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Pyramidal versus Tetrahedral Elements in Finite Element Applications

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A successful 3-D finite element code for Maxwell's equations must include all four kinds of geometrical shapes: tetrahedrons, hexahedrons, triangular prisms, and square-based pyramids. For cells of the first three types, the scalar shape functions, the high-order curl-conforming and divergence-conforming vector bases are well known and available for a long time [1]. On the contrary, it has been much more difficult to derive higher-order functions and bases of the same kind for pyramidal cells, so much so that those accessible in the literature until recently are obtained with extremely complex *ad hoc* techniques [2].

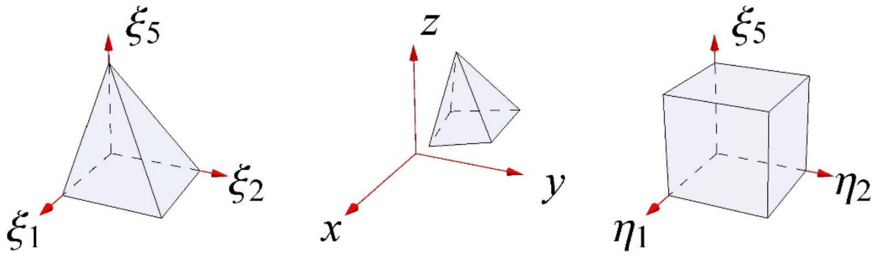

Figure 1. The shape functions map the parent pyramid on the left to the child pyramid in the center. In the grandparent space the pyramid is described by the cubic cell shown on the right, where the shape functions and the vector basis functions take polynomial form.

The peculiarity of pyramidal elements that complicates their definition is that the scalar functions (for example the shape functions) as well as the vector basis functions have fractional form in the so-called parent domain; therefore, it becomes necessary to introduce and work in a new, grandparent domain where these functions assume polynomial form. These facts recently led us to introduce a new paradigm by which we can derive the higher order bases of the pyramid in a simple and direct way, clearly defining their (polynomial) order, and demonstrating their completeness [3-6]. Using our new paradigm, the higher order functions are obtained simply by multiplying the known ones of the lowest possible order (given for example in [7]) by a complete set of scalar polynomials. The scalar sets can be of interpolatory kind to form shape functions and interpolatory vector bases, or of the hierarchical type such as those given in [3,4]. This multiplicative construction technique is the same used in [1] to build the higher order bases for tetrahedral, hexahedral, and prismatic cells.

However, since the vector functions based on the volume of the pyramid (i.e., the so-called bubbles) obtained with our new technique differ from those in [2], it is necessary to verify that our new high-order bases allow the same speed of convergence as those of other differently shaped elements (for example tetrahedrons), and that their use does not lead to spurious modes and/or solutions. At the conference we provide a comparison between pyramidal elements and tetrahedral elements in various finite element applications confirming that our new pyramidal elements are numerically equivalent to other known elements of different shape but identical order.

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