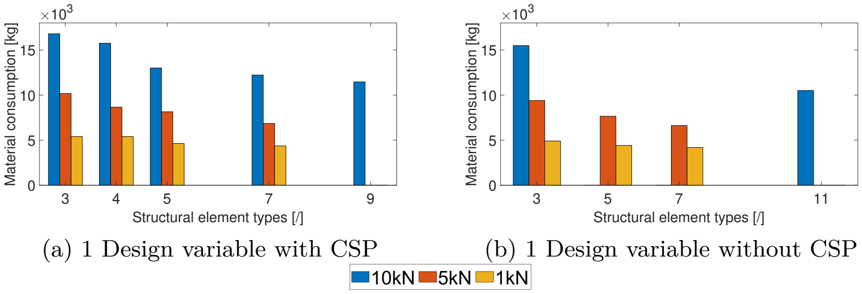
Table 2: Second case study optimization results in terms of use of material [kg].

In the second case study, the presented procedure was applied to optimize the gridshell derived from the base plan depicted in Figure 3(b). The optimizations were conducted using a design variable with the aim of minimizing both the number of structural elements of varying lengths and the material consumption, considering both waste and net quantities. The analyses were repeated for the same load cases as presented in the first case study, and the results are summarized in Table 2. Additionally, Figure 7 provides visual representations of the material quantities used, both in net terms (Figure 7(b)) and including the waste resulting from cutting industrial billets (Figure 7(a)). It can be observed that the increased complexity of the studied structure leads to a higher number of different types of structural elements compared to the simpler case. Furthermore, in Figure 8, the optimal structural forms for the case loaded with a nodal force of 50kN are depicted. Similarly to the previous case study, it is noticeable that as the structure becomes more low-profile, thicker structural elements and a greater amount of material are required. Conversely, as the structure becomes taller, the amount of used material decreases, but with a bigger number of different types of structural elements.



(a) $n_{type} = 3 \quad m = 52.0ton$ (b) $n_{type} = 5 \quad m = 41.9ton$

Fig. 7: Optimized structural shapes for case study 2 considering self-weight and a nodal overload of 50kN.



(a) 1 Design variable with CSP (b) 1 Design variable without CSP

Fig. 8: Second case study optimization results.

5 Conclusion

In this study, an optimization procedure combined with the form-finding process of gridshell structures is presented. Through the utilization of the i-MRA form-finding method, it was possible to optimize the structural form by minimizing the number of different types of structural elements and reducing material consumption. The utilization of the CSP allowed for the determination of the optimal cutting pattern, further minimizing waste during the production of the structural elements. The Pareto front obtained through the MOGA highlighted the trade-offs between different design criteria, such as structural performance, material usage, and aesthetic considerations. This provided designers with a range of optimal structural forms, enabling them to select the most appropriate solution based on specific project requirements. The presented case studies demonstrated how the optimization process could be applied to gridshell structures. The results indicated that the complexity of the structure influenced the number of different types of structural elements and the overall material consumption. Additionally, the relationship between structural height and material utilization was observed, with lower-profile configurations requiring thicker structural elements but a reduced number of element types, while taller configurations allowed for slimmer elements but involved a greater variety of element types. Finally, in order to make the i-MRA form-finding method used in this study accessible to professional designers, a component for the Grasshopper [10] parametric design environment was developed to facilitate the practical application of the proposed approach.

References

1. R. Cucuzza, M. M. Rosso, A. Aloisio, J. Melchiorre, M. L. Giudice, G. C. Marano, Size and shape optimization of a guyed mast structure under wind, ice and seismic loading, Applied Sciences 12 (10) (2022) 4875.
2. J. Melchiorre, A. M. Bertetto, G. C. Marano, Application of a machine learning algorithm for the structural optimization of circular arches with different cross-sections, Journal of Applied Mathematics and Physics 9 (5) (2021) 1159–1170.
3. J. Melchiorre, A. Manuello, F. Marmo, S. Adriaenssens, G. Marano, Differential formulation and numerical solution for elastic arches with variable curvature and tapered cross-sections, European Journal of Mechanics-A/Solids 97 (2023) 104757.
4. M. M. Rosso, R. Cucuzza, F. Di Trapani, G. C. Marano, Nonpenalty machine learning constraint handling using pso-svm for structural optimization, Advances in Civil Engineering 2021 (2021) 1–17.
5. M. M. Rosso, J. Melchiorre, R. Cucuzza, A. Manuello, G. C. Marano, Estimation of distribution algorithm for constrained optimization in structural design, WCCM-APCOM 2022 900 (2022).
6. G. C. Marano, M. M. Rosso, J. Melchiorre, Optimization as a tool for seismic protection of structures, in: World Conference on Seismic Isolation, Springer, 2022, pp. 100–113.
7. S. Adriaenssens, P. Block, D. Veenendaal, C. Williams, Shell structures for architecture: form finding and optimization, Routledge, 2014.
8. A. Manuello, J. Melchiorre, G. C. Marano, Improved multi-body rope approach for free-form grid shells, in: Italian Workshop on Shell and Spatial Structures (IWSS 2023), 2023.
9. MATLAB, version (R2022b), The MathWorks Inc., Natick, Massachusetts, 2022. URL https://www.mathworks.com/products/matlab.html
10. R. McNeel, et al., Rhinoceros 3d, version 7.0 (https://www.rhino3d.com/), Robert McNeel & Associates, Seattle, WA (2023).
11. A. Manuello, Multi-body rope approach for grid shells: form-finding and imperfection sensitivity, Engineering Structures 221 (2020) 111029.
12. A. Manuello, J. Melchiorre, L. Sardone, G. C. Marano, Multi-body rope approach for the form-finding of shape optimized grid shell structures, in: Proceedings of the 15th World Congress on Computational Mechanics, 2022.
13. K. Deb, Multi-objective optimisation using evolutionary algorithms: an introduction, Springer, 2011.
14. L. R. Ford Jr, D. R. Fulkerson, A suggested computation for maximal multi-commodity network flows, Management Science 5 (1) (1958) 97–101.
15. P. Gilmory, R. Gomory, A linear programming approach to the cutting stock problem-part ii, Operations Research 11 (6) (1963) 863–888.
16. M. Paradiso, F. Marmo, L. Rosati, Consistent derivation of a beam model from the saint venant's solid model, International Journal of Solids and Structures 159 (2019) 90–110.
17. M. Paradiso, S. Sessa, N. Vaiana, F. Marmo, L. Rosati, Shear properties of isotropic and homogeneous beam-like solids having arbitrary cross sections, International Journal of Solids and Structures 216 (2021) 231–249.