Multy Body Rope Approach and Funicular Prototype for a New Constructive System for Catenary Arches

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Abstract. The proposed method for the definition of the form is based on a multi-body rope approach (MRA) with masses connected by inextensible ropes characterized by a certain slack coefficient and by different constraint conditions. These parameters played a fundamental role in the definition of the shallowness ratio of the grid (rope), and therefore in the effect of the instability of the reversed shape under loading. In this paper, different MRA enhancing strategies were proposed and discussed. These approaches were used to determine the final shape of a funicular prototype for a pavilion inspired by Guarino Guarini's architecture. The prototype project is based on the principle of Gaudi's inverted catenary arch. It is composed of a series of arches, each one obtained hanging a metallic chain – which assumed the shape of a catenary curve – and then welding all the rings in order to fix its form. Different possible configurations will be described in this paper.

Keywords: Catenary arch · Form-finding · Funicularity · Multibody Rope Approac

1 Introduction

The aim of this paper is to describe an innovative method for the construction of catenary arches using iron chains. Over the years many optimization [1, 2] and form-finding methods have been developed for the purpose of obtaining the best structural shape of arcs and curved structures. A catenary is a curve corresponding to the shape of a hanging string, whose ends are fixed to supports and which is left free to hang under its self-load [3]. In these conditions, the chain form is optimized for purely tensile stresses. If this shape is fixed and then reversed, it becomes a frame that works under a sheer compression load: this is the principle of the reversed catenary [4], which was hypothesized for the first time by Robert

Hooke in 1675. The principle of the catenary was used in the past to verify the stability of arches since they can be considered stable if the line of force (which is a catenary) lies within the thickness of the arch itself [5]. Since a reversed catenary is shear-free, it is the ideal shape to build structures that work mostly in compression, such as masonry arches and vaults. One of the pioneers in the use of catenaries was the Spanish architect Antoni Gaudí, who was the first known architect to use a hanging model as an empirical optimization tool [6]. Several of Gaudí's architectures have been conceived using the catenary geometry as a design instrument: for example, the attic of Casa Milà is a vaulted space characterized by a sequence of diaphragm catenary arches, while the corridor of the School of Teresianas presents parabolic arched structures very similar to catenaries [7]. There are also some examples of the use of this peculiar shape in modern masonry structures. The St. George Orthodox Church, designed and built by Wallmakers in 2016, is a structure made of raw earth bricks and it is composed of a series of vaulted spaces whose form is based on the catenary geometry. The studio clearly stated that they were inspired by Gaudí's method of hanging ropes [8]. Another interesting application is represented by an experimental prototype of social housing realized by some researchers at the University of the Witwatersrand, who built a model of the house covered with a vaulted catenary-shaped ceiling. This allowed them to avoid steel reinforcements in the roof, to spare material [4]. A catenary arch is a complex curve, that is often created through an approximation and that can require the realization of a profile using CNC machines. The possibility to create a rigid catenary arch directly from a hanging chain would help to avoid the approximation and allows not necessarily use of CNC machines. These metal arches can be directly used to create architectural installations but can be also very useful in the realization of centrings for catenary vaulted spaces. In recent decades, advancements in computing technology have opened new avenues for research in structural engineering, particularly in the search for optimal structural forms. The emergence of digital tools, such as calculators and computer simulations, has greatly facilitated the exploration of various form-finding methods [9–12]. Among the diverse form-finding methods available, the Multibody Rope Approach (MRA) [13–15] has gained recognition for its ability to model structures as interconnected nodes connected by slack ropes. This approach treats the structure as a dynamic system and seeks to identify the funicular form that corresponds to the applied loading conditions. By considering the equilibrium of forces within the system, MRA determines the optimal configuration where the tension in the ropes is in balance, resulting in a structurally efficient and stable form. In this paper, the Multibody Rope Approach (MRA) is employed to determine the optimal structural form of a design inspired by the Chapel of the Holy Shroud by Guarino Guarini. The study focuses on the use of form-finding techniques to identify two distinct structural types: one composed of catenary chains and another representing a funicular form under the given loading conditions. After identifying these two structural configurations, the next step involves subjecting them to static self-weight loading analysis [16]. This analysis allows for the evaluation of the stress distribution within the structures and facilitates a comparison between the optimized funicular structure and the catenary-based structure, which is comparatively easier to construct. By quantifying the stress distances between the two structural types, the study aims to provide insights into the trade-off between structural performance and constructability. This analysis helps inform the decision-making process by highlighting the differences in stress distribution and potential structural implications of choosing one form over the other. Ultimately, the article contributes to the understanding of how form-finding techniques can assist in the design optimization process, considering both structural efficiency and practical feasibility.

2 The construction technique of the catenary frame

The constructive method presented has been conceived by prof. Giuseppe Fallacara to create rigid catenary arches directly "freezing" the shape of a hanging chain through welding. Some tests have been carried out using small pieces of chains, in order to evaluate the final outcome. The ends of the string were fixed to a horizontal bar and then the string itself was left hanging. It assumed naturally the configuration of a catenary curve, therefore all the joints were welded in order to fix the shape. After some successful results, an arch 3.10 m wide and 2.75 m tall has been realized. The experiment has been carried on using a galvanized iron chain, with a rod of 20 mm and rings 7 cm wide and 10.2 cm high (see Figure 1). Two important considerations must be done, in order to explain the good quality of the prototype. First of all, the iron bar used as support has been placed in order to be perfectly horizontal, otherwise, the curve wouldn't have been a precise catenary. On the second hand, the chosen chain has a mass of 6.3 kg/m and the high weight prevents significant deformations during the welding phase.

3 The assembly of the catenary arches

As previously written, these metallic catenary arches can be used to realize architectural installations. In this section, a first hypothesis will be described, together with some considerations linked to structural analysis.

3.1 A configuration inspired to Guarino Guarini

The first possible structure here described has been conceived as a tribute to the Italian architect Guarino Guarini, one of the main representatives of Italian Baroque. Guarini's vaults are generally characterized by lightness and transparency since they are usually formed by intersecting arches [17], and this principle is reprised in the design process of the described pavilion. The structure consists of three circular levels of catenary arches. The ground level is composed of six arched frames that intersect each other; the next level is as well composed

Fig. 1: The catenary arch obtained after the welding of the joints.

of six intersecting frames, that are rotated so that the bases of the arches are placed in correspondence with the intersection points of the previous level. The third level works the same way. The theme of staggered superimposed arches is inspired in particular by Guarini's dome of the Chapel of the Holy Shroud, which is characterized by a similar pattern (see Figure 2). As a matter of fact, this dome appears on the inside as a sequence of superimposed elliptical arches. In the case of the designed self-bearing frame, in which each arch unloads its weight on the arches below, the catenary configuration is supposed to produce mainly compressive stresses, even if it is not a perfectly optimized structure anymore. Its actual static behaviour will be described in the next paragraph.

3.2 Analysis through the multi-body rope approach

The structural form depicted in Figure 2, inspired by Guarino Guarini's Chapel of the Holy Shroud, was achieved through the application of the Multibody Rope Approach (MRA). In order to test the effectiveness of the method in defining catenary shapes, the MRA was initially applied to individual chains. By analyzing the behavior of these chains, the desired catenary shapes were obtained. Subsequently, these catenaries were superimposed to compose the complete geometry of the structure.

In addition to this approach, a comparative analysis was conducted by directly applying the MRA to the complete structure. This allowed for the determination of the funicular shape of the structure under the influence of self-weight

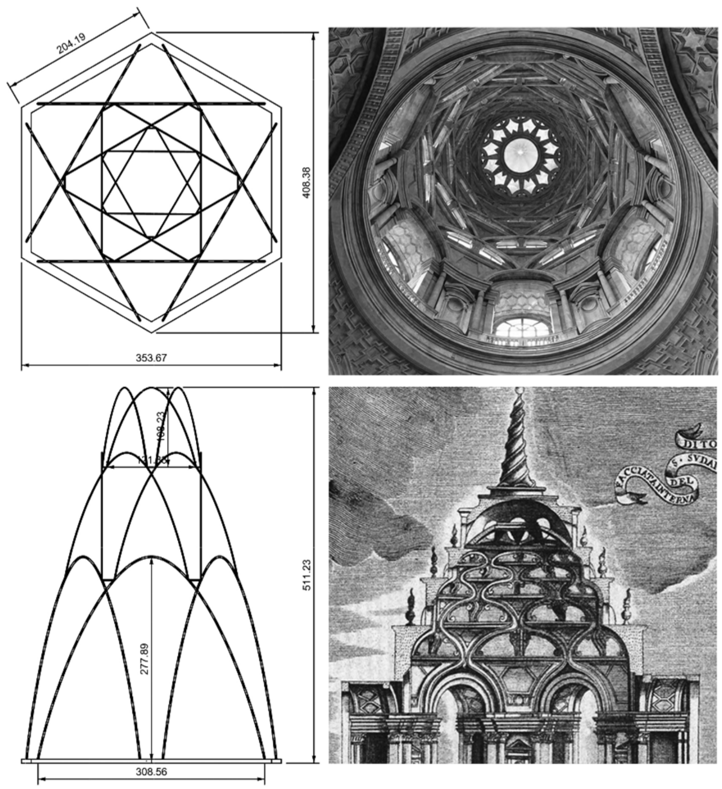


Fig. 2: A comparison between a scheme of the designed pavilion and the vault of the Chapel of the Holy Shroud.

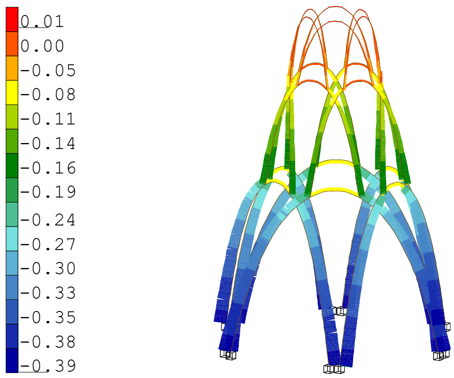
loading. By considering the complete structure, the MRA was able to accurately define the optimal form of the structure in relation to the imposed loads. The resulting structures were subjected to analysis using the FEM SOFiSTiK software [16], assuming that they were composed of a solid steel cable with a diameter of 3 cm. The analysis was performed considering the self-weight of the structure. Based on this assumption, the calculated load per linear meter was determined to be 5.5 kg/m.

The analysis results, including internal actions and Von Mises stress, are presented in Figure 3. It is evident from the results that the funicular structure proves to be the superior choice from a structural perspective. This is due to the fact that the funicular structure exhibits a bending moment of zero, which aligns with expectations for such a form. Consequently, the stress applied to the funicular structure is only 15% of that applied to the structure composed of catenaries. Taking into account the weight of the actual chain used in the construction of the structure, which has a weight per linear meter of 6.3 kg, an adjustment factor of 1.15 needs to be applied to the internal action and stress values shown in Figure 3. Multiplying the values in Figure 3 by this factor will provide the actual internal action and stress values experienced by the structure. Additionally, it is important to acknowledge that the structure composed of catenaries offers advantages from a constructional standpoint, aligning well with the technique described in section 2. This form proves to be more practical and easier to construct compared to the funicular structure. Moreover, despite the structural efficiency of the funicular form, it is worth noting that the stresses applied to both structures remain significantly below the yield strength of any type of structural steel. This indicates that both forms exhibit a satisfactory level of structural integrity and can safely withstand the imposed loads without reaching the limit of their material strength.

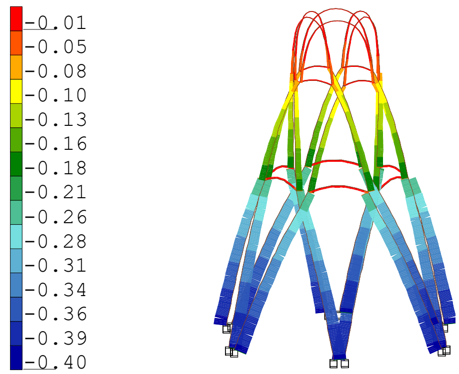
In this context, the catenary form represents a favourable compromise between structural efficiency and constructional simplicity. It provides a satisfactory level of structural performance while offering practicality and ease of construction. This makes it a viable and attractive option for realizing the desired architectural design, considering both the technical and practical aspects of the construction process.

3.3 Other possible configurations

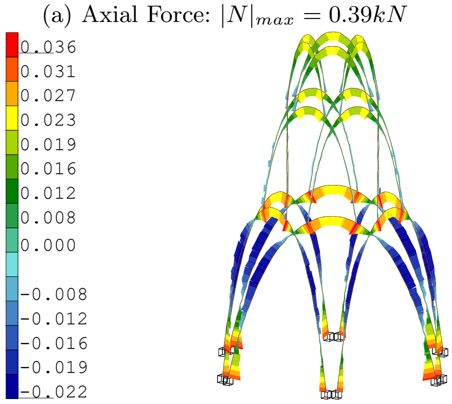
As emerged from the structural analysis, if the global structure consists of a composition of entwined catenary arches, it is no longer a frame optimized to work under sheer compression loads, since the optimal structure will be a funicular geometry. However, the installation previously described shows only one of the numerous possibilities offered by the assembly of catenary arches made of welded chains. First of all, the number of arches and levels can vary, producing more or less complex patterns. It is also possible to create totally different geometries, which don't refer necessarily to domes, but can be linked to other types of vaulted spaces. Fig. 4 shows another possible configuration, consisting



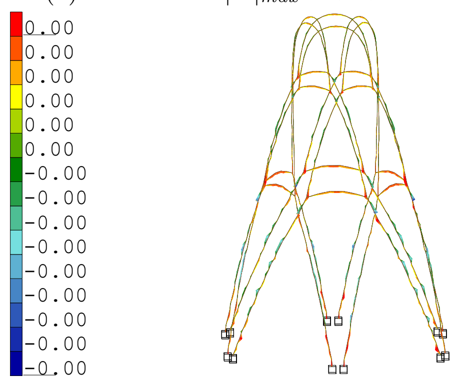
(a) Axial Force: $|N|_{max} = 0.39kN$



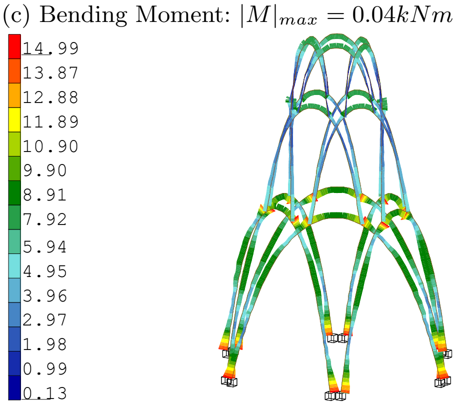
(b) Axial Force: $|N|_{max} = 0.40kN$



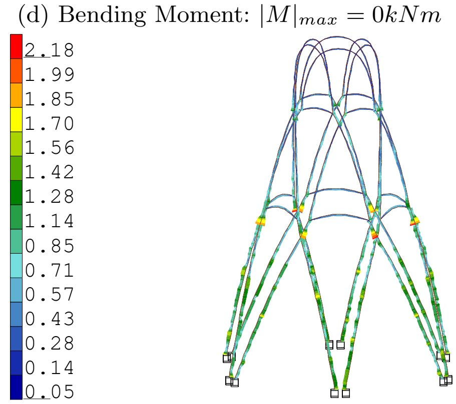
(c) Bending Moment: $|M|_{max} = 0.04kNm$



(d) Bending Moment: $|M|_{max} = 0kNm$



(e) V.M. Stress: $|\sigma^{VM}|_{max} = 14.99kPa$



(f) V.M. Stress: $|\sigma^{VM}|_{max} = 2.18kPa$

Fig. 3: Results of the static analysis on the two structural typologies inspired by the Chapel of the Holy Shroud by Guarini. Catenary shape on the left and funicular shape on the right.

of a “winged” structure. It has only two footholds, where all the arches are connected. The central arch is disposed on a vertical plane, while the lateral ones lie on oblique planes, creating overhangs. The entire structure is balanced thanks to a network of elastic ropes with a diameter of 7 mm, that connect the three curved frames and generate two symmetrical saddle-shaped profiles. The ground connection is resolved through two triangular frames connected transversally by an iron bar, which is in charge of nullifying the pressure produced by the arches.

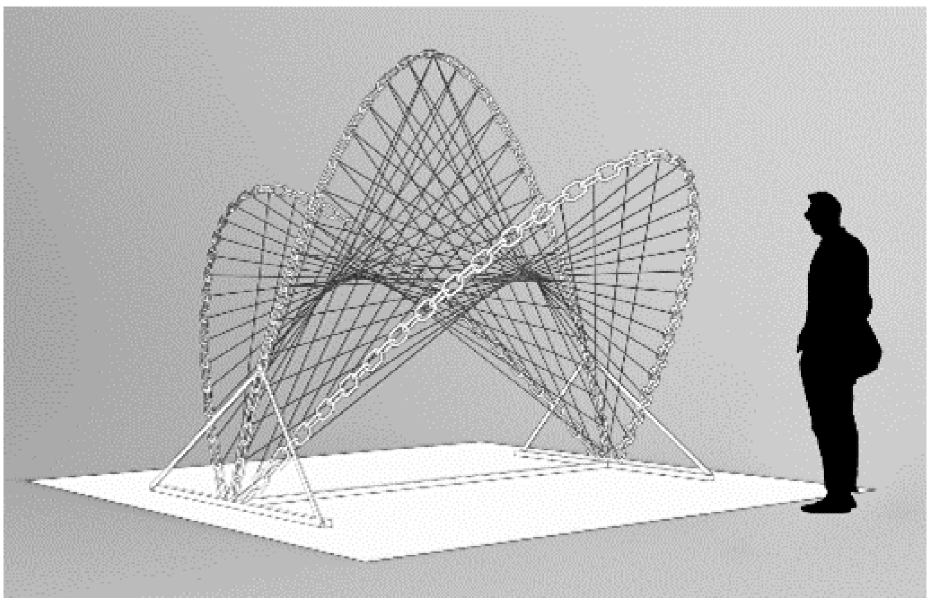


Fig. 4: 3D view of a winged structure composed of three catenary arches.

As it has been already said, there are much more configurations possible to realize this kind of structure, consisting essentially of metallic frames. However, the described constructive technique to obtain catenary arches through welding can be useful also for applications linked to the construction industry. For example, innovative centrings can be built using a series of arches connected by longitudinal bars. This could be useful for the realization of vaulted spaces that work only under compression loads. Creating centrings by “freezing” hanging chains would be more convenient than realizing wooden centrings, since cutting wood inevitably produces material waste. Moreover, the construction process itself would be more complex and would require CNC machines, differently from the proposed method, which requires only a welding phase.

4 Discussion and conclusions

The described method proposes an easy way to realize catenary profiles directly from hanging chains, avoiding a more complex fabrication through CNC machines or using approximations. Such a flexible method would allow also to realize non-symmetrical arches, varying the reciprocal height of supports. These welded chains can be directly used or employed as profiles or centrings for masonry arches or vaults. Despite the advantages explained, it is necessary to highlight some limits of this technique. First of all, the welded arches can really work under sheer compression only if the global structure is simple: as a matter of fact, a catenary is a curve optimized only under the self-weight load. A composition of more catenaries entwined or superimposed does not correspond to an optimized structure, since every arch is subject to other loads and not only to gravity, therefore shear stresses will be present. Moreover, arches made of a welded chain are simple to produce, but difficult to transport, especially if they have large dimensions. Therefore this method would be more convenient only if applied on-site. Furthermore, in section 3.2 of the study, it was observed that the shape obtained through the construction technique presented may not be the most structurally efficient one when compared to the funicular form obtained through the Multibody Rope Approach (MRA). However, it is important to highlight that the catenary form still represents a viable solution that balances both structural efficiency and constructive simplicity.

While the catenary form may not achieve the same level of structural optimization as the funicular form, it offers advantages in terms of construction feasibility and architectural considerations. The catenary shape aligns well with the construction technique discussed in section 2 and can be more easily realized using available construction materials and methods.

Despite its slight compromise in structural efficiency, the catenary form remains a valid solution that takes into account both the structural requirements and architectural design intent. It strikes a balance between structural performance and practical construction considerations, making it a suitable choice for many real-world applications where both factors need to be considered.

By considering the architectural and construction aspects alongside structural efficiency, the catenary form provides a satisfactory solution that meets the overall objectives of the project.

A possible future development of this research can deal with a deeper investigation of the use of welded chains to create centring systems for vaulted ceilings or even in the use of weights hunged in specific points of the chains which allow the creation of funicular shapes using the presented techniques.

References

1. J. Melchiorre, A. M. Bertetto, G. C. Marano, Application of a machine learning algorithm for the structural optimization of circular arches with different cross-sections, Journal of Applied Mathematics and Physics 9 (5) (2021) 1159–1170.

2. J. Melchiorre, A. Manuello, F. Marmo, S. Adriaenssens, G. Marano, Differential formulation and numerical solution for elastic arches with variable curvature and tapered cross-sections, European Journal of Mechanics-A/Solids 97 (2023) 104757.
3. M. Gohnert, R. Bradley, Catenary solutions for arches and vaults, Journal of Architectural Engineering 26 (2) (2020) 04020006.
4. M. Gohnert, I. Bulovic, R. Bradley, A low-cost housing solution: earth block catenary vaults, in: Structures, Vol. 15, Elsevier, 2018, pp. 270–278.
5. P. Block, M. DeJong, J. Ochsendorf, As hangs the flexible line: Equilibrium of masonry arches, Nexus Network Journal 8 (2) (2006) 13–24.
6. J. Tomlow, Gaudí's reluctant attitude towards the inverted catenary, Proceedings of the Institution of Civil Engineers-Engineering History and Heritage 164 (4) (2011) 219–233.
7. S. Huerta, Structural design in the work of gaudi, Architectural science review 49 (4) (2006) 324–339.
8. St george orthodox church / wallmakers, `https://www.archdaily.com/959657/st-george-orthodox-church-wallmakers?ad_source=search&ad_medium=projects_tab`, accessed: 2023/05/06.
9. H.-J. Schek, The force density method for form finding and computation of general networks, Computer methods in applied mechanics and engineering 3 (1) (1974) 115–134.
10. J. R. H. Otter, A. C. Cassell, R. E. Hobbs, POISSON, Dynamic relaxation, Proceedings of the Institution of Civil Engineers 35 (4) (1966) 633–656.
11. P. Block, J. Ochsendorf, Thrust network analysis: a new methodology for three-dimensional equilibrium, Journal of the International Association for shell and spatial structures 48 (3) (2007) 167–173.
12. A. Kilian, J. Ochsendorf, Particle-spring systems for structural form finding, Journal of the international association for shell and spatial structures 46 (2) (2005) 77–84.
13. A. Manuello, Multi-body rope approach for grid shells: form-finding and imperfection sensitivity, Engineering Structures 221 (2020) 111029.
14. A. M. Bertetto, F. Riberi, Form-finding of pierced vaults and digital fabrication of scaled prototype, Curved and Layered Structures 8 (1) (2021) 210–224.
15. A. Manuello, J. Melchiorre, L. Sardone, G. C. Marano, Multi-body rope approach for the form-finding of shape optimized grid shell structures, in: Proceedings of the 15th World Congress on Computational Mechanics, 2022.
16. SOFiSTiK, Text Editor 2023, Flataustr. 14, 90411 Nuremberg, 2022. URL `https://www.https://www.sofistik.com`
17. A. Mazziotti, G. Brandonisio, G. Lucibello, A. De Luca, Structural analysis of the basket dome in the chapel of the holy shroud by guarino guarini, International Journal of Architectural Heritage 11 (3) (2017) 324–338.