Investigation on the Loss Allocation Properties in Distribution Networks with Distributed Generation

Publisher:
IEEE

Published
DOI:10.1109/UPEC57427.2023.10294476

(Article begins on next page)

Investigation on the Loss Allocation Properties in Distribution Networks with Distributed Generation

Andrea Mazza, Soheil Saadatmandi, Salvador Safina and Gianfranco Chicco
Dipartimento Energia "Galileo Ferraris"
Politecnico di Torino
Torino, Italy
andrea.mazza@polito.it, soheil.saadatmandi@polito.it, salvador.safina@studenti.polito.it, gianfranco.chicco@polito.it

***Abstract*— The allocation of the losses that occur during the operation of the distribution system to the distribution network nodes is useful to understand whether there could be a deficit or an excess of load or local generation in the nodes. This concept is extended in this paper by looking at the zones of the distribution network in which there is an interaction among the generations and loads. The sign of the allocated losses is considered as a useful indicator of the need to provide more load or more local generation in the zone. Specific findings are extracted from the analysis of balanced and unbalanced distribution systems with time-varying load and distributed generation.**

Keywords—allocated losses, distribution system, network losses, photovoltaic, network zone.

I. Introduction

The distribution network losses are normally used as an indicator to be minimised in optimisation tools such as the ones used for optimal distribution system reconfiguration or optimal operational planning. Because of their dependence on the square of the branch currents, the losses have a non-linear nature with respect to the distribution system loads and generations and cannot be correctly assessed without executing the power flow calculations in the network. Moreover, the network losses cannot be associated to the network loads or generations in straightforward way. Early attempts to establish a conventional way to assign the network losses to the system nodes were based on postage stamp, allocation proportional to the load power, or allocation proportional to the square of the load power. The attempts to use any proportion with the load power could be applied in a system in which all loads are positive (e.g., in the traditional transmission systems [1] in which no reverse flow occurred), while cannot be meaningfully applied any longer to a system in which also local generation is available and the net power (i.e., load minus generation) taken from a node may be either positive or negative. Moreover, the role of the loss allocation in distribution systems (where the slack node is not an individual generator and is the prevailing source, so that no losses are allocated to the slack node) is different with respect to what happens in the transmission systems (where the slack node is a physical generator and shares part of the total system losses) [2].

In the early 2000's, the issue of loss allocation in distribution systems with distributed generation was addressed by proposing various methods. Some methods were based on the derivatives of the power flow equations [3], exploiting the Jacobian and Hessian matrices of the power flow equations to provide the loss allocation coefficients. In these methods, the sum of the loss allocation coefficients was not generally equal to the total network losses, requiring the calculation of new vectors to provide the reconciliation of the sum of the final vectors with the total network losses. In other cases, circuit-based methods were used to obtain the loss allocation coefficients without the need for reconciliation, using the coefficients of the bus impedance matrix [4] or of a modified bus admittance matrix [5]. Then, effective simplified circuit-based approaches were defined for *radial* networks, without needing the creation of the bus impedance or bus admittance matrix, applicable to balanced [6] and unbalanced [7] distribution systems. In these approaches, the existence and solution of the reactive power paradox and the loss partitioning paradox (in the three-phase representation) was discussed and solved, leading to final expressions in which the loss allocation is determined by using only the resistive parameter of the complex branch impedance. Successive developments have discussed the role of the losses in the neutral conductor [8]. Further notes are presented in [9].

In the approaches implemented for loss allocation to the system nodes, there are some distinguishing aspects to be considered:

- The losses are allocated to the *nodes* and not to the individual local load or generation connected to the node. Because of that, the information that can be obtained from the allocated losses refers to the *net power* and not to the individual components served at the node.
- The allocated losses can be *positive or negative*. In the three-phase system representation, the allocated losses can be positive or negative not only in different nodes, but also in different phases at the same node. The information on the numerical value and sign of the losses allocated to the network nodes has been used in [10] for the construction of specific scenarios to be adopted to investigate on the limits for the diffusion of distributed energy resources (DER) in the distribution system.
- The allocated losses are simpler to compute than the second-order sensitivities [11]. However, the losses allocated *cannot* be formally seen as sensitivity coefficients that indicate the variation of the total network losses with respect to the net load at a node. The sign of the allocated losses can be somehow considered as a qualitative indication of the convenience to increase/decrease the local load or the local generation to reduce the total network losses.

The latter point is further investigated in this paper. The benefits of considering the allocated losses for understanding the possible convenience of changing the load or generation are evaluated considering not only the node at which the load or generation is connected, but also the neighbouring nodes connected in the same zone of the network (e.g., a network feeder). In this way, it is possible to draw more general indications on the distribution system operation in specific zones of the network. In addition to the spatial variation, the variation in time of the allocated losses is investigated, to understand how the evolution of the allocated loads relates with the evolution of the loads and generations in the distribution system.

The next sections of this paper are organised as follows. Section II recalls the circuit-based methods used to compute the losses allocated to the nodes (and to the phases in three-phase systems). Section III sets out the framework of analysis of the losses allocated to the nodes in different network zones. Section IV presents the results of applications carried out on balanced and unbalanced distribution systems. The last section contains the concluding remarks.

II. Loss Allocation in Radial Distribution Systems

Let us consider a distribution system with radial structure, supplied from the slack node (denoted as node 0) with K nodes (slack excluded) and B branches. The loads in the distribution system are represented by their active and reactive net power (i.e., load minus generation) input to the node. The net power is variable in time, with constant average power at each time step $t = 1, \ldots, T$.

For the analysis of the radial system, two relevant sets are defined:

- the set $\mathbb{B}_k$ of the branches found in the path between node k and the root node 0, for $k = 1, \ldots, K$;
- the set $\mathbb{K}_b$ of the nodes that remain isolated if branch b is cut, that is, the nodes that are supplied through branch $b = 1, \ldots, B$.

The power flow calculations for radial distribution systems are carried out by using the backward-forward sweep (BFS) method (also applicable to weakly-meshed distribution systems by including an external iterative cycle) [12]. BFS is a circuit-based method that does not need the calculation of derivatives of the network variables, is easy to implement and has good convergence properties [13]. Starting from the power flow results, the losses allocated to the network nodes are determined from a circuit-based approach as indicated below.

A. *Calculation of the Losses Allocated to the Network Nodes for Radial Balanced Distribution Networks*

The loss allocation to the system nodes is computed following the Branch Current Decomposition for Loss Allocation (BCDLA) approach developed in [6]. From the power flow results, at time step t the relevant quantities are the *net current* inputs $\bar{I}_{k,t}$ to the nodes $k = 1, \ldots, K$, and the currents $\bar{I}_{b,t}$ flowing in the branches $b = 1, \ldots, B$ with resistance R_b. The losses allocated to node k are calculated as:

$$\lambda_{k,t} = \mathcal{R}e\left\{\bar{I}_{k,t}^{*} \sum_{b\in\mathbb{B}_k}\left(R_b \, \bar{I}_{b,t}\right)\right\} \quad (1)$$

A very useful property of the loss allocation mechanism is that the sum of the losses allocated to the nodes $k = 1, \ldots, K$ at time step t is exactly equal to the total losses in the whole network at time step t:

$$L_{\mathrm{tot},t} = \sum_{k=1}^{K} \lambda_{k,t} \quad (2)$$

B. *Calculation of the Losses Allocated to the Network Nodes for Radial Unbalanced Distribution Networks*

In unbalanced distribution networks, the hypothesis of single-phase equivalent circuit is no longer valid. Then, other types of conductors have to be considered, including the neutral conductor and the return path through the ground. In addition, the parameters of the phase conductors could be different to each other because of the mutual position of the phase conductors. Let us identify the phase conductors with the letters a, b and c.

There are two possible representations of the branch impedances:

1) Representation starting from the multi-conductor system modelled by using the *Carson equations* [14]. In this case, the self-impedances and mutual impedances of each conductor are determined, obtaining an impedance matrix $\boldsymbol{Z}_b$ of dimensions nxn for each branch b with n conductors. A typical example is the distribution system with the conductors at the three phases, the neutral and the ground. Under the hypothesis of equal neutral-to-ground voltages at the two terminals of the branch, the Kron reduction is applied to reach, for branch b, an impedance matrix described by a 3x3 square matrix indicated as $\boldsymbol{Z}_{abc.b}$. The real part of this matrix is denoted as $\boldsymbol{R}_{abc.b} = \mathcal{R}e\{\boldsymbol{Z}_{abc.b}\}$.

2) Representation of the branch impedance by using the symmetrical components introduced by Fortescue [15]. Considering the complex operator $\alpha = \mathrm{e}^{\mathrm{j}\frac{2\pi}{3}}$, the Fortescue transformation matrix is expressed as

$$\mathbf{T} = \frac{1}{3}\begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{bmatrix} \quad (3)$$

and the impedance matrix in symmetrical components is calculated as

$$\mathbf{Z}_{\mathrm{s},b} = \mathbf{T}\,\mathbf{Z}_{\mathrm{abc}.b}\,\mathbf{T}^{-1} \quad (4)$$

In particular, if all the diagonal terms of the matrix $\boldsymbol{Z}_{abc.b}$ are equal among each other and all the non-diagonal terms of the the matrix $\boldsymbol{Z}_{abc.b}$ are equal among each other, then the relation (2) corresponds to the diagonalisation of the matrix $\boldsymbol{Z}_{abc.b}$. In this case, the matrix $\boldsymbol{Z}_{s,b}$ contains three non-null diagonal entries defined at the positive, negative and zero sequences, while the non-diagonal entries are null:

$$\mathbf{Z}_{s,b} = \begin{bmatrix} \bar{Z}_b^{(+)} & 0 & 0 \\ 0 & \bar{Z}_b^{(-)} & 0 \\ 0 & 0 & \bar{Z}_b^{(0)} \end{bmatrix} \tag{5}$$

The diagonal representation of the matrix $\boldsymbol{Z}_{s,b}$ implies the full decoupling of the three sequence circuits, which can be constructed separately. In particular, impedance data at the three sequences can be directly provided for the distribution network [16]. If any of the previous properties is not satisfied, e.g., because of the different magnetic behaviour among the three phases occurring in untransposed lines [17], the matrix $\boldsymbol{Z}_{s,b}$ obtained from (4) is not diagonal and the effectiveness of the representation in symmetrical components is reduced by the absence of decoupling among the three sequences.

In this paper, the branch representation with the matrix $\mathbf{Z}_{\text{abc}.b}$ is taken as the reference. If the branch data are available in sequence component form, the matrix transformation (4) is applied in its reverse way to obtain $\mathbf{Z}_{\text{abc}.b} = \mathbf{T}^{-1}\,\mathbf{Z}_{\text{s},b}\mathbf{T}$.

For the analysis of the three-phase system, it is necessary to extend the notation referring to the branch current to the vector form in which the currents that flow towards the three phases of the load at node *k* at time step *t* are expressed as

$$\mathbf{i}_{k,t} = \begin{bmatrix} \bar{I}_{k,\text{a},t} & \bar{I}_{k,\text{b},t} & \bar{I}_{k,\text{c},t} \end{bmatrix}^{\text{T}} \tag{6}$$

Let us denote the losses allocated to the three phases of the load connected to node *k* at time step *t* as

$$\boldsymbol{\lambda}_{k,t} = \begin{bmatrix} \lambda_{k,\text{a},t} & \lambda_{k,\text{b},t} & \lambda_{k,\text{c},t} \end{bmatrix}^{\text{T}} \tag{7}$$

From the developments shown in [7], the allocated losses $\boldsymbol{\lambda}_{k,t}$ are determined by using the Resistive Component-based Loss Partitioning (RCLP) approach, based on the following equation:

$$\boldsymbol{\lambda}_{k,t} = \mathcal{R}e\left\{\mathbf{i}_{k,t} \otimes \sum_{b\in\mathbb{B}_k}\left(\mathbf{R}_{\text{abc}.b}\mathbf{i}_{b,t}^{*}\right)\right\} \tag{8}$$

where the symbol ⊗ represents the component-by-component (Hadamard) vector product, and the current $\mathbf{i}_{b,t}$ is calculated as the sum of the currents that flow in the nodes supplied through branch *b*:

$$\mathbf{i}_{b,t} = \sum_{k\in\mathbb{K}_b} \mathbf{i}_{k,t} \tag{9}$$

Also in this case, the sum of the losses allocated to each phase of each node corresponds to the total network losses:

$$L_{\text{tot},t} = \sum_{k=1}^{K} [1\ 1\ 1]\, \boldsymbol{\lambda}_{k,t} \tag{10}$$

C. Power Flow Solvers for Distribution Networks

There are many power flow solvers dedicated to the solution of electrical distribution systems. These solvers can generally be first categorised into commercial solvers and open access solvers. Recently, a new generation of open access solvers has been issued, by resorting to the latest developments in programming languages. Some solvers are available as specific packages in the Matlab® suite. Other solvers have been developed in the Python ambient, starting from PyPower, which has served as the basis for developing further solvers such as PyPSA (which incorporates models of renewable energy resources and storage), PandaPower (aimed at power system analysis and optimisation), Oemof/micrOgridS (focused on the modelling and optimisation of energy systems), and Steps (developed in C++ with modules to be used in Python for calculating power flow and dynamic simulations). A comparison among some solvers is presented in [18], considering the ambient in which the solvers have been developed. Beyond the open-source modules available in Matlab® and Python, there are other power flow solvers such as OpenDSS, Gridlab-D, and RAPSim. The various solvers have different characteristics. Each solver includes specific details in the modelling that could not be available in all other solvers. With reference to the calculations of interest in this paper, there are limitations for example from PandaPower [19], which until now considers only a symmetrical distribution of the conductors in the branches, and as such is unable to represent cases with different diagonal values in the branch impedance matrix.

The calculations indicated in the previous subsections are not available in any commercial or open-access solver. Therefore, the authors have implemented the calculation of the losses allocated to the nodes in the various programming languages (Matlab®, Python) as post-processing tools that use the results of the power flow solvers.

III. Interpretation of the Allocated Losses in Different Operational Cases of the Distribution Networks

A. Sensitivities and Allocated Losses

Let us first recall the concept that the allocated losses are not sensitivities. In sensitivity analysis, the sensitivity coefficient $\sigma_{k,t}^{(P)}$ represents (in the case of interest for this paper) the variation of the total active power losses with respect to the variation of the local load $P_{k,t}^{(\mathrm{d})}$ or generation $P_{k,t}^{(\mathrm{g})}$ at a given node k and time step t. This coefficient can be estimated by taking the power flow solution at an operating point as the reference ("ref") and calculating another power flow solution ("new") by imposing a small change in a component of the net power at a node, while all the other components do not change. The changes in the total losses are expressed by using the sensitivity coefficient with the following linearisation:

$$L_{\mathrm{tot},t}\Big|_{\mathrm{new}} - L_{\mathrm{tot},t}\Big|_{\mathrm{ref}} \cong \sigma_{k,t}^{(P)} \left(P_{k,t}^{(\mathrm{d})}\Big|_{\mathrm{new}} - P_{k,t}^{(\mathrm{d})}\Big|_{\mathrm{ref}} - P_{k,t}^{(\mathrm{g})}\Big|_{\mathrm{new}} + P_{k,t}^{(\mathrm{g})}\Big|_{\mathrm{ref}} \right) \qquad (11)$$

This formulation indicates that different situations could appear, with positive or negative sensitivities when the local load or generation changes.

The calculation of the sensitivity coefficients at every node requires the execution of multiple power flows to determine the new total losses after the variation of one component of the net power at a time. Then, the sensitivity coefficients can be estimated in a quantitative way. Conversely, the calculation of the allocated losses requires a single and simple calculation from (1) or (8), providing a qualitative information. The main information considered is the sign of the allocated losses, which is interpreted in a similar way as the sign of a sensitivity coefficient. If the losses allocated at a given node are positive, it is expected that the total losses could be reduced by reducing the net load at that node, i.e., by reducing the local load or increasing the local generation. On the other side, if the losses allocated at a given node are negative, it is expected that the total losses could be reduced by increasing the net load at that node, i.e., by increasing the local load or reducing the local generation.

B. *Conceptualisation*

Starting from the above concepts, the analysis has been extended to identify cases in which there is a consistent behaviour of the allocated losses for nodes located in the same zone. This corresponds to the fact that the excess of load (or the excess of generation) does not refer to a single node only, but what matters is the operational condition at groups of nodes in the same zone. In particular, the cases investigated refer to situations in which the negative losses allocated appear not only at generation nodes, but also at neighbouring nodes without local generation.

IV. Applications to Selected Cases in Balanced and Unbalanced Distribution Networks

A. *Balanced distribution system*

The balanced MV distribution system considered has been used in the project Atlantide [20]. The radial distribution system (20 kV) is supplied at 150 kV by the transmission system and contains 102 nodes divided into 7 feeders (Fig. 1). The system serves a rural area, with 67 agricultural load nodes, 40 residential load nodes, 14 industrial load nodes, and 5 nodes with photovoltaic generation (highlighted with blue rectangles). The daily load patterns of generation and demand correspond to the total energy 227 MWh, while the daily network energy losses are 3.4 MWh (i.e., 1.5% of the total load energy). The educational material shown is used by the Master students in Electrical Engineering at Politecnico di Torino in one of the classworks taught to the students in the computer laboratory.

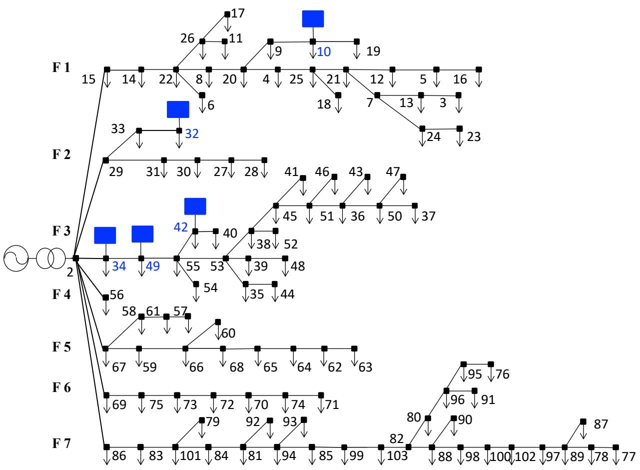

Fig. 1. The distribution system defined in the project Atlantide.

The information on the sign and magnitude of the allocated losses has been exploited to construct more challenging scenarios. The feeders with installed PV are three, namely F1, F2 and F3. It may happen that in the presence of quite high share of DG, unlike intuitive results, the generation nodes present *positive* allocated losses, whereas the load nodes are characterised by *negative* allocated losses. Let us consider the feeder F3 during the time interval (46–54), i.e., the quarters of hour around 12 am: the total generation and the total load are shown in Table I. The feeder presents non-negligible overgeneration, as part of the generation is in excess with respect to the needs of the feeder. The allocated losses calculated in the time interval (46–54) are shown in Table II. All the nodes with DG (namely, node 34, 49, and 42) present positive allocated losses, whereas the load nodes present negative allocated losses. This is aligned with the indications provided by [21]: for both node types, the product of net nodal power and the allocated losses is negative. Hence, a decrease of the system losses may be reached only by increasing the net nodal power, i.e., by decreasing the generation (or increasing the load) in the generation node and increasing the load in the load nodes. As a matter of example, Table III shows what happens by varying either the load of node 55 or the generation of node 49 of ±10 kW. The variation of the generation/load is beneficial for the entire system; however, the variations of the allocated losses in the other nodes have a different “direction”.

TABLE I. FEEDER F3: NET LOAD COMPONENTS, TIME STEPS 46–54

time step	generation [kW]	load [kW]	feeder net load [kW]
46	-3706	1181	-2526
47	-3916	1184	-2732
48	-4130	1156	-2974
49	-4340	1127	-3214
50	-4340	1153	-3187
51	-4340	1178	-3162
52	-4340	1180	-3159
53	-4339	1181	-3158
54	-4093	1182	-2911

TABLE II. FEEDER F3: ALLOCATED LOSSES, TIME STEPS 46–54

node	allocated losses [pu]	node	allocated losses [pu]
34	0.13411	*45*	-0.00266
35	-0.00305	*46*	-0.00279
36	-0.00253	*47*	-0.00124
37	-0.00247	*48*	-0.00652
38	-0.00138	*49*	**0.04551**
39	-0.00002	*50*	-0.00126
40	-0.00705	*51*	-0.00131
41	-0.00412	*52*	-0.00136
42	0.04255	*53*	-0.00004
43	-0.00247	*54*	-0.00639
44	-0.00303	*55*	**-0.00485**
Feeder losses [pu]	0.16762	**System losses [pu]**	0.34706

TABLE III. FEEDER F3: ALLOCATED LOSSES [PU], TIME STEPS 46–54

node	active power variation *ΔP*			
	node 55		*node 49*	
	-10 kW	*10 kW*	*-10 kW*	*10 kW*
34	0.13456	0.13367	0.13456	0.13367
35	-0.00308	-0.00303	-0.00307	-0.00303
36	-0.00255	-0.00251	-0.00254	-0.00251
37	-0.00249	-0.00245	-0.00248	-0.00245
38	-0.00139	-0.00137	-0.00139	-0.00138
39	-0.00002	-0.00002	-0.00002	-0.00002
40	-0.00709	-0.00700	-0.00709	-0.00701
41	-0.00416	-0.00409	-0.00415	-0.00409
42	0.04280	0.04229	0.04277	0.04233
43	-0.00249	-0.00245	-0.00249	-0.00246
44	-0.00305	-0.00300	-0.00304	-0.00301
45	-0.00268	-0.00264	-0.00268	-0.00265
46	-0.00281	-0.00277	-0.00281	-0.00277
47	-0.00125	-0.00123	-0.00124	-0.00123
48	-0.00657	-0.00647	-0.00657	-0.00648
49	0.04575	0.04527	**0.04631**	**0.04472**
50	-0.00127	-0.00125	-0.00126	-0.00125
51	-0.00132	-0.00130	-0.00132	-0.00130
52	-0.00137	-0.00135	-0.00137	-0.00136
53	-0.00004	-0.00004	-0.00004	-0.00004
54	-0.00644	-0.00635	-0.00643	-0.00635
55	**-0.00434**	**-0.00535**	-0.00488	-0.00482
Feeder losses [pu]	0.16870	0.16656	0.16874	0.16651
System losses [pu]	0.34811	0.34602	0.34815	0.34598

In fact, while the allocated losses in the nodes where the load/generation is varied according to the sign of the net power and the allocated losses decrease (same behaviour of the entire system), other nodes experience an increase of the allocated losses.

This means that the load/generation variation according to the sign of the net power and the allocated losses on one node is beneficial for the system, but it changes the allocated losses of the other feeder nodes, i.e., changes the contribution allocated to the other nodes of the entire system losses. This aspect has to carefully considered in case the allocated losses are considered as indicators for sharing the system losses among the nodes: in fact, the load/generation variation in one node influences a zone (and not only that particular node), and hence the impact of every demand response action taken to improve the distribution system performance (i.e., system losses reduction) must be priorly and carefully evaluated.

Let us now consider the feeder F1: the active power at node 19 is varied with step 1 kW in the range (-75–75) kW. On the horizontal axis, the value $\Delta P = 0$ kW indicates the nominal active power of the node 19 at time step 54, i.e., $P_{54}^{(19)}$= 44.82 kW. The value of the applied load variation and the corresponding allocated losses are shown in Fig. 2. The sign of the allocated losses changes with the net load variation: the figure shows that a minimum value of allocated losses exists. The minimum value of allocated losses is -5.48e^{-6} pu and is reached with a positive load variation ΔP =12 kW, i.e., with a nodal load equal to 56.82 kW. Hence, a load increase is beneficial for the allocated losses. Higher load increases do not enable a further allocated losses improvement, despite the allocated losses are negative and net load is positive.

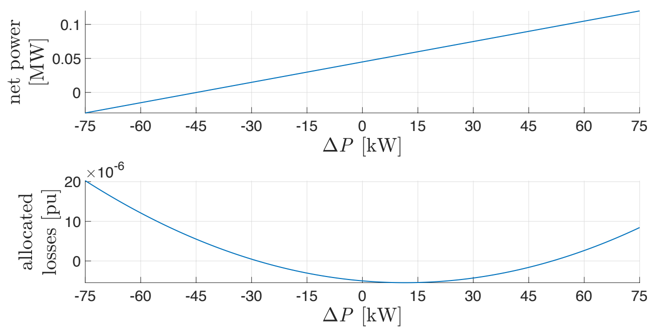

Fig. 2. Node 19: net power and allocated losses variations (time step 54).

B. *Unbalanced distribution system*

The analysis of an unbalanced distribution system has been carried out on the IEEE 906-bus Low-Voltage (LV) network, also known as the "IEEE European Low Voltage Test Feeder" [22], shown in Fig. 3. The network is radial, with rated frequency 50 Hz and rated voltage 416 V and is supplied by a Medium Voltage (MV) node through a MV/LV transformer (11 kV). The 55 single-phase loads are distributed among the different phases of the network. The reference data include the daily load profiles for the 55 loads, with time resolution of 1 min, i.e., with 1440 points for each load profile.

The reference IEEE 906-bus LV network has neither local generation nor storage. To analyse situations with significant contribution of the local generation, the network has been divided into four zones with different generation capacity installed. Photovoltaic (PV) generation is considered in all cases. In Zone 1 there is no local generation. In Zone 2 there is three-phase generation with rated power in each phase equal to 70% of the rated power of the single-phase load connected to the node where the local generation is installed (recalling that all loads in the network are single-phase). In Zone 3 there is a mix of single-phase and three-phase generation. The rated power of the single-phase generation is equal to the rated power of the load connected to the same node, whereas the rated power of the three-phase generation is equal to the rated power of the single-phase load connected to the same node. In Zone 4, there is only single-phase generation in some nodes, with rated power double with respect to the rated power of the load connected to the same node. Table IV shows the detail on the location and rated power of the PV systems. The PV generation inserted in the system is relatively high, so that in the mid hours of a clear sky day there is an overall excess of generation in the network, leading to the presence of reverse power flow with power injected in the MV system at the supply point (Fig. 4).

Fig. 5 shows the losses allocated to the nodes in which there is only generation (connected to phase B). Excluding the night, in the first period (from hour 8:15 to hour 9:38) the losses allocated are negative. Then, until hour 18:08, the allocated losses are prevailingly positive, while in the final period of PV generation (until hour 19:27) the allocated losses are again prevailingly negative.

TABLE IV. LOCATION AND RATED POWER OF THE PV GENERATION ADDED TO THE NETWORK

Zone	Node(s)	Phase(s)	PV rated power (kW)
1	(none)	(none)	(none)
2	522	a, b, c	1.8
	539	a, b, c	4.5
	458	a, b, c	7.2
3	900	a, b, c	9.0
	817	a, b, c	8.1
	860	a	4.5
	614	c	3.7
4	899	b	5.7
	886	b	4.0
	780	c	4.4

	813	b	6.5

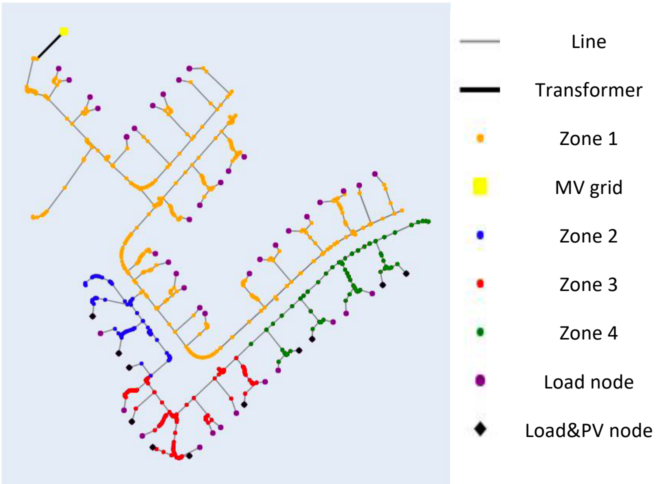

Fig. 3. Structure of the IEEE 906-bus network.

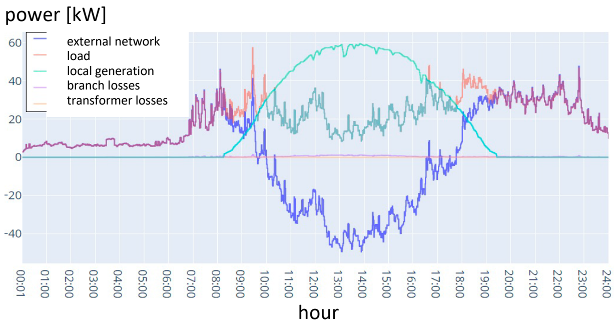

Fig. 4. Power balance in the IEEE 906-bus network.

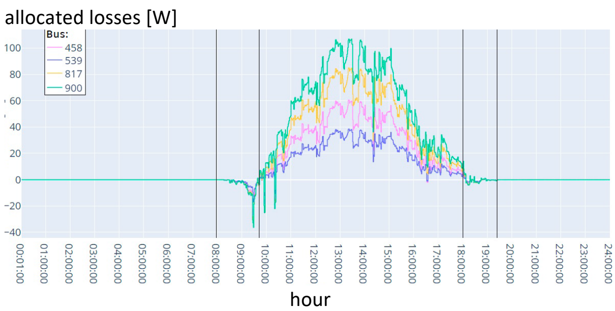

Fig. 5. Losses allocated to the nodes with generation only (Phase B).

V. Conclusions

The losses allocated to the nodes of the distribution systems (and to the phases for unbalanced systems) incorporate interesting and useful information on the convenient direction for changing the net power load in the system. The allocated losses can be positive or negative. In particular, the negative losses allocated are not strictly related to generation nodes, as they depend on the zone in which there could be an excess of generation. Moreover, the sign of the allocated losses has to be interpreted when there are cases of reverse power flow, that sometimes leads to results that could seem counterintuitive. For unbalanced systems, the situation has to be interpreted by taking into account the loads and generations connected to the different phases.

Some particular cases have been shown and discussed in this paper. In the presence of negative losses allocated to a node, a demand increase in that node could lead to reducing the total network losses, however, the losses allocated to the other nodes increase.

This paper has not addressed the economic aspects referring to the allocated losses. These aspects become important when different economic value is given to the cost of the energy given to the load and the revenue obtained from the local energy production. In this case, obtaining the same net power variation by increasing load or decreasing the local generation (or vice versa, by decreasing load and increasing the local generation) has different economic implications. These aspects will be the subject of further investigations.

Acknowledgment

Soheil Saadatmandi acknowledges the PhD grant received in application of the Italian Ministry Decree DM 1061/2021.

REFERENCES

[1] A.G. Expósito, J.M.R. Santos, T.G. Garcia, and E.A.R. Velasco, "Fair allocation of transmission power losses," *IEEE Trans. on Power Systems*, vol. 15, no. 1, pp. 184–188, 2000.

[2] E. Carpaneto, G. Chicco and J. Sumaili Akilimali, "Characterization of the loss allocation techniques for radial systems with distributed generation," *Electric Power Systems Research*, vol. 78, no. 8, pp. 1396–1406, 2008.

[3] J. Mutale, G. Strbac, S. Curcic, and N. Jenkins, "Allocation of losses in distribution systems with embedded generation," *IEE Proc.-Gener. Transm. Distrib.*, vol. 147, no. 1, pp. 7–14, 2000.

[4] A.J. Conejo, F.D. Galiana, and I. Kockar, "Z-Bus loss allocation," *IEEE Trans. on Power Systems*, vol. 16, no. 1, pp. 105–109, 2000.

[5] J.S. Daniel, R.S. Salgado, and M.R. Irving, "Transmission loss allocation through a modified Ybus," *IEE Proc.-Gener. Transm. Distrib.*, vol. 152, no. 2, pp. 208–214, 2005.

[6] E. Carpaneto, G. Chicco, and J. Sumaili Akilimali, "Branch Current Decomposition Method for Loss Allocation in Radial Distribution Systems with Distributed Generation," *IEEE Trans. on Power Systems*, vol. 21, no. 3, pp. 1170–1179, 2006.

[7] E. Carpaneto, G. Chicco, and J. Sumaili Akilimali, "Loss Partitioning and Loss Allocation in Three-Phase Radial Distribution Systems with Distributed Generation," *IEEE Trans. on Power Systems*, vol. 23, no. 3, pp. 1039–1049, 2008.

[8] M. Usman, M. Coppo, F. Bignucolo, R. Turri and A. Cerretti, "Multi-Phase Losses Allocation Method for Active Distribution Networks Based on Branch Current Decomposition," *IEEE Trans. on Power Systems*, vol. 34, no. 5, pp. 3605–3615, 2019.

[9] S. Mishra and A.P. Hota, "Loss Allocation Schemes in Power Distribution Networks-A Bibliographic Review," *2018 International Conference on Recent Innovations in Electrical, Electronics & Communication Engineering (ICRIEECE)*, Bhubaneswar, India, pp. 281–286, 2018.

[10] A. Mazza, E. Carpaneto, G. Chicco and A. Ciocia, "Creation of Network Case Studies with High Penetration of Distributed Energy Resources", *Proc. UPEC 2018 Conference*, Glasgow, UK, 4-7 September 2018.

[11] H.M. Ayres, D. Salles, and W. Freitas, "A Practical Second-Order Based Method for Power Losses Estimation in Distribution Systems With Distributed Generation," *IEEE Trans. on Power Systems*, vol. 29, no. 2, pp. 666–674, 2014.

[12] D. Shirmohammadi, H.W. Hong, A. Semlyen, and G.X. Luo, "A compensation-based power flow method for weakly meshed distribution and transmission networks," *IEEE Trans. on Power Systems*, vol. 3, no. 2, pp. 753–762, 1988.

[13] E. Bompard, E. Carpaneto, G. Chicco, and R. Napoli, "Convergence of the backward/forward sweep method for the load-flow analysis of radial distribution systems," *Electrical Power & Energy Systems*, vol. 22, no. 7, pp. 521–530, 2000.

[14] J.R. Carson, "Wave propagation in overhead wires with ground return," *Bell System Technical Journal*, New York, vol. 5, 1926.

[15] C.L. Fortescue, "Polyphase Power Representation by Means of Symmetrical Coordinates," *Transactions of the American Institute of Electrical Engineers*, vol. *XXXIX*, no. 2, pp. 1481–1484, 1920.

[16] G. Chicco and A. Mazza, "100 Years of Symmetrical Components," *Energies*, vol. 12, art. 450, 2019.

[17] P. Paranavithana, S. Perera, R. Koch, and Z. Emin, "Global voltage unbalance in MV networks due to line asymmetries," *IEEE Trans. on Power Delivery*, vol. 24, no. 4, pp. 2353–2360, 2009.

[18] G.A. Sigurosson and M.F. Abdel-Fattah, "Smart grids simulation tools: Overview and recommendations," *IEEE 62nd International Scientific Conference on Power and Electrical Engineering of Riga Technical University (RTUCON)*, Riga, Latvia, 2021.

[19] PandaPower, [Online] http://www.pandapower.org.

[20] F. Pilo, G. Pisano, S. Scalari, D. Dal Canto, A. Testa, R. Langella, R. Caldon, and R. Turri, "ATLANTIDE — Digital archive of the Italian electric distribution reference networks," *CIRED Workshop Integration of Renewables into the Distribution Grid*, 29-30 May 2012.

[21] A. Mazza, G. Chicco, "Losses Allocated to the Nodes of a Radial Distribution System with Distributed Energy Resources – A Simple and Effective Indicator," *2019 International Conference in Smart Energy System and Technologies*, Porto (Portugal), 9-11 September 2019.

[22] IEEE, The IEEE European Low Voltage Test Feeder, [Online] https://cmte.ieee.org/pes-testfeeders/resources/.