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Modeling the Co-evolution of Climate Impact and Population Behavior: A Mean-Field Analysis ⋆

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Abstract: Motivated by the climate crisis currently ravaging the planet, we propose and analyze a novel framework for the coupled evolution of anthropogenic climate impact and human environmental behavior. Our framework includes a human decision-making process that captures social influence, government policy interventions, and the cost of acting environmentally friendly, modeled within a game-theoretic paradigm. By taking a mean-field approach at the limit of large populations, we derive the equilibria and their local stability characteristics. Subsequently, we study global convergence, showing that the system converges to a periodic solution for almost all initial conditions. Numerical simulations confirm our findings and suggest that the level of environmental impact might become dangerously high before the system reaches the periodic solution, calling for the design of optimal control strategies to influence the system trajectory.

Keywords: multi-agent systems, modeling and decision making in complex systems, climate.

1. INTRODUCTION

With the rapid rise in temperature and extreme weather conditions registered in the last few years all around the globe, it is hard to deny that climate change is a serious threat to all life on our planet. In a myriad of ecosystems, the climate change crisis has already been the cause of substantial damages and often irreversible losses of biodiversity (Pörtner et al., 2022).

To mitigate the consequences of the climate crisis, a collective adoption of environmentally responsible behavior is necessary (Otto et al., 2020). However, even though there is an increasing global awareness that the climate crisis is real, dangerous, and occurring right now, this awareness has not yet translated into sufficiently resolute actions capable of decreasing carbon dioxide emissions. On the contrary, preliminary data for the year 2022 suggest a relative increase in global fossil CO_2 emissions of 1.0% compared to 2021 (Friedlingstein et al., 2022), thereby reaching an atmospheric CO_2 concentration of 417.2 ppm, which is 51% above pre-industrial levels.

To predict whether sustainable practices will be collectively adopted, it is necessary to develop accurate models of the individual-level mechanisms that drive people to make behavioral decisions. In the design of such models, evolutionary game theory has emerged as a powerful framework (Hofbauer et al., 1998). Of particular interest are feedback-evolving games in which the behavior of individuals influences the surrounding environment, which in turn impacts the behavioral decision-making process (Gong et al., 2022; Weitz et al., 2016). Feedback-evolving games have proved useful in explaining dynamical phenomena in biological systems, such as resource harvesting and plant nutrient acquisition (Tilman et al., 2020). However, such models inherently oversimplify the complex and evolving nature of human behavior by assuming that individual decision-making is governed by a game whose payoff matrix depends linearly on the surrounding environment. Hence, such frameworks are not amenable to nonlinear features due to the role of social influence, and they do not explicitly consider the (potentially time-varying) implementation of policy interventions. Therefore, feedback-evolving games are limited in their practical applicability.

To address this gap, we propose a novel mathematical network model for the co-evolution of anthropogenic environmental impact and human behavior, where the decision-making process of individuals includes factors such as social influence, policy interventions, and the cost of acting environmentally responsible. We propose a behavioral revision process in which individuals tend to imitate individuals with a higher payoff (Hofbauer et al., 1998) while also preferring to conform to the behavioral norm of their social environment (Cialdini and Goldstein, 2004). We formulate our model as a continuous-time Markov process. By taking a mean-field approach in the limit of

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large populations (Van Mieghem et al., 2009), we derive a deterministic approximation of our stochastic model and analyze the obtained system. We start by deriving local stability properties of the system equilibria, and subsequently, we study the system's global asymptotic behavior through an argument based on the Poincaré-Bendixson theorem. Specifically, we prove that for (almost) every initial condition in the domain's interior, the system converges to a periodic solution. Finally, we provide numerical simulations to illustrate our findings and explore feedback control policies to mitigate risky system trajectories.

Notation: Let $\mathbb{R}$, $\mathbb{R}_{\geq 0}$, and $\mathbb{R}_{>0}$ denote the set of real, real nonnegative, and strictly positive real numbers, respectively. We state that an event E is triggered by a *Poisson clock* with a rate of $\rho_E(t)$ if and only if (iff)

$$\lim_{\Delta t \searrow 0} \frac{\mathbb{P}\big[E \text{ occurs during } (t, t+\Delta t)\big]}{\Delta t} = \rho_E(t).$$

2. MODEL

We consider a population of n individuals, denoted by $\mathcal{V} := \{1, \dots, n\}$. Each individual is represented by a vertex in a directed network $\mathcal{G} := (\mathcal{V}, \mathcal{E})$, where $(i,j) \in \mathcal{E}$ iff j has a social influence on the behavior of i. The neighbor set of $i \in \mathcal{V}$ is denoted by $\mathcal{N}_i := \{j \in \mathcal{V} : (i,j) \in \mathcal{E}\}$, with cardinality $d_i := |\mathcal{N}_i|$. The environmental behavior of an individual $i \in \mathcal{V}$ at time $t \in \mathbb{R}_{\geq 0}$ is captured by $x_i(t) \in \{0,1\}$, which represents whether i is displaying environmentally responsible behavior ($x_i(t) = 1$), or environmentally irresponsible behavior ($x_i(t) = 0$). The states of all individuals are gathered into an n-dimensional vector $X(t) := [x_1(t), x_2(t), \dots, x_n(t)] \in \{0,1\}^n$, which represents the behavior of the entire population at time t.

2.1 Environmental Impact

For the past 50 years, anthropogenic CO_2 (i.e., the increase in the atmospheric value of CO_2 with respect to the preindustrial value) has increased exponentially (Hofmann et al., 2009; Friedlingstein et al., 2022). Therefore, we choose to model the evolution of the anthropogenic environmental impact $\varepsilon \in \mathbb{R}_{\geq 0}$ through the following linear, non-autonomous ordinary differential equation (ODE):

$$\dot{\varepsilon} = r(t)\varepsilon, \tag{1}$$

where the rate of growth or decay at time t is given by

$$r(t) := \gamma \bar{x}_0(t) - \tau. \tag{2}$$

Here, the effect of environmentally irresponsible behavior is modeled by $\gamma \bar{x}_0(t)$, with $\gamma \in \mathbb{R}_{>0}$ and where

$$\bar{x}_0(t) := \tfrac{1}{n}\big|\{i \in \mathcal{V} \;:\; x_i(t) = 0\}\big| \tag{3}$$

denotes the fraction of people who act irresponsibly at time t. The parameter $\tau \in \mathbb{R}_{>0}$ represents efforts to reduce environmental impact via, e.g., massive tree-planting projects or negative emissions technologies.

2.2 Environmental Behavior

Inspired by the decision-making process proposed in (Ye et al., 2021; Frieswijk et al., 2022) that employs an evolutionary game-theoretic mechanism (Hofbauer et al., 1998), each individual $i \in \mathcal{V}$ has *incentives* for acting environmentally responsibly or not. For any $i \in \mathcal{V}$, we define the incentive functions for responsible and irresponsible behavior as

$$\iota_1^{(i)}(X(t), \varepsilon(t)) := \frac{1}{d_i} \sum_{j \in \mathcal{N}_i} x_j(t) + \mu\varepsilon(t) + \alpha\,, \tag{4a}$$

$$\iota_0^{(i)}(X(t)) := \frac{1}{d_i} \sum_{j \in \mathcal{N}_i} \big(1 - x_j(t)\big) + \kappa - \sigma\,, \tag{4b}$$

respectively. The behavioral incentives include several terms whose meanings are detailed below.

- **Social influence.** The first element in (4a)–(4b) represents social influence, where the incentive of $i \in \mathcal{V}$ for certain behavior is higher when more of i's neighbors act accordingly. This reflects the tendency of individuals to conform to their social environment (Cialdini and Goldstein, 2004). Field experiments showed that social norms influence the behavior of individuals, e.g., curbside recycling behavior (Schultz, 1999).
- **Environmental response.** The term $\mu\varepsilon(t)$ models the population response to the environmental impact $\varepsilon(t)$, which is reflected in, e.g., global food price inflation and shortages (Pörtner et al., 2022). Here, we assume that the population response increases linearly with the impact, regulated by $\mu \in \mathbb{R}_{>0}$; the higher the value of μ, the faster the population reacts to environmental changes. However, one may consider more complex and nonlinear response functions, similar to (Ye et al., 2021).
- **Cost.** The higher costs of responsible behavior represent a barrier to individuals' "green" behavior (Young et al., 2010). Thus, the direct and indirect economic costs of acting responsibly—captured by $\kappa \in \mathbb{R}_{>0}$—bolster the incentive for irresponsible behavior.
- **Environmental subsidies.** Government subsidies stimulate responsible behavior, modeled by reducing the cost of responsible behavior κ by $\sigma \in [0, \kappa]$.
- **Awareness campaigns.** Besides the cost, another barrier to ecologically sustainable behavior is a lack of available information on how to act responsibly (Young et al., 2010). Awareness campaigns—modeled by the parameter $\alpha \in \mathbb{R}_{\geq 0}$—boost public knowledge and thereby increase the incentive for responsible behavior.

Individuals change behavior according to a stochastic adaptation of classical *imitation dynamics*, often employed in evolutionary game theory (Hofbauer et al., 1998; Como et al., 2020). In particular, an individual $i \in \mathcal{V}$ who acts irresponsibly at time t ($x_i(t) = 0$) will adopt responsible behavior if triggered by a Poisson clock with a rate of

$$\rho_{01}^{(i)}(X(t), \varepsilon(t)) = \frac{1}{d_i} \sum_{j \in \mathcal{N}_i} x_j(t)\,\iota_1^{(j)}(X(t), \varepsilon(t))\,, \tag{5a}$$

while an $i \in \mathcal{V}$ who acts responsibly ($x_i(t) = 1$) will cease to do so if triggered by a Poisson clock with a rate of

$$\rho_{10}^{(i)}(X(t)) = \frac{1}{d_i} \sum_{j \in \mathcal{N}_i} \big(1 - x_j(t)\big)\,\iota_0^{(j)}(X(t))\,. \tag{5b}$$

The revision protocol driven by the rates in (5) has an intuitive interpretation. Individuals interact with neighbors and revise their own behavior by imitating neighbors with a probability proportional to the incentive associated with that behavior, similar to classical imitation dynamics (Hofbauer et al., 1998). The proposed *conformity-driven imitation framework* combines incentive-driven behavior with individuals' propensity to conform to the behavioral norm of their social environment (Cialdini and Goldstein, 2004).

In summary, the behavioral-environmental feedback model is characterized by the coupling between i) the environment $\varepsilon(t) \in \mathbb{R}_{\geq 0}$ whose evolution is captured by (1) with the rate of growth/decay in (2), and ii) the behavior $X(t)$ of a network of n individuals that is updated according to the revision protocol in (5) with incentives in (4).

3. MEAN-FIELD DYNAMICS

All Poisson clocks are independent. Hence, the population's behavioral state $X(t) \in \{0,1\}^n$ evolves according to a non-homogeneous continuous-time Markov process (Levin et al., 2006). Specifically, for any $i \in \mathcal{V}$, the transition rate matrix is given by

$$Q_i(X(t), \varepsilon(t)) = \begin{bmatrix} -\rho_{01}^{(i)}(X(t), \varepsilon(t)) & \rho_{01}^{(i)}(X(t), \varepsilon(t)) \\ \rho_{10}^{(i)}(X(t)) & -\rho_{10}^{(i)}(X(t)) \end{bmatrix}, \quad (6)$$

where the first and second row/column correspond to the state $x_i = 0$ and $x_i = 1$, respectively. Thus, the probability that any $i \in \mathcal{V}$ transitions from behavior $y \in \{0,1\}$ to $z \in \{0,1\}$ at time t, with $y \neq z$, is given by

$$\mathbb{P}[x_i(t+\Delta t) = z \,|\, x_i(t) = y] = \big(Q_i(t)\big)_{yz}\Delta t + o(\Delta t),$$

where $o(\Delta t)$ is the Landau little-o notation for $\Delta t \searrow 0$. From the explicit expression of the transition rate matrix $Q_i(X(t), \varepsilon(t))$, we observe that all its entries depend on other individuals' behavior through the dependency on the state of the neighboring nodes in (5) and (4). Moreover, the transition rates are non-homogeneous since the first row depends on $\varepsilon(t)$. The complexity of the transition matrix $Q_i(X(t), \varepsilon(t))$ and the fact that the size of the state space $\{0,1\}^n$ increases exponentially with the population size n make a direct analysis of the non-homogeneous Markov process $X(t)$ unfeasible for large-scale populations.

Following Van Mieghem et al. (2009), we take a mean-field approach in the limit $n \to \infty$; that is, instead of studying the evolution of the state of all individuals (collected in $X(t)$), we study the probability for any $i \in \mathcal{V}$ to act irresponsibly and responsibly, defined as $p_0^{(i)}(t) := \mathbb{P}[x_i(t) = 0]$ and $p_1^{(i)}(t) := \mathbb{P}[x_i(t) = 1]$, respectively. For any $i \in \mathcal{V}$, the evolution of the mean-field dynamics for $p_0^{(i)}(t)$ and $p_1^{(i)}(t)$ is governed by a system of (non-autonomous) ODEs, obtained by noting that $\mathbb{E}[x_i(t) = 1] = p_1^{(i)}(t)$. By replacing $X(t)$ with the vector $p_1(t) := [p_1^{(1)}(t), \dots, p_1^{(n)}(t)]$ in (6) and by using the Chapman-Kolmogorov equation (Levin et al., 2006), we obtain $[\dot{p}_0^{(i)}\ \dot{p}_1^{(i)}] = [p_0^{(i)}\ p_1^{(i)}]Q_i(p_1(t), \varepsilon(t))$ and

$$\begin{aligned} \dot{p}_0^{(i)} &= -\rho_{01}^{(i)}(p_1(t), \varepsilon(t))p_0^{(i)} + \rho_{10}^{(i)}(p_1(t))p_1^{(i)}, \\ \dot{p}_1^{(i)} &= \rho_{01}^{(i)}(p_1(t), \varepsilon(t))p_0^{(i)} - \rho_{10}^{(i)}(p_1(t))p_1^{(i)}, \end{aligned} \quad (7)$$

for any $i \in \mathcal{V}$. Note that (7) is non-autonomous due to the dependency of $\rho_{01}^{(i)}$ on $\varepsilon(t)$ and, ultimately, on t, while the other terms depend only on t through the state $p_1(t)$.

Despite such a dependency on $\varepsilon(t)$, we can provide a general invariance result for (7), provided that $\varepsilon(t)$ is bounded and Lipschitz. Note that if $\varepsilon(t)$ is defined via (1), then it is necessarily Lipschitz since it is the solution of an ODE. The following lemma shows that—for any function $\varepsilon(t)$ that has these properties—$(p_0^{(i)}\ p_1^{(i)})$ is well-defined as a probability vector for all $t \in \mathbb{R}_{\geq 0}$ and for all $i \in \mathcal{V}$.

Lemma 1. Assume that $\varepsilon(t)$ is Lipschitz-continuous and bounded for all $t \in \mathbb{R}_{\geq 0}$. Then, for all $i \in \mathcal{V}$, the set

$$\left\{(p_0^{(i)}\ p_1^{(i)}) : p_0^{(i)}, p_1^{(i)} \geq 0,\ p_0^{(i)} + p_1^{(i)} = 1\right\}$$

is positive invariant under (7).

Proof. Consider any $i \in \mathcal{V}$. First, note that $\dot{p}_0^{(i)} + \dot{p}_1^{(i)} = 0$, so $p_0^{(i)} + p_1^{(i)} = 1$ for all $t \in \mathbb{R}_{\geq 0}$. Second, since the vector field in (7) is Lipschitz-continuous, we can apply Nagumo's Theorem (Blanchini, 1999). Next, we check the dynamics at the domain boundary. Note that $\dot{p}_0^{(i)} \geq 0$ if $p_0^{(i)} = 0$, and $\dot{p}_1^{(i)} \geq 0$ if $p_1^{(i)} = 0$, so $p_0^{(i)}, p_1^{(i)} \geq 0$ for all $t \in \mathbb{R}_{\geq 0}$. □

It follows directly from Lemma 1 that only one of the two equations in system (7) is sufficient to describe the behavioral evolution of an individual $i \in \mathcal{V}$. Hence, the mean-field dynamics of the population behavior ultimately consist of an n-dimensional set of non-autonomous ODEs.

Next, let us define the average probability for a randomly selected individual to act responsibly at time t,

$$x(t) := \frac{1}{n}\sum_{i \in \mathcal{V}} p_1^{(i)}(t). \quad (8)$$

Let $\bar{x}_1(t) := \frac{1}{n}\big|\{i \in \mathcal{V} : x_i(t) = 1\}\big|$ denote the fraction of individuals who behave responsibly at time t. For large-scale populations, the fraction $\bar{x}_1(t)$ can be approximated by the macroscopic variable $x(t)$ with arbitrary accuracy (while the two quantities coincide in the limit $n \to \infty$) for any finite time horizon (Kurtz, 1971; Zino et al., 2017). This allows us to accurately study the population behavior from a macroscopic perspective.

In the mean-field approach, the system's evolution is captured by a coupling between i) the system of n independent ODEs that governs the behavioral evolution of all individuals in (7), and ii) the mean-field dynamics of the environmental impact, obtained via (1) and (2) by replacing $\bar{x}_0(t)$ with the macroscopic variable $1 - x(t)$:

$$\begin{aligned} \dot{p}_1^{(i)} &= \rho_{01}^{(i)}(p_1(t), \varepsilon(t))(1 - p_1^{(i)}) - \rho_{10}^{(i)}(p_1(t))p_1^{(i)}, \ \forall i \in \mathcal{V}, \\ \dot{\varepsilon} &= \Big(\gamma\Big[1 - \frac{1}{n}\sum_{i \in \mathcal{V}} p_1^{(i)}(t)\Big] - \tau\Big)\varepsilon. \end{aligned} \quad (9)$$

The system in (9) is an autonomous system of $n+1$ ODEs. In the rest of the paper, we study this system under the following simplifying assumption, which allows for a theoretical analysis of the model.

Assumption 1. For any $i \in \mathcal{V}$, we assume that i is influenced by the entire population, i.e., $\mathcal{N}_i = \mathcal{V}$ for all $i \in \mathcal{V}$.

Under Assumption 1, the incentive functions in (4) become

$$\begin{aligned} \iota_1(x(t), \varepsilon(t)) &:= x(t) + \mu\varepsilon(t) + \alpha, \\ \iota_0(x(t)) &:= 1 - x(t) + \kappa - \sigma, \end{aligned} \quad (10)$$

where the index i was discarded, as the incentives are no longer individual-dependent.

Before analyzing system (9), we will make some realistic assumptions on the incentive functions in (10) and the mean-field rate of growth/decay $\bar{r}(x(t)) := \gamma(1 - x(t)) - \tau$. First, we would like to point out that, currently, none of the negative emission technologies has been demonstrated to be effective at a sufficiently large scale (Rau, 2019). Hence, it is natural to assume that the environmental impact increases if the entire population behaves irresponsibly;

that is, if $x(t) = 0$ at time t, then $\bar{r}(x(t)) > 0$. Second, it is reasonable to assume that if there is no environmental impact (i.e., $\varepsilon(t) = 0$), then $\iota_0(t) > \iota_1(t)$ for any $x \in [0, 1]$. These two observations directly lead to the following.

Assumption 2. Let (i) $\tau < \gamma$ and (ii) $\kappa > \sigma + \alpha + 1$.

Under Assumption 1, we derive the macroscopic mean-field evolution of the population in the following proposition.

Proposition 1. Under Assumption 1 and in the limit of large-scale populations $n \to \infty$, the mean-field evolution of the macroscopic variable x and the environmental impact ε is governed by the following autonomous planar system:

$$\begin{aligned} \dot{x} &= x(1-x)(2x + \mu\varepsilon + \alpha + \sigma - \kappa - 1)\,, \\ \dot{\varepsilon} &= (\gamma(1-x) - \tau)\varepsilon\,. \end{aligned} \tag{11}$$

Proof. Under Assumption 1, (5) reduces to $\rho_{01}^{(i)} = x(x + \mu\varepsilon + \alpha)$ and $\rho_{10}^{(i)} = (1-x)(1-x+\kappa-\sigma)$ for all $i \in \mathcal{V}$. Using this, (11) immediately follows from (8) and (9) by substituting (9) in $\dot{x}(t) = \frac{1}{n}\sum_{i\in\mathcal{V}} \dot{p}_1^{(i)}(t)$. □

To show that system (11) is well-defined, we will first show that the environmental impact is bounded from above for any initial condition $x(0) \in (0, 1)$.

Lemma 2. Under (11), there exists an $\bar{\varepsilon} \in \mathbb{R}_{>0}$ such that $\varepsilon(t) \le \bar{\varepsilon}$ for all $(x(0), \varepsilon(0)) \in (0, 1) \times \mathbb{R}_{\ge 0}$ and $t \in \mathbb{R}_{\ge 0}$.

Proof. Observe that $\dot{\varepsilon} < 0$ for any $x > \hat{x}$ and $\varepsilon > 0$, where $\hat{x}$ solves $\gamma(1-\hat{x})-\tau = 0$, and $\hat{\varepsilon}$ solves $\mu\hat{\varepsilon}+\alpha+\sigma-\kappa-1 = 0$. Then, $\dot{x} > 0$ for any $x \in (0, 1)$ and $\varepsilon > \hat{\varepsilon}$. We will use proof by contradiction to show that $\varepsilon(t)$ is bounded for all $t \in \mathbb{R}_{\ge 0}$ and for all $(x(0), \varepsilon(0)) \in (0, 1) \times \mathbb{R}_{\ge 0}$. Assume that $\varepsilon(t)$ is unbounded. Then, for any $M > 0$ there exists a time t such that $\varepsilon(t) > M$. Let us consider a time $\bar{t}$ such that $\varepsilon(\bar{t}) > \hat{\varepsilon}$ and $x(\bar{t}) \in (0, 1)$, so $\dot{x}(\bar{t}) > 0$. Note that $\dot{x}(\bar{t}) \ge q(1-x)$ for some constant $q > 0$ and $x \in (0, 1)$. By the Grönwall-Bellman inequality (Pachpatte, 1997), $x(t) \ge x(\bar{t})e^{q(t-\bar{t})}$ for all $t \ge \bar{t}$. Hence, there exists a t^* such that $x(\bar{t}+t^*) > \hat{x}$, so $\dot{\varepsilon}(\bar{t}+t^*) < 0$. Note that for any $x \in [0, 1]$ and $\varepsilon \in \mathbb{R}_{>0}$, we have $\dot{\varepsilon} < \gamma\varepsilon$, so the Grönwall-Bellman inequality yields $\varepsilon(t) < \varepsilon(\bar{t})e^{\gamma(t-\bar{t})}$ for all $t \ge \bar{t}$. Thus, $\varepsilon(\bar{t}+t^*) < \varepsilon(\bar{t})e^{\gamma t^*}$. Since $\dot{\varepsilon}(\bar{t}+t^*) < 0$, there does not exist a time t such that $\varepsilon(t) > M$ for any $M > \varepsilon(\bar{t})e^{\gamma t^*}$. □

Using the above result, the following lemma shows that (11) is well-defined for all $t \in \mathbb{R}_{\ge 0}$.

Lemma 3. The set $(x, \varepsilon) \in [0, 1] \times \mathbb{R}_{\ge 0}$ is positive invariant under (11).

Proof. It follows directly from Lemma 1 and 2 that for any initial condition with $x(0) \in (0, 1)$, $(x, \varepsilon) \in [0, 1] \times \mathbb{R}_{\ge 0}$ for all $t \in \mathbb{R}_{\ge 0}$. Next, we study the boundary behavior. For $x(0) = 0$, the dynamics reduce to $\dot{\varepsilon} = (\gamma - \tau)\varepsilon$ and $\dot{x} = 0$. This is solved by $\varepsilon(t) = \varepsilon(0)e^{(\gamma-\tau)t}$ and $x(t) = 0$, which is in the set for all $t \in \mathbb{R}_{\ge 0}$. For $x(0) = 1$, the dynamics reduce to $\dot{x} = 0$ and $\dot{\varepsilon} = -\tau\varepsilon$, which converges exponentially to the origin, belonging to the set. □

4. MAIN RESULTS

In this section, we perform an analysis of the planar mean-field system in (11) to fully unveil its asymptotic behavior. We start by characterizing the equilibria of (11) and establishing their local stability properties, which are presented in the following proposition.

Proposition 2. Under Assumption 2, the system in (11) has three equilibria:

i) $(x, \varepsilon) = (0, 0)$, which is a saddle point;
ii) $(x, \varepsilon) = (1, 0)$, which is a saddle point;
iii) $(x, \varepsilon) = (1 - \frac{\tau}{\gamma}, \frac{1}{\mu}[\frac{2\tau}{\gamma} + \kappa - \sigma - \alpha - 1])$, (12)
which is an unstable spiral.

Proof. Let Assumption 2 hold. Solving $\dot{x} = 0$ yields $x = 0$, $x = 1$ or $x = \frac{1}{2}(-\mu\varepsilon - \alpha - \sigma + \kappa + 1)$. If $x = 0$ or $x = 1$, then the only solution to $\dot{\varepsilon} = 0$ is $\varepsilon = 0$, giving equilibria $(x, \varepsilon) = (0, 0)$ and $(x, \varepsilon) = (1, 0)$.
Now consider $x = \frac{1}{2}(-\mu\varepsilon - \alpha - \sigma + \kappa + 1)$. By solving $0 = \dot{\varepsilon} = (\gamma(1-x) - \tau)\varepsilon$, we find the equilibrium in (12). Note that $\varepsilon = 0$ is not an option for $x \in [0, 1]$, as this gives $x = \frac{1}{2}(-\alpha - \sigma + \kappa + 1) > 1$, by Assumption 2(ii).
Next, we examine the local stability. First, consider equilibrium (12). Linearizing (11) around this equilibrium is equivalent to linearizing the system $(\dot{\tilde{x}}, \dot{\tilde{\varepsilon}})$ around the origin, with $\tilde{x} := x - 1 + \frac{\tau}{\gamma}$ and $\tilde{\varepsilon} := \varepsilon - \frac{1}{\mu}[\frac{2\tau}{\gamma} + \kappa - \sigma - \alpha - 1]$. Doing so yields

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{\varepsilon}} \end{bmatrix} = \begin{bmatrix} 2\frac{\tau}{\gamma}(1 - \frac{\tau}{\gamma}) & \mu\frac{\tau}{\gamma}(1 - \frac{\tau}{\gamma}) \\ -\frac{1}{\mu}(2\tau + \gamma[\kappa - \sigma - \alpha - 1]) & 0 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{\varepsilon} \end{bmatrix},$$

where the Jacobian matrix has eigenvalues

$$\lambda_\pm = \frac{\tau}{\gamma}(1 - \frac{\tau}{\gamma}) \pm \sqrt{\frac{\tau^2}{\gamma^2}(1-\frac{\tau}{\gamma})^2 - \frac{\tau}{\gamma}(1-\frac{\tau}{\gamma})(2\tau+\gamma[\kappa-\sigma-\alpha-1])}.$$

Note that the radicand of $\lambda_\pm$ is negative iff

$$\tau^2 + \gamma(2\gamma - 1)\tau + \gamma^3[\kappa - \sigma - \alpha - 1] > 0.$$

The equation $\tau^2 + \gamma(2\gamma - 1)\tau + \gamma^3[\kappa - \sigma - \alpha - 1] = 0$ is solved by

$$\tau_\pm = -\tfrac{1}{2}\gamma(2\gamma - 1) \pm \tfrac{1}{2}\sqrt{\gamma^2(2\gamma - 1)^2 - 4\gamma^3[\kappa - \sigma - \alpha - 1]}.$$

Observe that $\tau_\pm \notin \mathbb{R}_{>0}$, due to Assumption 2(ii), so

$$\tau^2 + \gamma(2\gamma - 1)\tau + \gamma^3[\kappa - \sigma - \alpha - 1] > 0$$

for all $\tau \in \mathbb{R}_{>0}$. Thus, the radicand of $\lambda_\pm$ is negative and $\mathrm{Re}(\lambda_+) = \mathrm{Re}(\lambda_-) = \frac{\tau}{\gamma}(1 - \frac{\tau}{\gamma}) > 0$ by Assumption 2(i), which implies that equilibrium (12) is an unstable spiral.
Next, consider $(x, \varepsilon) = (0, 0)$. Linearizing the system in (11) around $(x, \varepsilon) = (0, 0)$ yields a Jacobian matrix with eigenvalues $\gamma - \tau > 0$ and $\alpha + \sigma - 1 - \kappa$. By Assumption 2(ii), $\alpha + \sigma - 1 - \kappa < -2 < 0$, so $(x, \varepsilon) = (0, 0)$ is a saddle point.
Finally, we consider $(x, \varepsilon) = (1, 0)$. Linearizing (11) around $(x, \varepsilon) = (1, 0)$ gives a Jacobian matrix with eigenvalues $-\tau < 0$ and $\kappa - (\sigma + \alpha + 1) > 0$ (by Assumption 2(ii)), so $(x, \varepsilon) = (1, 0)$ is a saddle point. □

Using the local stability properties of the system equilibria, we derive the following (almost) global convergence result.

Theorem 1. If Assumption 2 holds, there exists a limit cycle attracting all solutions of (11) that start in the interior of $[0, 1] \times \mathbb{R}_{\ge 0}$, excluding equilibrium (12).

Proof. Let $\kappa > \sigma + \alpha + 1$ (by Assumption 2). To prove that the system in (11) converges to a limit cycle, we first need to study the behavior of (11) close to the boundary of its domain. Consider the boundary $x = 1$. Let us assume that there exists a trajectory that reaches $x = 1 - \epsilon$ at a time t_0, where $\epsilon \in (0, \frac{\tau}{\gamma})$ is arbitrarily infinitesimally small. Note that for $x \in [1 - \epsilon, 1)$ and $\varepsilon \le \frac{1}{\mu}(\kappa - \sigma - \alpha - 1)$, we have $2x + \mu\varepsilon + \alpha + \sigma - \kappa - 1 < 1 + \mu\varepsilon + \alpha + \sigma - \kappa \le 0$, which

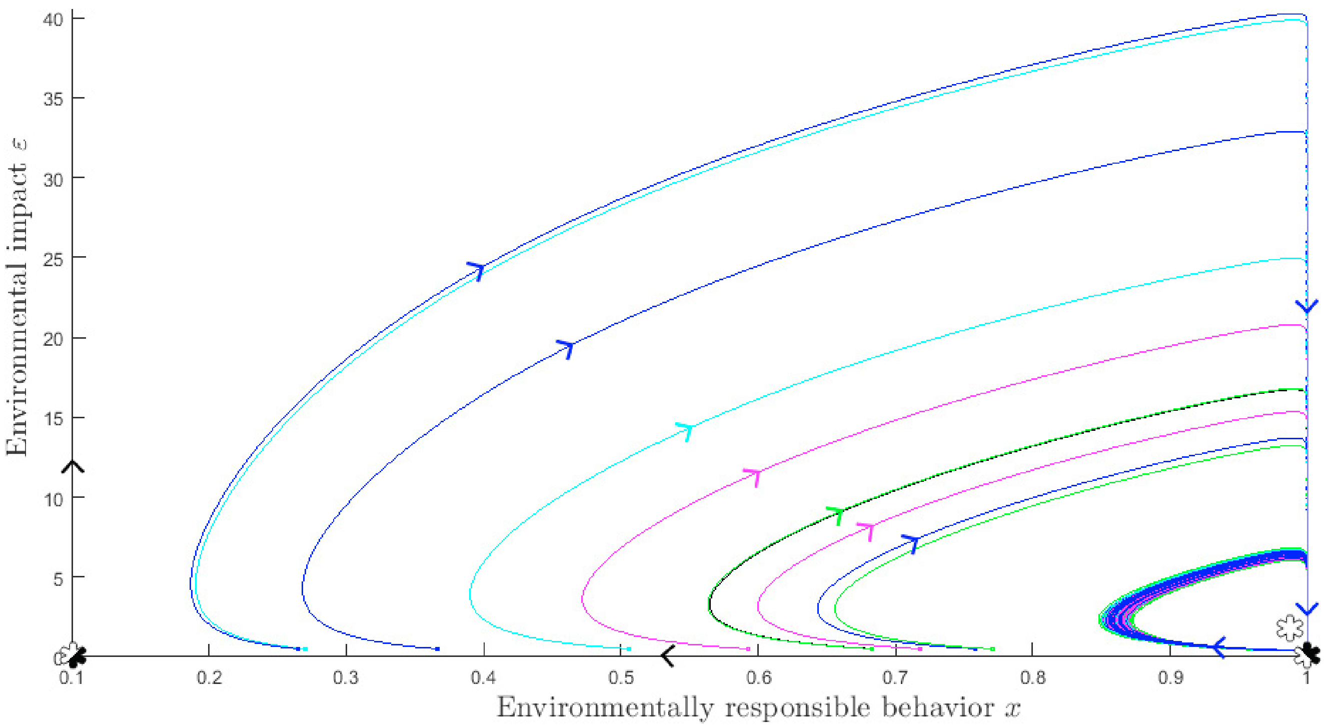

Fig. 1. Simulated trajectories of the system in (11) for parameter values $\alpha = 0.3$, $\sigma = 0.6$, $\kappa = 3$, $\gamma = 10$, $\tau = 0.1$ and $\mu = 0.6$. Saddle points and unstable equilibria are marked with black-white and white asterisks, respectively.

implies that $\dot{x} = x(1-x)(2x+\mu\varepsilon+\alpha+\sigma-\kappa-1) < 0$. Hence, $\dot{x}$ can only be positive for $\varepsilon > \frac{1}{\mu}(\kappa-\sigma-\alpha-1)$. Let us consider any trajectory with $x(t_0) = 1-\epsilon$ that enters

$$\mathcal{R} := [1-\epsilon, 1] \times \left(\tfrac{1}{\mu}(\kappa-\sigma-\alpha-1), \varepsilon(t_0)\right].$$

We will now show that it is not possible for a trajectory in $\mathcal{R}$ to reach the boundary $x = 1$. First, note that for $\varepsilon > 0$ and $x > 1-\frac{\tau}{\gamma}$, we have $\dot{\varepsilon} = (\gamma(1-x)-\tau)\varepsilon < 0$, so $\dot{\varepsilon} < 0$ for $(x,\varepsilon) \in [1-\epsilon, 1] \times \mathbb{R}_{>0}$ and the trajectory cannot exit $\mathcal{R}$ from the the top. Next, let us define $u(t) = 1 - x(t)$. Since $\varepsilon(t) \leq \varepsilon(t_0)$ for any $t \geq t_0$, it follows that $\dot{x} \leq p(1-x)$ for some constant $p > 0$, which is equivalent to $-\dot{u} \leq -p(-u)$. By the Grönwall-Bellman inequality (Pachpatte, 1997), we have $-u(t) \leq -u(t_0)e^{-p(t-t_0)}$, or equivalently,

$$x(t) \leq 1 - \epsilon e^{-p(t-t_0)} < 1,$$

for any $t \geq t_0$. Next, note that in $\mathcal{R}$,

$$\dot{\varepsilon} \leq -(\tau-\epsilon\gamma)\varepsilon < -\tfrac{1}{\mu}(\tau-\epsilon\gamma)(\kappa-\sigma-\alpha-1).$$

Thus, $|\dot{\varepsilon}| > \frac{1}{\mu}(\tau-\epsilon\gamma)(\kappa-\sigma-\alpha-1)$. The length of the ε-axis in $\mathcal{R}$ is less than $\varepsilon(t_0) - \frac{1}{\mu}(\kappa-\sigma-\alpha-1)$. Hence,

$$\exists\ \tilde{t} \leq \tfrac{\mu\varepsilon(t_0)-(\kappa-\sigma-\alpha-1)}{(\tau-\epsilon\gamma)(\kappa-\sigma-\alpha-1)}$$

such that $\varepsilon(t_0+\tilde{t}) \leq \frac{1}{\mu}(\kappa-\sigma-\alpha-1)$. At time $t_0+\tilde{t}$, we have $x(t_0+\tilde{t}) \leq 1-\epsilon e^{-p(\tilde{t})} < 1$ and the trajectory is in the region $\mathcal{S} := [1-\epsilon, 1] \times \left[0, \frac{1}{\mu}[\kappa-\sigma-\alpha-1]\right]$. Since $\dot{x} < 0$ for $\varepsilon \leq \frac{1}{\mu}(\kappa-\sigma-\alpha-1)$, the trajectory will move away from the boundary and cannot reach $x = 1$. Similarly, we can show that any trajectory starting in the interior cannot reach the boundaries $\varepsilon = 0$ and $x = 0$. This implies that it is impossible to reach the boundary equilibria if the initial conditions of the system are in the interior of $[0,1] \times \mathbb{R}_{\geq 0}$.

Lastly, consider the set $(x,\varepsilon) \in (0,1) \times \mathbb{R}_{>0}$. Since the unique equilibrium in the interior is unstable, there does not exist a homoclinic orbit. Moreover, Lemma 2 guarantees that all solutions are bounded. Hence, by the generalized Poincaré-Bendixson theorem (Teschl, 2012), every non-empty compact ω-limit set of an orbit is periodic. □

Below, we report a brief characterization of the system behavior if the initial condition is on the domain boundary.

Proposition 3. Under Assumption 2, the following holds:

i) If $x(0) = 1$ and $\varepsilon(0) \geq 0$, then the solution of (11) converges to the equilibrium $(1, 0)$;
ii) If $x(0) < 1$ and $\varepsilon(0) = 0$, then the solution of (11) converges to the equilibrium $(0, 0)$;
iii) If $x(0) = 0$ and $\varepsilon(0) > 0$, then the solution of (11) diverges toward $(0, \infty)$.

Theorem 1 is illustrated by Fig. 1, which shows that all of the simulated trajectories converge to a periodic solution. Additionally, Fig. 1 suggests that before the trajectory reaches the natural oscillations in the limit cycle, the environmental impact might increase to an alarmingly high level during the transient phase, dependent on the initial system conditions. This observation calls for the design of optimal control strategies to influence the system trajectory in the transient regime. By letting the control parameters κ and α be time-varying, control strategies can be developed in terms of environmental subsidies and awareness campaigns. Specifically, the use of feedback control schemes—where we let α and κ depend on t through $\varepsilon(t)$—might be extremely beneficial toward mitigating the increase in environmental impact during the transient phase. This intuition is bolstered by simulations of the proposed control scheme in Fig. 2—implementing a control action that is linearly (or even super-linearly) proportional to the environmental impact seems to be highly beneficial in reducing the trajectory peaks.

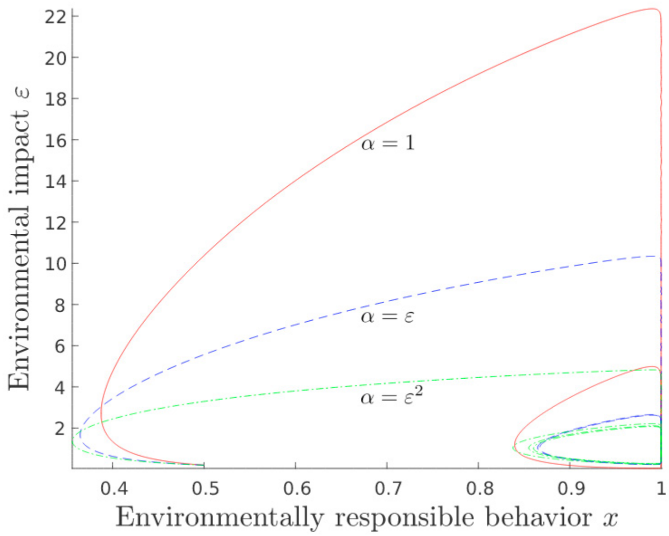

Fig. 2. Simulated trajectories of the system in (11) for different choices of the control function α, for $\sigma = 0.6$, $\kappa = 3$, $\gamma = 10$, $\tau = 0.1$ and $\mu = 0.6$.

5. CONCLUSION

We proposed a novel stochastic network model that captures the coevolution of human environmental behavior and environmental impact. Our modeling framework includes a variety of factors such as policy interventions, negative emission technologies, social influence, a behavioral response to increases in environmental impact, and the cost of environmentally friendly behavior. By employing a mean-field approach, we derived a deterministic approximation of the system in the limit of large-scale populations, for which we performed a complete asymptotic analysis. Specifically, we proved global convergence to a periodic solution for almost all initial conditions.

Our modeling framework and results open up the path for several directions of future research. First, our theoretical results are derived under the simplifying assumption of an all-to-all network of interactions. To better approximate real-world scenarios, the model can be studied while employing non-trivial social networks. Second, we assumed a linear behavioral response to the environmental impact, but more complex nonlinear functions may be considered. In particular, one may consider extending the framework to a multi-population scenario with cautious and reckless subpopulations, modeled by assigning different environmental response functions. By including a degree of homophily, i.e., a tendency of people to interact with like-minded individuals, one can explore the role of a polarized network structure in the evolution of environmental population behavior. Third, as we discussed through numerical simulations, future efforts should be placed on investigating the possibility of mitigating extreme system trajectories via time-varying control policies and, in particular, state-dependent policies, where the effort placed by public authorities is defined as a feedback function of the environmental state—thereby finding a way to guarantee that the environmental impact stays less than a critical threshold above which the planet becomes unsuitable for life.

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