A Minimum-propellant Pontryagin-based Nonlinear MPC for Spacecraft Rendezvous in Lunar Orbit

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A Minimum-propellant Pontryagin-based Nonlinear MPC for Spacecraft Rendezvous in Lunar Orbit

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Abstract— We propose a Nonlinear Model Predictive Control approach to spacecraft rendezvous in non-Keplerian Lunar orbits. The approach is based on the Pontryagin Minimum Principle and allows the accomplishment of minimum-propellant maneuvers. The relative motion between the chaser and the target is described by the nonlinear and unstable dynamics of the circular restricted three body-problem. In the proposed formulation, we design a minimum-propellant controller, which leads to a bang-bang behavior of the control signal. Under suitable assumptions, simplified dynamics is employed as prediction model, in order to reduce the complexity of the controller algorithm but, at the same time, without penalizing the controller tracking performance. The proposed approach's effectiveness is validated by a simulation example.

I. INTRODUCTION

In space engineering, the rendezvous and docking (RdV) maneuver is one of the most critical in-orbit operations. The RdV consists in guiding and controlling a spacecraft (SC) (called the chaser) so that it achieves a very close distance to a passive target while possibly (nearly) nullifying the relative velocity between the two (see, e.g. [1], [2]). The RdV maneuver is essential for space exploration since it is required for complex - manned and unmanned - missions.

In the last decades, RdV operations have been catching increasing attention within the space research and industry, in the framework of multi-purpose space servicing vehicles for in-orbit servicing and/or active debris removal. Therefore, the new generation of guidance and control systems must be able to guarantee a high technological standard to autonomously perform complex tasks in space such as trajectory planning, obstacle avoidance, and constraints satisfaction, with high accuracy and robustness with respect to external disturbances and model uncertainties.

Particularly, Moon missions have recently gained interest, as witnessed by the well-known NASA Artemis projects. A key aspect for the success of future missions (e.g. the Gateway mission) is the capability to autonomously perform RdV, which implies the necessity of highly accurate control algorithms to accomplish the last and the most safety-critical phase of the RdV, called close-range rendezvous. As in the classical RdV maneuver, the approach is performed through a

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series of Hold points [3] and, in the last kilometers, the Guidance, Navigation, and Control loop is closed with a controller based on the feedback of the relative state measurements. The relative dynamics formulation in the Circular Restricted Three Body Problem (CR3BP) was previously investigated in the literature by [4] and [5], among others. Many controllers could be used to perform such maneuver: in [6] and [7] a non-linear State Dependent Riccati Controller is used; a stochastic robust linear time-variant MPC is employed in [8] and, in [9], the rendezvous is instead performed exploiting the natural dynamics.

In this context, Nonlinear Model Predictive Control (NMPC) has attracted visible attention among the aerospace academy and industry, being a flexible and reliable control approach in a broad range of practical applications. The success of NMPC is mainly due to its capability to provide optimal control signals for multidimensional nonlinear systems under state, input, and output constraints [10]. For this reason, in recent years, predictive control (both linear and nonlinear) applications to three-body dynamics have gained great attention for multi-fold purposes. Examples include the station keeping [11]–[13], orbital transfers [14], and, indeed, the rendezvous [15] (and the references therein)- [16].

Unfortunately, nowadays, the literature on control strategies for Lunar RdV is limited. The predictive approaches presented in [16] and [8] are confined to linear or linear time-varying cases. On the other hand, the non-linear State Dependent Riccati Controller - still dealing with nonlinearities - does not allow to merge into a single algorithm the trajectory planning and the control. Moreover, as highlighted in [6] and [7], this latter approach presents an unstable pole in the origin. Finally, controllers based on natural dynamics in [9] do not have the flexibility for handling possible trajectory constraints and unknown disturbances. For this reason, NMPC appears as a suitable guidance and control technology for future Lunar missions, being able to overcome the above cited limitations.

In this paper, we propose a minimum-propellant NMPC, based on the Pontryagin optimality principle, for a spacecraft rendezvous in Lunar orbit. In detail, we propose a control synthesis for the last phase of the RdV maneuver, being the unique closed-loop portion of the full RdV baseline, when the relative chaser-target distance is less than $1 \ km$ [6].

In the proposed work, the chaser relative dynamics (i.e., the plant) and the prediction model used for the controller design and validation are slightly different. Indeed, under suitable assumptions, the relative dynamics formulation of the CR3BP [4] is employed as plant, while a simplified

dynamics as prediction model. As result, the proposed controller features less algorithmic complexity, without affecting the precision in tracking the reference. Another interesting feature of the presented approach consists of the NMPC algorithm, based on the Pontryagin Minimum Principle (PMP) [17]. An advantage of this approach is the possibility to obtain a control law which is an explicit representation of the state and the costate, even if the system dynamics and/or constraints are nonlinear. On the other hand, one needs to solve a two-points boundary value problem (TPBVP) in order to find the state and the costate. Having an explicit optimal control in terms of state and covector is useful for further and more advanced NMPC schemes, based on the intrinsic characteristics of the dynamics manifolds.

To sum up, the novel contribution of the paper consists of proposing a novel NMPC strategy for autonomous spacecraft, allowing the accomplishment of challenging rendezvous maneuvers in non-Keplerian Lunar orbits. The key features of the strategy are as follows: i) the definition of a strategy which allows joining into a single algorithm the guidance and control tasks in a highly nonlinear scenario: the SC is able to autonomously plan and perform the required maneuvers with a strongly reduced human effort; ii) the design of a nonlinear minimum-propellant receding horizon control for spacecraft RdV in the context of CR3BP, the proposed techniques are applied to a more complex scenario with respect to the classical RdV ones, without significant modification of the control algorithm design (see, e.g., [18]); iii) the solution of the optimal control problem by means of PMP, so that, the optimal input is an explicit representation of the state and the costate. The advantages of the proposed methodology are i) the PMP-NMPC flexibility in handling complex aerospace dynamic systems; ii) unlike other control approaches (see, e.g., [6], [7]), the PMP-NMPC capability to jointly manage the trajectory planning and control; iii) the tracking performance and a reduced computational effort with respect the classical numerical methods for NMPC optimal control problem solution (see, e.g., [19]). Moreover, the PMP-based explicit optimal control law is useful for further control design that can exploit the stability of the dynamics manifolds.

Note that, an extended version of the paper can be found in [20].

The work is organized as follows. In Section II we present the nonlinear dynamics describing the relative motion between the chaser and the target in a Lunar orbit environment. In Section III we propose the NMPC algorithm whose optimal control problem is solved by means of the PMP. The effectiveness of the proposed methodology is shown in the example of Section IV. Finally, the conclusions are drawn in Section V.

II. SPACECRAFT DYNAMICS IN LUNAR ORBIT AS CR3BP

The restricted three-body equations are a commonly used approximation of the dynamics of a spacecraft subject to the Earth and Moon gravity. This hypothesis implies that the spacecraft motion is regulated solely by the influence
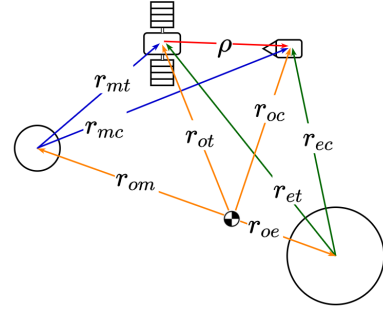

Fig. 1.   Scheme of vector notations for the chaser relative motion.

of two main gravitational masses, called primaries, while the contribution of all the other masses is neglected. As a result, the dynamics of the spacecraft can be expressed by the following, with $N = 2$:

$$\mathbf{F} = m\,\mathbf{a} = -\sum_{i=1}^{N} G\frac{M_i\,m}{||r_i||^2}\bar{\mathbf{r}}_i \qquad (1)$$

where M_i are the masses of primaries, r_i are the distances of the spacecraft from the bodies, $\bar{\mathbf{r}}_i$ is the versors along the direction of the joining between the gravitational bodies and the moving object, and G is the universal gravity constant.

The equations of motion can be rewritten under the following assumption:

Assumption 1: The system is described with respect to a co-rotating reference frame, called synodic. M_1 and M_2 are revolving in a circular motion around the common center of mass. All the quantities are normalized according to the characteristics of the system.

Figure 2 shows the synodic reference frame where the distances of the primaries from the center of mass (O_s) are μ and $1 - \mu$, respectively the normalized position of the two primaries (being μ the system's gravitational parameter, $\mu = M_2/(M_1 + M_2)$) and ω is the angular velocity of the revolving system, which is normalized to 1.

Under these assumptions, the dynamics featuring the CR3BP is described by

$$\begin{cases} \ddot{x} - 2\dot{y} = -\dfrac{\partial U}{\partial x} \\ \ddot{y} + 2\dot{x} = -\dfrac{\partial U}{\partial y} \\ \ddot{z} = -\dfrac{\partial U}{\partial z} \end{cases} \qquad (2)$$

where the effective potential, U, is given by

$$U = -\frac{x^2 + y^2}{2} - \frac{1-\mu}{\|\mathbf{r}_{et}\|_2} - \frac{\mu}{\|\mathbf{r}_{mt}\|_2} - \frac{\mu(1-\mu)}{2} \qquad (3)$$

and $[x, y, z]^T$ are the components of the spacecraft position (henceforth, r_{ot}) in the synodic reference frame, r_{mt} the vector joining the chaser with the Moon's center, and r_{et} is the chaser-Earth's center position vector (see Figure 1). The CR3BP formulation proves the existence of five equilibrium points, named Lagrangian points, numbered from L1 to L5, around which periodic families of solutions exist. Due to
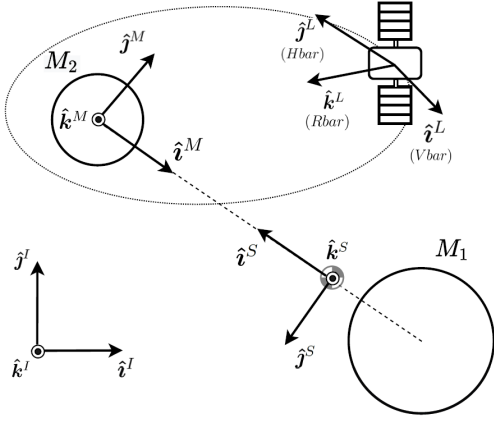
Fig. 2. Representation of the reference frame systems employed in the paper.

their stability and visibility properties, the most plausible to be used in future missions are the so-called L2 Near Rectilinear Halo orbits [21]. Therefore, the presented work is focused on this particular family. For the sake of clarity, two definitions are given here: the closest point of the periodic non-keplerian orbit to the Moon is defined as perilune, while the furthest point is named apolune.

A. Rendezvous maneuver and Relative Dynamics

In the context of the non-keplerian dynamics, the RdV maneuver is not already standardised as in the two-body dynamics, even if some references can be found in [3]. Mainly, the idea is to replicate the same procedure used around Earth for rendezvous and docking: there are two vehicles, one passive (target), which is located on a non-keplerian orbit, and one active (chaser) which completes the RdV maneuver.

We remind that the proposed control synthesis copes with the last phase of the RdV operations, since it is the only portion of the maneuver of the full RdV baseline whose controller is in closed-loop. The final RdV phase takes place when the relative chaser-target distance is less than $1\ km$.

The presented work proposes the synthesis of a nonlinear MPC controller to accomplish RdV maneuver in the proximity of the Moon, therefore a local reference system shall be defined in order to model the dynamics of the relative motion of the target with respect to the chaser. Herein, a definition of the Local-Vertical-Local-Horizon (LVLH) is provided, as well as the definition of the relative motion dynamics $L : \{O_t; \{\hat{i}\}^L, \{\hat{j}\}^L, \{\hat{k}\}^L\}$ or equivalently $L : \{O_t; V_{bar}, H_{bar}, R_{bar}\}$. L is centered in the target. Its basis vectors are defined as:

$$\left\{ \hat{\mathbf{j}}^L \times \hat{\mathbf{k}}^L \quad -\frac{\mathbf{r}_{mt} \times \dot{\mathbf{r}}_{mt}}{\|\mathbf{r}_{mt} \times \dot{\mathbf{r}}_{mt}\|} \quad -\frac{\mathbf{r}_{mt}}{\|\mathbf{r}_{mt}\|} \right\}. \quad (4)$$

Therefore, as introduced in [4] the relative position of the chaser ρ with respect to the target, in the LVLH reference frame, is described by the following nonlinear affine-in-the-input system of differential equations:

$$\ddot{\rho} = -2[\mathbf{\Omega}_{IL}]\dot{\rho} - [\dot{\mathbf{\Omega}}_{IL}]\rho - [\mathbf{\Omega}_{IL}]^2\rho +$$
$$+ \mu\left(\frac{\mathbf{r}_{ot} - \mathbf{r}_{om}}{\|\mathbf{r}_{ot} - \mathbf{r}_{om}\|^3} - \frac{\rho + \mathbf{r}_{ot} - \mathbf{r}_{om}}{\|\rho + \mathbf{r}_{ot} - \mathbf{r}_{om}\|^3}\right) + \quad (5)$$
$$+ (1 - \mu)\left(\frac{\mathbf{r}_{ot} - \mathbf{r}_{oe}}{\|\mathbf{r}_{ot} - \mathbf{r}_{oe}\|^3} - \frac{\rho + \mathbf{r}_{ot} - \mathbf{r}_{oe}}{\|\rho + \mathbf{r}_{ot} - \mathbf{r}_{oe}\|^3}\right) + u$$

where $\mathbf{\Omega}_{IL}$ is the angular velocity of the LVLH frame with respect to an inertial frame, in agreement with the formulation introduced by [4], and u is the control input. Moreover, the operator $[\mathbf{\Omega}_{IL}]$ consists of the skew-symmetric matrix for the vector cross-product. Note that, if not differently specified, all the quantities in (5) are expressed in LVLH frame.

III. NONLINEAR MODEL PREDICTIVE CONTROL SETTING

Henceforth, the relative motion dynamics in (5) is simplified under some suitable assumptions, so that it can be employed as prediction model within the NMPC loop.

Assumption 2: The RdV begins at least 6 hours prior to the passage at the apolune where the dynamics is slower and the influence of the non-linearities is reduced [22]. Moreover, the NMPC prediction interval is a few minutes long.

Given the above assumptions, some terms in (5) can be considered constant along the prediction time window. In particular, $\mathbf{\Omega}_{IL}$ is assumed not to vary along the prediction horizon (and so, dropping its time derivative in (5)), together with $\mathbf{r}_{ot}$, $\mathbf{r}_{om}$, and $\mathbf{r}_{oe}$. This simplifies the overall complexity of the optimal control problem, without reducing the generality and the optimality of the NMPC solution. In few words, we have considered that, along the prediction time window, the spacecraft, the chaser, and the Moon displacements – with respect to Earth – are negligible. To sum up, the dynamics in (5) is employed as the plant, fed with the optimal control law. The simplified dynamics coming from Assumption 2 is used as the prediction model of the system.

By accounting for the relative motion in (5), together with Assumption 2, the chaser-target dynamics is described by a set of nonlinear time-invariant affine-in-the-input differential equation as:

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t), \quad t \geq 0, \quad (6)$$

where $x(t) \in \mathbb{R}^{n_x}$, $u(t) \in \mathbb{R}^{n_u}$ are, respectively, the state (chaser relative position and velocity), and the input (thrust acceleration) vectors at time t. Moreover, $f \in \mathcal{C}^1(\mathbb{R}^{n_x} \to \mathbb{R}^{n_x}$ and $g \in \mathcal{C}^1(\mathbb{R}^{n_x} \to \mathbb{R}^{n_x \times n_u})$.

At each k-th time instant, the state is measured in real-time - with a sampling time T_S - and a prediction $\hat{x}$ of the system state is performed over a finite time interval $[t_k, t_k + T_p]$, where T_p is the prediction horizon. The prediction is obtained by integration of (6). Based on the state measurement, we compute the control input $u(t)$ along the prediction window. To this end, fixed a prediction horizon $T_p \geq T_s$ and solve

the Bolza-type optimization problem:

$$u^* = \arg\min J\big(x(t), u(t)\big)$$

subject to:

$$\dot{\hat{x}}(\tau) = f(\hat{x}(\tau)) + g(\hat{x}(\tau))\hat{u}(\tau), \quad \hat{x}(t_k) = x(t_k)$$
$$\hat{x}(\tau) \in \mathcal{X} \subset \mathbb{R}^{n_x}, \; \hat{u}(\tau) \in \mathcal{U} \subset \mathbb{R}^{n_u}, \quad \forall \tau \in [t_k, t_k + T_p],$$
$$\hat{u}(\cdot) \in \mathcal{PC}\big([t_k, t_k + T_p]\big).$$
(7)

$\mathcal{X}$ and $\mathcal{U}$ are set describing possible constraints on the state and input, respectively, while $\mathcal{PC}\big([t_k, t_k + T_p]\big)$ is the space of piece-wise continuous functions. Upon finding the optimal solution $(\hat{x}^*, \hat{u}^*)$, the control input $u(t) \in [t_k, t_{k+1}]$ is defined as the constant value

$$u(t) \equiv \hat{u}^*(t_k) \quad \forall t \in [t_k, t_{k+1}).$$
(8)

This operation is performed at each sampling instant t_k, where $k = 0, 1, \ldots$. Note that, as in the usual scenario for the MPC setting, the receding horizon strategy is employed.

In the framework of the space RdV, a particular kind of cost function - which promotes the minimum propellant expenditure - is employed. As already presented in [18], in the following, we minimize the $\mathcal{L}_1$ norm of the input, since it effectively leads to a minimum-propellant controller [23]. As a consequence, the resulting input signal will be bang-bang in time. Associating to each solution $\hat{x}$ the corresponding tracking error $\tilde{x}(\tau) \doteq \hat{x} - x_r$ (where x_r is the reference signal), the following functional is introduced:

$$J = \int_{t_k}^{t_k+T_p} \big(\tilde{x}^T(\tau)\mathbf{Q}\tilde{x}(\tau) + \|\mathbf{R}\hat{u}(\tau)\|_2\big) \, d\tau \\ + \tilde{x}^T(t_k + T_p)\mathbf{P}\tilde{x}(t_k + T_p)$$
(9)

where $\mathbf{Q}, \mathbf{P} \geq 0 \in \mathbb{R}^{n_x \times n_x}$ and $\mathbf{R} > 0 \in \mathbb{R}^{n_u \times n_u}$ are suitable diagonal matrices.

For the application at hand, the admissible control set is defined as $\mathcal{U} = \{u \in \mathbb{R}^{n_u} : \|u\|_2 \leq u_{max}\}$ where u_{max} is the maximum thrust acceleration deliverable by the engines. Unlike the input, in this preliminary work, the state is considered unconstrained, then, we assume that $\mathcal{X} \equiv \mathbb{R}^{n_x}$. Note that, the integration of state constraints can be carried out by following the methodology described in [18] and [19]. Note also that, some brief notions about control algorithm stability and convergence can be found in [20].

A. Pontryagin-based NMPC Solution

To solve the optimal control problem (7), we employ the well-established Pontryagin principle [17]. The necessary condition for optimality requires the introduction of the Hamiltonian

$$H(x, u, \lambda) = \tilde{x}^T\mathbf{Q}\tilde{x} + \|\mathbf{R}u\|_2 + \lambda^T[f(x) + g(x)u] \in \mathbb{R}. \quad (10)$$

where $\lambda \in \mathbb{R}^{n_x}$ is the vector of costates (or covector).

Denoting, for brevity, $t^f \doteq t_k + T_p$, the necessary conditions of optimality in (7) are as follows [24]. If $(\hat{x}^*, \hat{u}^*)$ is an optimal solution, then a costate function $\lambda^* : [t_k, t^f] \to \mathbb{R}^{n_x}$ exists such that

$$\dot{\lambda}^*(t) = -\nabla_x H(\hat{x}^*(t), \hat{u}^*(t), \lambda^*(t)), \quad (11)$$
$$\lambda^*(t^f) = \nabla_x \tilde{x}^T(t^f)\mathbf{P}\tilde{x}(t^f), \quad (12)$$
$$H(\hat{x}^*(t), \hat{u}^*(t), \lambda^*(t)) = \min_{u \in U_C} H(\hat{x}^*(t), u, \lambda^*(t)) \quad (13)$$
$$\forall t \in [t_k, t^f], \quad t^f \doteq t_k + T_p. \quad (14)$$

Adding constraints (7) to this system, one obtains a two-point boundary value problem (TPBVP) with $2n$ scalar differential equations for the vector-function $(\hat{x}^*(t), \lambda^*(t)) \in \mathbb{R}^{2n}$, n scalar boundary conditions at time t_k (namely, $\hat{x}^*(t_k) = x_k$) and n boundary conditions at time t^f.

B. Rendezvous Optimal Control Problem

In order to present the Pontryagin-based solution for the RdV dynamics, let λ_r and λ_v be defined as the co-vectors associated with the chaser relative position ρ and the relative velocity $\dot{\rho}$, respectively. Hence, by considering (10), together with cost function (9), the Hamiltonian for the RdV optimal control problem is

$$H_{RdV} = \tilde{x}^T\mathbf{Q}\tilde{x} + \|\mathbf{R}u\|_2 + \lambda_r^T\dot{\rho} + \lambda_v^T\ddot{\rho}. \quad (15)$$

We recall now the notion of primer vector, denoted by p, introduced by [25]. The velocity covector represents the engines optimal fire direction so that $p \doteq -\lambda_v$. Hence, the thrust unit verctor is $\bar{u} = p/P$, being $P = \|p\|_2 = -\lambda_v^T\bar{u}$ the primer vector magnitude. Hence, the Hamiltonian must be minimized over the choice of the thrust magnitude $\Gamma \doteq \|u\|_2$.

We refer to [20] for all the mathematical manipulations and we highlight that Γ appears linearly in the final formulation of the Hamiltonian. Thus, the optimal control problem solution would lead to an input signal with an infinite magnitude. Nevertheless, if the admissible input set is bounded, the minimization of the Hamiltonian will depend only on the algebraic sign of the Γ coefficient, which, in aerospace literature is defined as switching function $\Upsilon = P - \|\mathbf{R}\|_2$. The sign of Υ defines the policy for the engines power on/off and the thrust is allowed to assume only the maximum or zero value. The direction of the thrust is driven by $p = -\lambda_v$. Therefore, the optimal control policy is:

$$u^* = \begin{cases} \Gamma_{max}\dfrac{p}{\|\lambda_v\|_2} & \text{if } \Upsilon > 0, \\ 0 & \text{if } \Upsilon \leq 0. \end{cases} \quad (16)$$

Remark 1: Note that, when Υ vanishes the problem of singular control must be tackled. In these situations, the optimal control must be searched to find an explicit expression of u by nullifying the time derivatives of $\nabla_u H$, until u appears (see, e.g. [26]). As consequence, the optimal control does not lie anymore on the boundary of $\mathcal{U}$ but it can assume any value inside the set. Nevertheless, for the applications dealt with in this work, it is important to stress that the bang-bang control problem is also driven by the necessity to cope with some technological limitations of the actuators whose output must have a switch on/off behaviour. For this reason, in this

peculiar application, a suitable - but sub-optimal - choice to deal with the singular control is to set the corresponding value to zero.

We conclude the PMP-based NMPC solution by presenting the so-called Euler-Lagrange equations which describe the time variation of the costate as in (11):

$$\dot{\lambda} = \begin{bmatrix} \dot{\lambda}_r \\ \dot{\lambda}_v \end{bmatrix} - 2\mathbf{Q}\tilde{x} \qquad (17)$$

where

$$\dot{\lambda}_r = \left[\left[\dot{\mathbf{\Omega}}_{IL} \right]^T \left[\dot{\mathbf{\Omega}}_{IL} \right]^T + \sum_{i=1}^{2} \frac{\mathbf{M}_i}{\|\rho + \mathbf{r}_{ot} - \mathbf{r}_{oi}\|^3} \cdot \right. \qquad (18a)$$
$$\left. \cdot \left(\mathbf{I} - 3 \frac{(\rho + \mathbf{r}_{ot} - \mathbf{r}_{oi})^T (\rho + \mathbf{r}_{ot} - \mathbf{r}_{oi})}{\|\rho + \mathbf{r}_{ot} - \mathbf{r}_{oi}\|^2} \right) \right] \lambda_v$$

$$\dot{\lambda}_v = -\lambda_r + 2 \left[\mathbf{\Omega}_{IL} \right]^T \lambda_v. \qquad (18b)$$

where $\mathbf{I}$ is the identity matrix, $\mathbf{M} = \{\mu, 1 - \mu\}$, and $r_{oi} = \{\mathbf{r}_{om}, \mathbf{r}_{oe}\}$. Note that, referring to (18a) and (18b), we remind that the time variation of the costate is evaluated in the time window $[t_k, t_k + T_p]$. Then, according to Assumption 2, $\mathbf{\Omega}_{IL}$, $\mathbf{r}_{ot}$, $\mathbf{r}_{oe}$, and $\mathbf{r}_{om}$ are constant.

IV. SIMULATION ANALYSIS

The proposed control technique is then applied in a specific test case: for realistic purposes, the target orbit was selected to be the same as the Lunar Gateway (mission Artemis IV) [21], using as numerical values for the simulation of the environment: $\mu = 0.01215$, Earth-Moon distance $L = 384400.0 km$ and the Earth-Moon synodic revolution period $T = 2360591.424s$. The rendezvous maneuver is accomplished at least 6 hours before reaching the aposelene to mitigate the effect of the non-linear dynamics, in particular, the focus is posed on the close RdV, therefore the chaser shall be located on a hold point (quasi-zero relative velocity) at $-5 \ km$ along V-bar. The time to accomplish the full mission shall be less than 4h. For the application at hand, the initial conditions are set as $\rho_0 = [-5, 0.1, 0.1]^T \ [km]$ and $\dot{\rho}_0 = [2e-5, 2e-5, 2e-5]^T \ [km/s]$ while the reference is a constant zero vector both for position and velocity, except for the components ρ_x set at $-5 \ m$, in agreement with [7]. Concerning the NMPC parameters, we have that $T_s = 2 \ s$ and $T_p = 45 \cdot T_s$, while, the cost function matrices are $\mathbf{R} = \mathbf{I}_{3 \times 3}$ (whereas $\mathbf{I}$ is the identity matrix), $\mathbf{P} = \text{diag}(8.05e10, 8.05e10, 8.05e10, 1, 1, 1)$, and $\mathbf{Q} = \text{diag}(5e14, 5e14, 5e14, 9e7, 9e7, 9e7)$, chosen by a trial and error procedure. Finally, the thrust acceleration provided by the engines is allowed to vary within the set $\mathcal{U} = \{u : \|u\|_2 \leq 0.02 \ m/s^2\}$ which approximately corresponds to a maximum thrust force of $10 \ N$. We are now in position to present the outcomes of the simulations. The orbital simulator and the NMPC algorithm are implemented in the Matlab/Simulink environment. The optimal control problem, formulated as a TPBVP, is solved by means of *bvp5c* Matlab function.

Concerning the tracking performance of the controller, Figure 3 shows how both the chaser position and velocity

components own excellent convergence properties. Indeed, whereas the final tracking error on x-axis is below $1 \ m$, the displacements on y/z-axis are in the order of a few millimetres. Finally, the residual velocity components are all below the threshold of cm/s. Note that, small oscillations of the chaser nearby the reference can make the two SC collide. In order to avoid collision a keep-out-zone can be included within the NMPC optimization problem, with the same methodology proposed in [18]. The implementation of such constraint is a topic of on-going work.
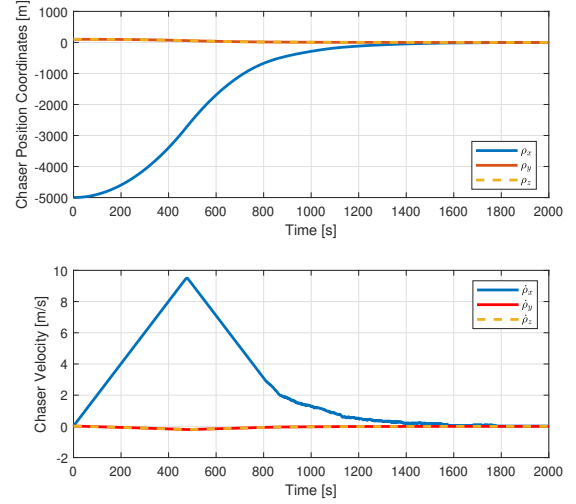

Fig. 3. Time evolution of the chaser position/velocity components. The components along the z-axis are dashed in order to visualize the overlapping with the y-axis ones.

By evaluating the input time history, it is useful to stress the following point. According to (16), the bang-bang behaviour of the thrust activity is evident in the last subplot of Figure 4. Indeed, we remind bang behaviour is meant to refer to the magnitude acceleration and not component-wise. For this reason, the single components of the thrust can assume any value within the input set, being the input constraint always satisfied. Moreover, even though the thrust acceleration evolution presents a high-frequency behaviour, the issue can be mitigated through a proper control dispatch during the SC engines configuration design. To conclude, the overall impulse $I_u = \int_{t_0}^{t_F} \|u\|_2 dt$ delivered by the engines is $I_u = 28 \ m/s^2 \cdot s$. This latter result is comparable with the results obtained with SDRE - State Dependent Riccati Equations - controller formulated in [7].

Note that, the graphical representation of time evolution of the switching function along the chaser approaching maneuver can be found in [20].

V. CONCLUSION

The problem of obtaining a Pontryagin-based control law for the Nonlinear Model Predictive Control applied to a Lunar rendezvous in Halo orbit has been addressed. The optimal control law is an explicit function of both the state and costate, which time evolution is described by the so-called
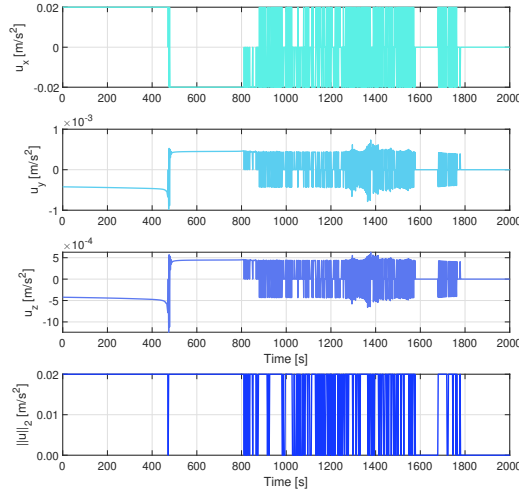
Fig. 4. Thrust delivered by the chaser engines along the trajectory in terms of components and total magnitude. The bang-bang behaviour is evident in the last subplot where $\|u\|_2$ is plotted. The engines can assume only an on-off policy.

Euler-Lagrange equations. For control design purposes, we used different dynamics for the prediction model with respect to the plant one (described as the chaser/target relative motion in the Circular Restricted Three Body Problem). In detail, given a short prediction horizon, some time-varying parameters of the system's dynamics can be kept constant along the prediction time window. We employed a cost function which takes into account the $\mathcal{L}_1$-norm of the input, leading to a minimum-propellant optimal control law. As a consequence, the input signal is bang-bang in time. The results of the simulations confirmed the effectiveness of the NMPC algorithm presented in the paper. Finally, the integration of nonlinear/non-convex state constraints and the implementation of a robust version of the controller are topics of on-going research.

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