Postbuckling Echoes of iMRA Introduced Variation in Gridshells Mechanical Behaviour

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Abstract. The collapse of a structure serves as a profound testament to its design quality. Forensic studies in structural engineering have revealed that collapses are intricate events with various interconnected metrics. The introduction of resilience and robustness concepts has revolutionized structural engineering, surpassing the conventional approach of designing for maximum load. This paradigm shift is especially crucial for shells and membranes. Postbuckling investigation emerged as a significant area of study since 1970, offering insights into shell design and stability. This paper focuses on the effect of the improved Multibody Rope Approach (i-MRA) for form finding on the postbuckling behaviour of gridshell structures. The i-MRA exhibits superior postbuckling behaviour, which directly relates to the method's underlying mechanics. The investigation centres around the mosque roofing in Dakar, Senegal. A comprehensive analysis was conducted, encompassing linear buckling analysis and geometrically nonlinear analysis with imperfections (GNIA) under incremental loading. The study successfully captured crucial insights into the behaviour of the structure throughout both the buckling and postbuckling phases. Additionally, a standard square-plan gridshell was analyzed to validate the findings and evaluate their generality. The research findings demonstrate the efficacy of the i-MRA and provide valuable insights for the design and stability assessment of gridshell structures.

Keywords: Form-finding · Instability · Gridshell · Snap-through · Buckling.

1 Introduction

As karmic as it seems, the way of a structure collapses undoubtedly defines how well, or bad, it was designed in the first place. One of the biggest advancements, at least in cultural terms, in structural engineering has been the introduction of the concepts of resilience and robustness, overcoming the idea of designing towards a maximum load [1]. And if this is true, that a number expressed kilonewtons cannot solely describe a safe design, for common civil structures, it is a certainty for shells and membranes. Since 1970, postbuckling investigation became a hot topic in structural mechanics for its relevance to shell design [2], [3]. Using an inverse logic, a design method that evidences a *better* postbuckling behaviour represents an improved procedure to conceive safer and more performing structures (in terms of stability) [4–6]. For the case of gridshells, hybrid little sisters of classical shells, we will show how the improved Multibody Rope Appoach (i-MRA) [7], an improved version of the Multibody Rope Approach (MRA) [8–10] for form finding, manifests a preferable postbuckling behaviour; and that this is strictly connected with the mechanics behind the method itself. The discovery of a qualitative improvement in the mechanisms of structural instability in structures generated using the i-MRA occurred while investigating the structural behaviour of the roofing of the mosque in Dakar, Senegal, with particular focus on the postbuckling phases. The roofing of the mosque, designed by the architectural firm "Fragomeli+Partners," serves as an intriguing case study. What sets this structure apart is its complex and captivating geometry, characterized by non-optimal curvatures [11], from a structural perspective. The structural analysis of the mosque's roofing was conducted through a comprehensive approach that encompassed both linear buckling analysis and geometrically nonlinear analysis with imperfections (GNIA) under incremental loading [12]. This methodology allowed for a thorough examination of the structure's behaviour, capturing crucial insights at different stages of instability [13]. The initial step involved a linear buckling analysis, which provided valuable information regarding the critical load multiplier for buckling. Building upon the linear buckling analysis, a geometrically nonlinear analysis with imperfections was performed using GNIA. This advanced technique enabled a more detailed assessment of the structural response, specifically focusing on the postbuckling phase. By applying incremental loading, the study measured the displacements corresponding to each load increment, capturing the progressive behaviour of the structure as it moved beyond its stable state. To simulate realistic conditions and account for imperfections that could influence structural stability, the imperfections imposed in the analysis were proportionate to the first mode of buckling. This comprehensive analysis provides valuable insights into the behaviour of the mosque's roofing throughout both the buckling and postbuckling phases. Lastly, to inspect the generality of the obtained conclusions, the proposed methodology was applied to investigate a standard square-plan gridshell. This additional study aimed to evaluate the consistency of the findings and ascertain if the observed trends hold true for a more common gridshell configuration.

2 The postbuckling behaviour of the Dakar Mosque roof

In this section the postbuckling behaviour of the roof of a mosque planned for Dakar, Senegal, designed by the architectural firm "Fragomeli+Partners" in collaboration with "Wafai Architecture" is presented. The roof of the Dakar

Mosque, illustrated in Figure 1, showcases a unique geometry and is constructed using a gridshell system composed of tubular profiles CHS 60x3.6. The tubular profiles utilized in the gridshell are made from S355 structural steel and have a length of 2.00m. This gridshell structure spans a rectangular area measuring 63x58m without any intermediate supports. Consequently, it appears to be solely constrained at its edges, allowing for an open and expansive interior space.

Fig. 1: Dakar Mosque

The roof of the mosque represents an interesting case study as its structural form is defined by architectural design, which imposes constraints on the shapes achievable through traditional form-finding methods. These constraints have a tangible impact on both the quantitative and qualitative aspects of the structural response, indicating a distinct alteration in the structural stability characteristics. In fact, the roof exhibits a curvature that poses a structural weakness, resulting in a sub-optimal structural configuration. However, the use of MRA (Multibody Rope Approach) and i-MRA (improved Multibody Rope Approach) techniques allows for the definition of a form that, given the initial architectural design, achieves a structurally viable configuration while maintaining a harmonious geometry. The MRA method employs a modelling approach that represents the structure as a network comprising loose ropes and masses concentrated at the nodes. This representation allows for the depiction of the structural geometry as the inverted configuration of a hanging net. By adopting this approach, the MRA method facilitates the attainment of a funicular structural configuration, which minimizes bending actions on the structural elements. However, the quadrangular mesh of the suspended net leads to each mass being connected by four ropes, thereby constraining the three degrees of freedom associated with displacements in three-dimensional space. Consequently, certain constraints represented by the ropes become redundant, resulting in some of the

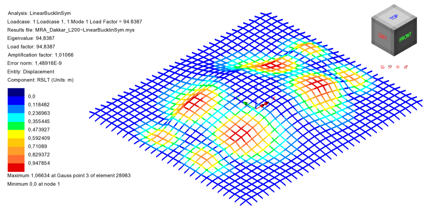
		MRA		i-MRA		Difference %	
λ	**Mode**	**Sym**	**Asym**	**Sym**	**Asym**	**Sym**	**Asym**
100	**1**	82.3	69.9	63.1	40.8	23.3%	41.6%
	2	84.9	78.1	67.1	42.3	21.0%	45.8%
	3	90.7	86.2	68.0	44.9	25.1%	47.9%
150	**1**	15.1	12.4	12.2	8.3	19.5%	32.6%
	2	15.6	13.7	12.3	8.5	21.1%	38.2%
	3	16.4	14.8	12.5	8.9	23.8%	40.2%

Table 1: Dakar Mosque linear buckling eigenvalues considering symmetric and asymmetric load condition

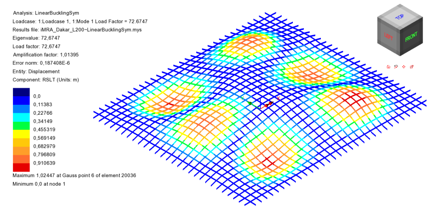
ropes remaining slack in the final configuration. On the other hand, the i-MRA method aims to reduce the length of the slack ropes, ensuring that each rope is under tension in the final configuration. Regarding the specific case of the Dakar mosque, it is apparent that the geometries obtained using the two form-finding methods differ significantly. More specifically, the MRA-derived geometry ensures that only the ropes associated with the central archway along the longer side of the structure become tensioned. This establishes a structural hierarchy wherein the central arch supports the entire roof, constituting a secondary hierarchical structure suspended from the primary one. In the case of the geometry obtained through the i-MRA method, the final shape exhibits a smoother profile that closely resembles the curved surface of the architectural design. As a result, no prominent structural hierarchies are evident, and all elements appear to collaborate in bearing the loads The different in the structure geometry is evident in the linear buckling analysis, which reveals distinct values for the load multiplier for buckling between the two structural forms, as reported in Table 1.

Figure 2 presents the first buckling mode corresponding to the two different geometries, considering both a symmetrically distributed load on the roof nodes (Figures 2(a) and 2(b)) and an asymmetrically distributed load on half of the roof (Figures 2(c) and 2(d)). In the case of the MRA configuration, the observed buckling behavior closely resembles that of lowered arches. On the other hand, the i-MRA configuration exhibits a buckling mode that bears similarity to the typical buckling pattern observed in plates.

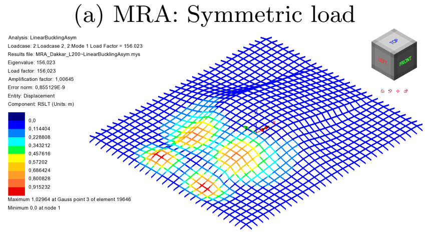
In order to examine the postbuckling behaviour of the structure, a geometrically nonlinear analysis with imperfections (GNIA) using incremental load analysis was conducted. Imperfections were introduced to account for potential defects that may occur during the production and assembly phases. These defects were modelled based on the deformation corresponding to the first mode of linear buckling, scaled to represent a maximum displacement within the accepted tolerance of 1 cm during assembly. Considering the impact of imperfections is essential for gridshell structures, as they can lead to significant deviations from ideal behaviour and affect structural stability. Figure 3 illustrates the displacement-load paths for the two geometries, considering both the symmetric and asymmetric load configurations. Notably, the disparity in postbuckling behaviour becomes more prominent when analyzing the asymmetrical loading case. Consequently, a detailed analysis focusing on this specific scenario is presented in Figure 4.



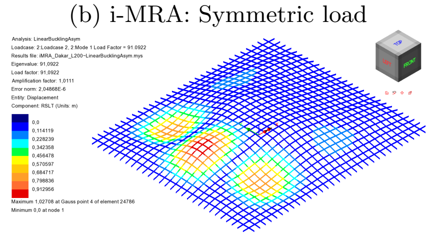
(a) MRA: Symmetric load



(b) i-MRA: Symmetric load



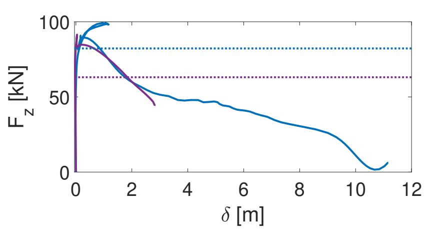
(c) MRA: Asymmetric load



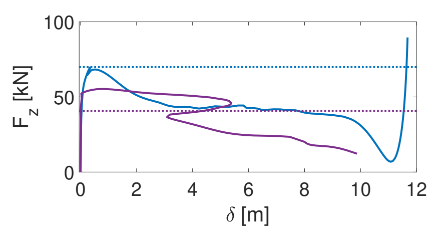
(d) i-MRA: Asymmetric load

Fig. 2: Dakar Mosque: linear buckling mode comparison

The graph illustrates the displacements and stresses experienced by the structure along the load path, highlighting a notable difference in post-peak behaviour. Specifically, for the MRA configuration, a behaviour indicative of the interaction between linear buckling and snap-through phenomena is observed. Initially, the structure undergoes a partial collapse in the loaded section of the main arch during the early stages. After reaching the peak load, the collapse becomes catastrophic, resulting in a complete overturning of the roof. In contrast, the postbuckling behaviour of the i-MRA configuration demonstrates improved performance, despite the lower peak load. In this case, the post-peak phase is characterized by a plateau, where the partial collapse experienced in the initial stages gradually expands. Notably, there is no sharp transition between partial and global collapse, as observed in the MRA case.



(a) Symmetric incremental load



(b) Asymmetric incremental load

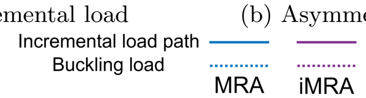


Fig. 3: Displacement-load path of the central node of the Dakar mosque gridshell considering an incremental load geometric nonlinear analysis

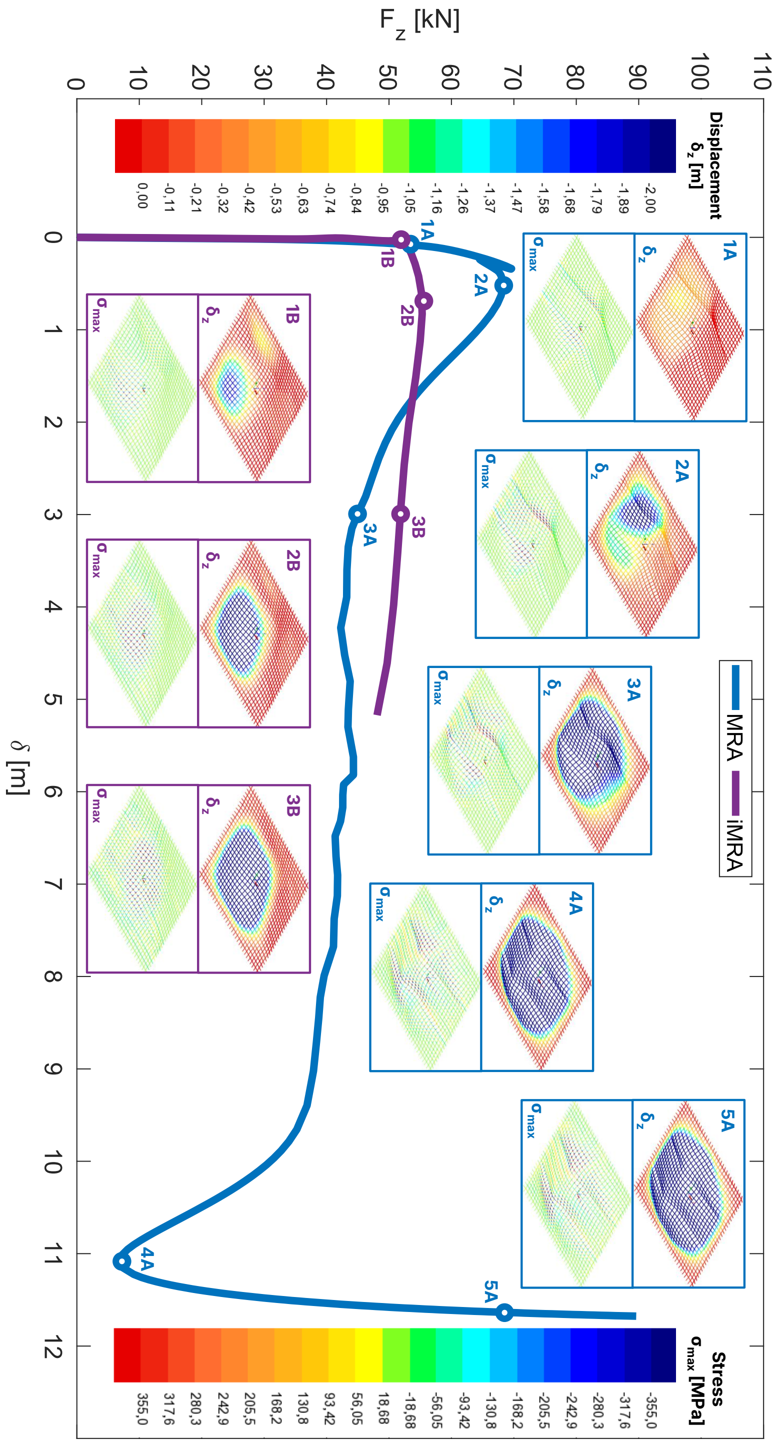


Fig. 4: Postbuckling behaviour of the Dakar Mosque

3 Are the effects of i-MRA relevant in standard gridshell structures?

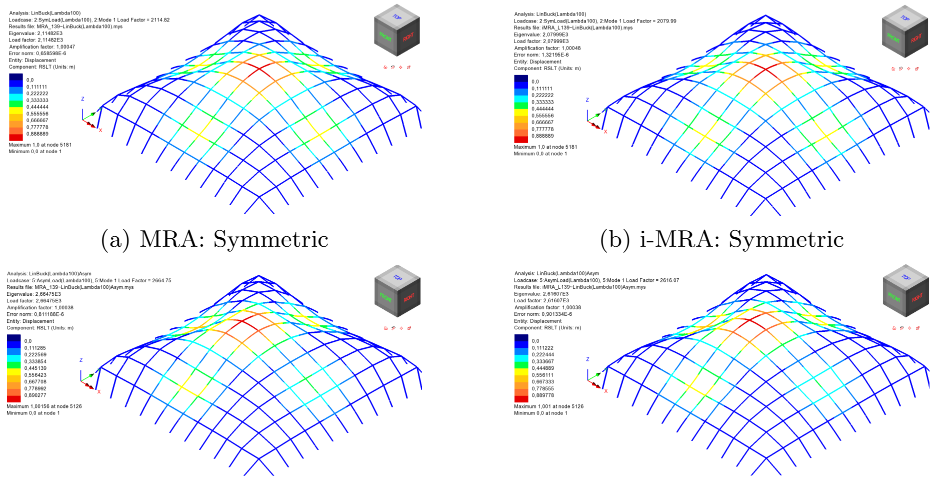
The case study of the roof covering of the Dakar Mosque has revealed that reducing slack through the application of i-MRA has a profound impact on the structural behavior, particularly during the post-buckling phase. It is worth noting that the analyzed case represents a structural anomaly in which the behaviour of the structure is heavily influenced by a combination of factors, including the architectural design, the big free-span, and variations in curvature along the roof covering profile.

	Symmetric load $F_z[kN]$						**Asymmetric load $F_z[kN]$**					
	MRA			**i-MRA**			**MRA**			**i-MRA**		
$\lambda\backslash\eta$	**0.142**	**0.205**	**0.282**	**0.142**	**0.205**	**0.282**	**0.142**	**0.205**	**0.282**	**0.142**	**0.205**	**0.282**
50	2187	3861	6980	2189	3797	6956	1523	2671	4681	1524	2625	4656
100	146	256	440	146	252	439	101	176	294	101	173	292
150	26	51	87	26	50	87	18	35	58	18	34	58

Table 2: Mode 1: Euler linear buckling eigenvalue for square gridshell for both vertical symmetric and asymmetric load cases.

In order to assess the influence of applying i-MRA to a standard gridshell, this section presents an instability analysis of a square plan gridshell with a side length of 15m, considering various slenderness ratios λ of structural elements and dimensionless heights η. In particular, the slenderness ratio $\lambda = \sqrt{\frac{AL^2}{J}}$ is defined as a function of the cross-section area A, the length of the structural elements L and the flexural inertia J while η is the ratio between the maximum height of the structure and the maximum base span. Specifically, the analysis involved calculating the linear buckling loads, as shown in Table 2. In this case, it is evident that the application of i-MRA does not introduce significant variations compared to the use of MRA, both geometrically and in terms of structural behaviour. Actually, the load multiplier for linear buckling is practically identical in both configurations, as is the deformation corresponding to the first mode of instability shown in Figure 5, whether under symmetric loading or asymmetric loading applied only to one half of the roof covering.

Lastly, Figure 6 displays the results of an in-depth analysis of the post-buckling behaviour of the structure. The figure illustrates the displacement-load paths obtained from an incremental load analysis conducted on the structure, considering the presence of a geometric defect that induces unstable behavior. The geometric imperfection is determined by the deformation of the structure in its first buckling mode. The inclusion of this defect aims to address the most critical scenario involving imperfections introduced during the production or construction of the structural elements. The analysis shows that even during the post-buckling phase, structures generated using the two form-finding methods do not appear to have significant differences in their structural behavior. Both under symmetric and asymmetric loading conditions, it is clear that the load paths exhibit similar trends. In particular, it can be observed from Figures 6(d)



(a) MRA: Symmetric (b) i-MRA: Symmetric

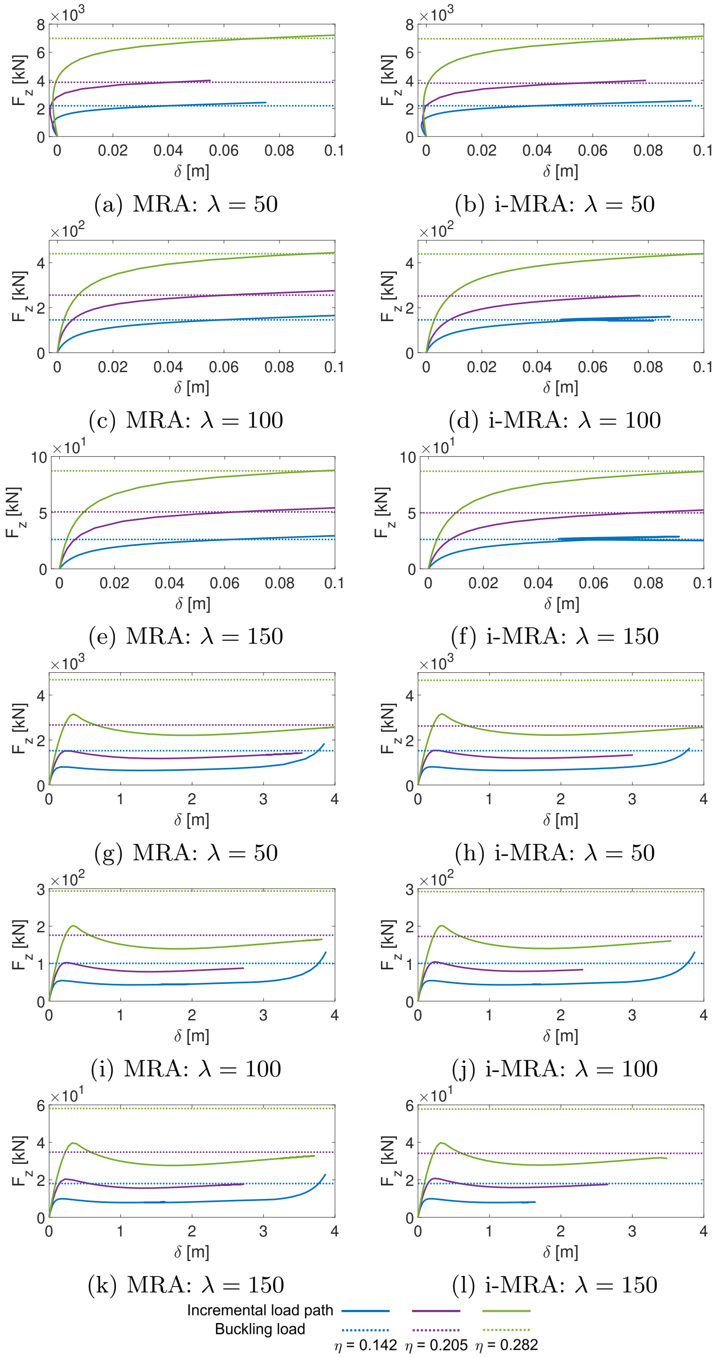
(c) MRA: Asymmetric (d) i-MRA: Asymmetric

Fig. 5: Comparison of linear buckling mode 1 for square gridshell with $\eta = 0.205$ and $\lambda = 100$

and 6(f) that for structures generated with i-MRA and slenderness ratios λ of 100 and 150, the load path actually follows the bifurcation predicted by the linear buckling load. Confirming the presence of a load path bifurcation in this structure. When asymmetrically loaded, all the curves exhibit a peak followed by large displacements at loads considerably lower than those predicted by the buckling multipliers (dashed lines). In the analyzed geometries, the imperfections introduced by i-MRA are found to be less significant compared to potential defects during assembly. Imperfections greatly influence the behavior of these structures, altering the instability mechanism and reducing the instability load. Further investigations are needed to explore how combining deformations from different buckling modes can modify the structural behavior.

4 Conclusions, so far...

This paper provides a comparative analysis of two form-finding methods, the Multibody Rope Approach (MRA) and its enhanced version i-MRA, with a focus on their effects on structural stability. The study aims to compare the buckling mechanisms and postbuckling behavior of structures generated by these methods, specifically examining the influence of geometric variations introduced by i-MRA on structural instability phenomena. To investigate these aspects, the authors examined the roofing of a mosque in Dakar, Senegal. Surprisingly, the introduction of constraints in the geometry through i-MRA, even if resulted in a reduction of the critical load, improved the post-buckling behaviour of the structure in qualitative terms. Rather than exhibiting a catastrophic snap, the structure showed a gradual collapse when computed using i-MRA, contrasting



(a) MRA: $\lambda = 50$
(b) i-MRA: $\lambda = 50$
(c) MRA: $\lambda = 100$
(d) i-MRA: $\lambda = 100$
(e) MRA: $\lambda = 150$
(f) i-MRA: $\lambda = 150$
(g) MRA: $\lambda = 50$
(h) i-MRA: $\lambda = 50$
(i) MRA: $\lambda = 100$
(j) i-MRA: $\lambda = 100$
(k) MRA: $\lambda = 150$
(l) i-MRA: $\lambda = 150$

Fig. 6: Displacement-load path of the central node of the square gridshell considering an incremental load geometric nonlinear analysis

with the behaviour observed in structures generated by the MRA method. Furthermore, the study revealed that for standard geometries, the introduction of geometric variations did not yield significant differences in terms of buckling behaviour. However, due to the highly nonlinear and complex nature of buckling phenomena, relying solely on the analysis of simple standard structures is inadequate to capture the effects of geometric variations introduced by different form-finding methods. To accurately evaluate structural integrity, it was crucial to consider a severe case involving suboptimal curvatures and other imperfections. In such scenarios, the reduction of slack introduced by i-MRA demonstrates a qualitative improvement in the structural response to buckling phenomena.

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