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Analysis of composite beams, plates, and shells using Jacobi polynomials and NDK models

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Keywords: Finite element method; beam, plate, and shell models; Jacobi polynomials; Node-dependent Kinematics; Carrera unified formulation.

Abstract. In this work, hierarchical Jacobi-based expansions are explored for the static analysis of multilayered beams, plates, and shells as structural theories as well as shape functions. Jacobi polynomials, denoted as $P_p^{(\gamma,\theta)}$, belong to the family of classical orthogonal polynomials and depend on two scalars parameters γ and θ, with p being the polynomial order. Regarding the structural theories, layer wise and equivalent single-layer approaches can be used. It is demonstrated that the parameters γ and θ of the Jacobi polynomials are not influential for the calculations. These polynomials are employed in the framework of the Carrera Unified Formulation (CUF), which allows to generate of finite element stiffness matrices straightforwardly. Furthermore, Node-dependent Kinematics is used in the CUF framework to build global-local models to save computational costs and obtain reliable results simultaneously.

Introduction

As modern engineering requires complicated and computationally expensive structural static analyses, appropriate 1D and 2D structural theories and Finite Element (FE) shape functions can be adopted to diminish the computational costs. The Carrera Unified Formulation (CUF) [1] is a versatile method to build 1D and 2D models. The governing equations can be derived and expressed in a compact way and are invariant from the adopted structure theory.

Considering the beam theories, Euler-Bernoulli Beam Model (EBBM) [2] and Timoshenko Beam Model (TBM) [3] are the classical formulations. For both, the cross-section is considered to be rigid in its plane. For EBBM, the shear deformation is neglected, while it is considered constant along the cross-section in the case of TBM. In the domain of CUF theories, Carrera and Giunta [4] used Higher Order Theories (HOT) derived from the Taylor polynomials. Furthermore, Carrera et al. [5] used Lagrange-like expansions over the cross-section. Concerning the FE models, Carrera et al. [1] used two-, three- and four-node Lagrange-like shape functions in the CUF framework.

As far as 2D plate and shell FEs are considered, Thin Plate Theory (TPT) and Thin Shell Theory (TST) are the classical models, see Kirchhoff [6]. The line remains orthogonal to the plate/shell reference surface in these models. When the transverse shear deformation is added, the Reissner–Mindlin [7,8] (also known as First-Order Shear Deformation Theory, FSDT) theory can be built. Carrera [9] proposed general HOT from the Taylor polynomials for the analysis of plates and shells for the CUF framework. The classical theories can be derived through penalization techniques from first-order Taylor. Furthermore, Carrera et al. [5] used Lagrange-like expansions along the thickness direction. Finally, Carrera et al. [1] used four-, eight- and nine-node FEs to study composite plates.

Two approaches can be used when dealing with laminates: Equivalent Single Layer (ESL) and Layer-Wise (LW) models. In the first one, the number of unknowns is unaffected by the number of layers, while in the second one, they depend on the layers, see Carrera [10].

Jacobi polynomials are utilized for building shape functions and structural theories for the analysis of beams, plates, and shells in CUF. Carrera et al. [11] first used these polynomials to build structural theories in the framework of CUF. They originate several polynomials changing the two parameters γ and θ, i.e., Legendre, Chebyshev, see the book of Abramowitz and Stegun [12]. Szabo et al. [13] proposed a hp-version of FE derived from Legendre (i.e., γ and θ equal to zero) polynomials for beam, plate, and solid. Zappino et al. [14] compared Legendre and Lagrange shape functions for 2D plate elements. Concerning the expansion functions in CUF, Pagani et al. [15] used Legendre for 2D cross-section in beam formulation, while Carrera et al. [16] studied plates with 1D expansions from Chebyshev polynomials.

Using enhanced models improves solutions' accuracy, but it increases costs. It is possible to use refined models in specific parts and low-fidelity models for the other part of the structure without using any mathematical artifices. Carrera and Zappino [17] first presented a global-local analysis for beams called Node-Dependent Kinematics (NDK). This was extended to laminated composite plates and shells by Zappino et al. [18] and Li et al. [19], respectively.

Hierarchical Jacobi polynomials for beams, plates, and shells

In the framework of CUF, Hierarchical Jacobi (HJ) polynomials have been used to build shape functions and structural theories for beams, plates, and shells. These elements have the interesting capability to use hierarchical features.

Jacobi polynomials are formulated using recurrence relations, see [12]. The Jacobi polynomials are described by the following expression:

$$P_p^{(\gamma,\theta)}(\zeta) = \left(A_p + B_p\right)P_{p-1}^{(\gamma,\theta)}(\zeta) - C_p P_{p-2}^{(\gamma,\theta)}(\zeta) \tag{1}$$

where γ and θ are two scalar parameters, and p stands for the polynomial order. The formula is evaluated in the natural plane $\zeta = [-1, +1]$. The first values are $P_0^{(\gamma,\theta)}(\zeta) = 1$ and $P_1^{(\gamma,\theta)}(\zeta) = A_0\zeta + B_0$. The explicit expressions of the scalars A_p, B_p, and C_p can be found in [12].

One-dimensional functions. It is possible to use HJ polynomials for building theories of structure along the thickness (z-axis) for plate and shell formulations, see Fig. 1 (a). Similarly, Jacobi-like shape functions can be adopted along the y-axis for the beam formulation, see Fig. 1 (b). For both cases, the building procedure is the same. However, for the sake of simplicity, one-dimensional shape functions are first considered. In this case, two kinds of polynomials are used along the y axis: vertex (or node) and edge. Basically, there are two vertexes and a number of edge modes that depends on the polynomial order of the chosen elements.

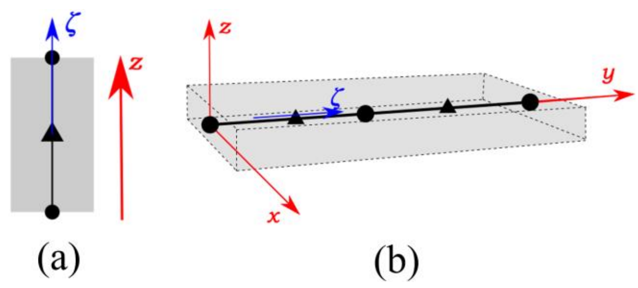


Figure 1: circle represents a vertex expansion, whereas triangle is an edge expansion. Theory of structure for plate and shell (a), shape functions for beam (b).

Given that formulas are formally identical for both cases $L_m(\zeta)$ is used to indicate the expansions. When the HJ polynomials are used as structural theories, $P_\tau(\zeta)$ is used, while $N_i(\zeta)$ is adopted for the shape functions. The hierarchic functions are defined as follows:

$$\begin{aligned} L_1(\zeta) &= \frac{1}{2}(1-\zeta) \\ L_2(\zeta) &= \frac{1}{2}(1+\zeta) \\ L_m(\zeta) &= \phi_{m-1}(\zeta), m = 3,4,\ldots,p+1 \end{aligned} \tag{2}$$

with

$$\phi_j(\zeta) = (1-\zeta)(+\zeta)P_{j-2}^{(\gamma,\theta)}, j = 2,3,\ldots,p \tag{3}$$

where p indicates the polynomial order. Given the following property

$$L_m(-1) = L_m(+1) = 0, m \geq 3 \tag{4}$$

The function $L_m(\zeta)$, m= 3, 4, … are named bubble functions or edge expansions.

Two-dimensional functions. It is possible to use HJ polynomials for building theories of structure in the cross-section (x-z plane) for beam formulation, see Fig. 2 (a). Similarly, Jacobi-like shape functions can be adopted over the x-y plane for the plate and shell formulations, see Fig. 2 (b). For both cases, the building procedure is the same. For the sake of simplicity, two-dimensional shape functions are first considered. In this shape functions, three kinds of polynomials are used: vertex, edge, and internal. There are four vertex modes that vanish at all nodes but one. Contrarily, the number of edge modes changes according to the polynomial order of the FE, and they vanish for all sides of the domain but one. Finally, the internal modes are included from the fourth-order polynomial. They vanish at all sides. See [16] for more information.

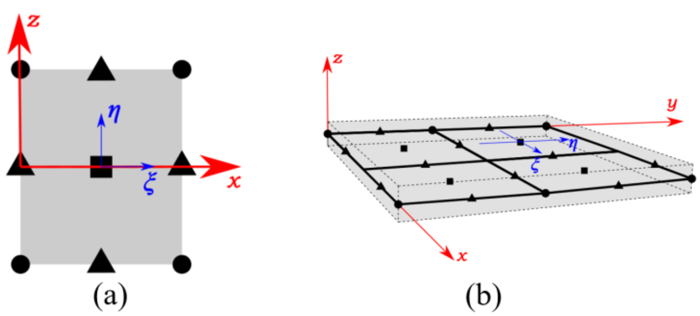


Figure 2: circle represents a vertex expansion, whereas triangle is an edge expansion and square indicates an internal expansion. Theory of structure for beam (a), shape functions for plate and shell (b).

Given that formulas are formally identical for both cases $L_m(\zeta)$ is used to indicate the expansions. When the HJ polynomials are used as structural theories, $P_\tau(\zeta)$ is used, while $N_i(\zeta)$ is adopted for the shape functions. The vertex modes are written as follows:

$$L_m(\xi,\eta) = \frac{1}{4}(1-\xi_m\xi)(1-\eta_m\eta), m = 1,2,3,4 \tag{5}$$

where ξ and η are calculated in the natural plane between -1 and +1, and ξ_m and ξ_m and η_m are the vertexes. From $p \geq 2$, the edge modes arise in the natural plane as follows

$$
\begin{aligned}
L_m &= \frac{1}{2}(1-\eta)\phi_p(\xi), m = 5,9,13,18, \dots \\
L_m &= \frac{1}{2}(1+\xi)\phi_p(\eta), m = 6,10,14,19, \dots \\
L_m &= \frac{1}{2}(1+\eta)\phi_p(\xi), m = 7,11,15,20, \dots \\
L_m &= \frac{1}{2}(1-\xi)\phi_p(\eta), m = 8,12,16,21, \dots
\end{aligned} \tag{6}
$$

where p represents the polynomial degree of the bubble function ϕ_j. Internal expansions are inserted for $p \geq 4$, they vanish at all the edges of the quadrilateral domain. There are $(p-2)(p-3)/2$ internal polynomials. By multiplying 1D edge modes, L_m internal expansions are built. For instance, considering the fifth-order polynomials, three internal expansions are found, which are

$$
\begin{aligned}
L_{17} &= \phi_2(\xi)\phi_2(\eta),\ 2+2=4 \\
L_{22} &= \phi_3(\xi)\phi_2(\eta),\ 3+2=5 \\
L_{23} &= \phi_2(\xi)\phi_3(\eta),\ 2+3=5
\end{aligned} \tag{7}
$$

Refined element based on CUF

In this section, the Carrera Unified Formulation (CUF) is presented for beams, plates, and shells. Multilayered beam, plate and shell structures are shown in Fig. 3.

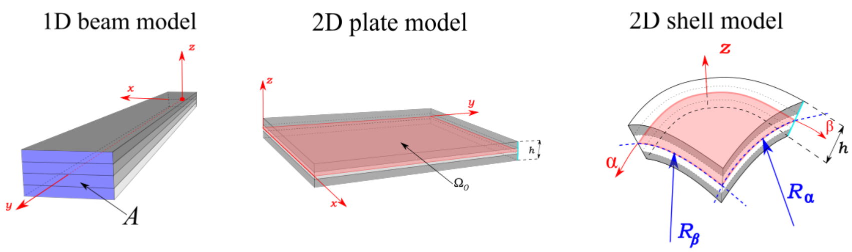


Figure 3: Generic multilayered beam, plate, and shell structures.

Concerning the beam model, the cross-section A lays on the $x-z$ plane of a Cartesian reference frame, whereas the beam axis is placed along the y direction. Contrarily, the plate model uses the z coordinate along the thickness direction, and the coordinates x and y indicate the in-plane mid-surface Ω_0. The 2D shell uses a curvilinear reference system (α, β, z) to account for the curvatures R_α and R_β. The displacement vector for the models is introduced in the following

$$
\mathbf{u}^k(x,y,z) = \{u_x^k, u_y^k, u_z^k\}^T,\ \mathbf{u}^k(\alpha,\beta,z) = \{u_\alpha^k, u_\beta^k, u_z^k\}^T \tag{8}
$$

where k indicates the layer. The stress, $\boldsymbol{\sigma}^k$, and strain, $\boldsymbol{\epsilon}^k$, vectors are defined as

$$
\begin{aligned}
\boldsymbol{\sigma}^k &= \{\sigma_{xx}^k, \sigma_{yy}^k, \sigma_{zz}^k, \sigma_{xz}^k, \sigma_{yz}^k, \sigma_{xy}^k\}^T \quad \boldsymbol{\epsilon}^k = \{\epsilon_{xx}^k, \epsilon_{yy}^k, \epsilon_{zz}^k, \epsilon_{xz}^k, \epsilon_{yz}^k, \epsilon_{xy}^k\}^T \\
\boldsymbol{\sigma}^k &= \{\sigma_{\alpha}^k, \sigma_{\beta\beta}^k, \sigma_{zz}^k, \sigma_{\alpha z}^k, \sigma_{\beta z}^k, \sigma_{\alpha\beta}^k\}^T \quad \boldsymbol{\epsilon}^k = \{\epsilon_{\alpha\alpha}^k, \epsilon_{\beta\beta}^k, \epsilon_{zz}^k, \epsilon_{\alpha z}^k, \epsilon_{\beta z}^k, \epsilon_{\alpha\beta}^k\}^T
\end{aligned} \tag{9}
$$

The displacement–strain relations are expressed as

$$
\boldsymbol{\epsilon}^k = \mathbf{b}\mathbf{u}^k \tag{10}
$$

where $\mathbf{b}$ is the matrix of differential operators, see [1] for more information. The constitutive relation for linear elastic orthotropic materials reads as:

$$\boldsymbol{\sigma}^k = \mathbf{C}^k \boldsymbol{\epsilon}^k \tag{11}$$

where $\mathbf{C}^k$ is the material elastic matrix, see Bathe [20] for the explicit form.

The 3D displacement field $\mathbf{u}^k(x,y,z)$ of the 1D beam and 2D plate and $\mathbf{u}^k(\alpha,\beta,z)$ shell models can be expressed as a general expansion of the primary unknowns in Table 1.

Table 1: CUF Formulation. τ=1, …, M.

Formulation	3D Field	CUF Expansion	
1D beam	$\mathbf{u}^k(x,y,z)$	$F_\tau^k(x,z)$	$\mathbf{u}_\tau^k(y)$
2D plate	$\mathbf{u}^k(x,y,z)$	$F_\tau^k(z)$	$\mathbf{u}_\tau^k(x,y)$
2D shell	$\mathbf{u}^k(\alpha,\beta,z)$	$F_\tau^k(z)$	$\mathbf{u}_\tau^k(\alpha,\beta)$

F_τ are the expansion functions of the generalized displacements $\mathbf{u}_\tau^k$ the summing convention with the repeated indexes τ is assumed and M denotes the order of expansion. Thanks to this formalism, it is possible to choose a generic structural theory freely. As explained in the previous sections, Taylor, Lagrange, and Jacobi polynomials can be used. Furthermore, the last two polynomials can be adopted in both ESL and LW approaches.

The Finite Element Method (FEM) is adopted to discretize the generalized displacements $\mathbf{u}_\tau^k$. Thus, recalling equations described in Table 1, they are approximated as displayed in Table 2.

Table 2: Finite element method. i=1, …, N_n

Formulation	3D Field	FEM + CUF Expansion		
1D beam	$\mathbf{u}^k(x,y,z)$	$N_i(y)$	$F_\tau^k(x,z)$	$\mathbf{u}_{\tau i}^k$
2D plate	$\mathbf{u}^k(x,y,z)$	$N_i(x,y)$	$F_\tau^k(z)$	$\mathbf{u}_{\tau i}^k$
2D shell	$\mathbf{u}^k(\alpha,\beta,z)$	$N_i(\alpha,\beta)$	$F_\tau^k(z)$	$\mathbf{u}_{\tau i}^k$

N_i stand for the shape functions, the repeated subscript i indicates summation, N_n is the number of the FE nodes per element and $\mathbf{u}_{\tau i}^k$ are the following vectors of the FE nodal parameters:

$$\mathbf{u}_{\tau i}^k = \left\{u_{x_{\tau i}}^k, u_{y_{\tau i}}^k, u_{z_{\tau i}}^k\right\}^T,\ \mathbf{u}_{\tau i}^k = \left\{u_{\alpha_{\tau i}}^k, u_{\beta_{\tau i}}^k, u_{z_{\tau i}}\right\}^T \tag{12}$$

A further step can be made if the cross-sections functions are anchored to the nodes of beam elements. In this way, the so-called Node-dependent kinematics (NDK) method can be performed. Substantially, each FE node has its own structural theories. Thence, the 3D field is modified as

$$\mathbf{u}^k = N_i F^{ki} \mathbf{u}_{\tau i}^k \tag{13}$$

FE governing equations

The Principle of Virtual Displacements is used for a static analysis, and it reads:

$$\int_{V_k} \delta\boldsymbol{\epsilon}^T \boldsymbol{\sigma} \mathrm{d}V_k = \int_{V_k} \delta\mathbf{u}^{kT} \mathbf{p}^k \mathrm{d}V_k \tag{14}$$

where $\mathbf{p}$ is the external load. When a cartesian frame is used $\mathrm{d}V_k = \mathrm{d}x\mathrm{d}y\mathrm{d}z$, where for a curvilinear reference system $\mathrm{d}V_k = H_\alpha H_\beta \mathrm{d}\alpha\mathrm{d}\beta\mathrm{d}z$. The left-hand side is the variation of the internal work, while the right-hand side is the virtual variation of the external work. The real and the virtual systems are used, and displacements and virtual displacement are written as

$$\mathbf{u}^k(x,y,z) = N_i F_\tau^{ki} \mathbf{u}_{\tau i}^k, \qquad \delta \mathbf{u}^k(x,y,z) = N_j F_s^{kj} \delta \mathbf{u}_{sj}^k \tag{15}$$

By using the CUF-type displacement functions in Eq. (13), the geometric relations in Eq. (10), and constitutive equations Eq. (11), the following expression can be obtained:

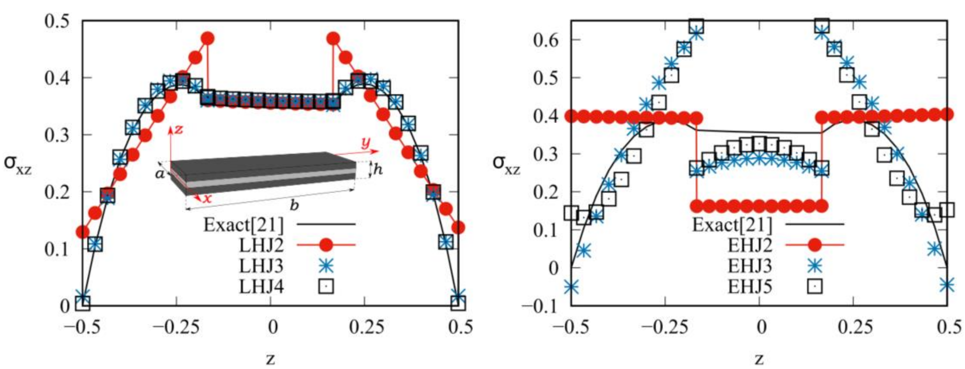
$$\mathbf{K}_{ij\tau s}^k \mathbf{u}_{\tau i}^k = \mathbf{P}_{sj}^k \tag{16}$$

where $\mathbf{K}_{ij\tau s}^k$, a 3X3 matrix, is the fundamental nucleus (FN) of stiffness matrix, and $\mathbf{P}_{sj}^k$, a 3X1 vector, represents the FN of the load vector. See [1] for the explicit form of the components of the stiffness matrix for each formulation and the assembly procedure.

Results

Concerning the structural theories, T*P* indicates Taylor with order *P*, and LHJ*P* indicates layer-wise Jacobi of *P*th polynomial order, whereas EHJ*P* stands for equivalent single-layer Jacobi. For the shape functions only the acronym, J*P* is used.

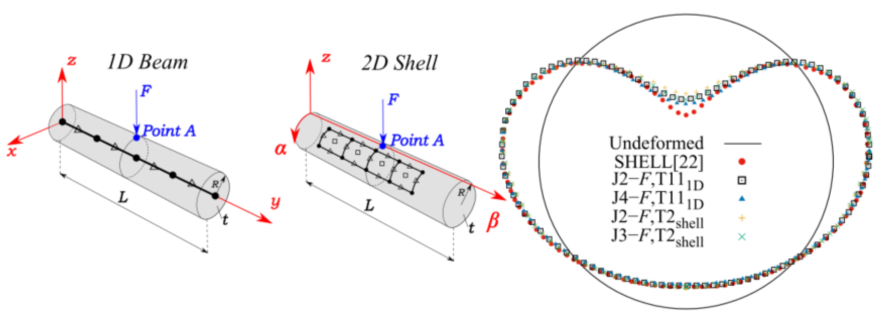
A three-layered composite plate subjected to a sinusoidal pressure (see Fig. 4) with b/h=4 is studied with Jacobi-like polynomials along the thickness, see Pagano [21]. Nine-node Lagrangian shape functions are used for the FE mesh. The CUF based results were presented in [11].



Model	DOF
Exact[21]	—
LJ2	4557
LJ3	6510
LJ5	9765
EJ2	1953
EJ3	2604
EJ5	3906

Figure 4: Shear stresses in *[a/2, 0, z]* of three-layer composite plate for LW and ESL.

As a second example, a thin-walled cylinder is analysed by using beam and shell formulations, see Fig. 5. This case is taken form Carrera et al. [22]. In this case, Jacobi-like shape functions are adopted, while Taylor polynomials are used as structural theories.



Model	DOF
SHELL[22]	49500
J2,T11$_{1D}$	18954
J4,T11$_{1D}$	18954
J2,T2$_{shell}$	22320
J3,T2$_{shell}$	18540

Figure 5: Comparison for beam and shell formulations for thin-walled cylinder. Deformed cross-section at the midspan of the hollow cylinder.

It is shown that the parameters γ and θ of the Jacobi polynomials are not influential for the calculations. Thence, Legendre-like polynomials can be adopted without loss of generality.

Finally, a cantilever beam is considered, see [23, 24]. Fig. 6 shows the axial stresses near the clamped section. NDK models with Legendre-Legendre combination are compared with uniform models. In this case, the following notation is HLE5$^{\times a}$-HLE1$^{\times b}$, where a and b represent the number of nodes of the beam elements adopting the corresponding kinematics. Forty cubic Lagrange-like finite elements are used along y axis.

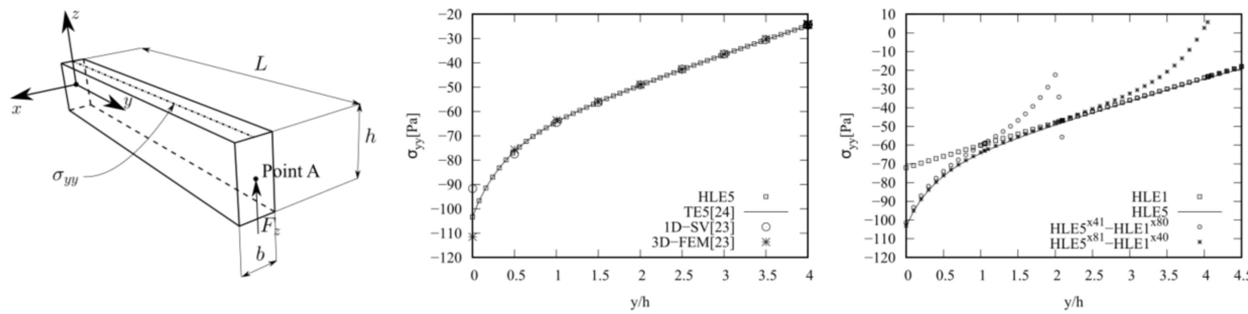


Figure 6: End-effects analysis for compact beam. Stresses evaluated in *[0, y, h/2]*.

In proximity of the clamped section, the results calculated with NDK models are near to those referred to uniform kinematic HLE5 model.

Conclusions

The Carrera Unified Formulation (CUF) permits to build a huge number of models, by adopting different shape functions and structural theories in a hierarchical and coherent manner. In the present work, Jacobi polynomials have been included as shape functions and structural theories in analysis of beam, plates, and shells. It has demonstrated, however, that γ and θ of the Jacobi polynomials are not influential for the calculations. Concerning the structural theories, using the equivalent single layer approach for the Lagrange and Jacobi-based expansions is useful to reduce the computational time. Furthermore, it is possible to use an advanced global-local analysis, that is Node-Dependent Kinematics, which can link different structural theories in the same finite element.

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