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Efficient Solution of Multi-Scale Problems with Localized Mesh Refinement Schemes and Huygens' Surfaces

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Abstract—**This paper presents a framework composed by Domain Decomposition Method (DDM) that integrates a local mesh refinement scheme based on Huygens' equivalence principle and the integration of multibranch basis functions for the efficient analysis of multiscale problems. The proposed method allows the improvement of the precision of the solution while the solving time is reduced via the decomposition of the problem.**

I. INTRODUCTION

When dealing with complex real-life problems, computational electromagnetic analysis becomes a challenge. These structures [1] combine large regions with a small level of details and areas with tiny details, like antennas, that need dense meshes to properly address the elements' behavior. This gives place to multiscale problems, requiring heavy computational requirements when using widespread methods, such as the surface-integral equation method of moments (SIE-MoM).

Mesh refinement methods [2,3] constitute an attractive approach for this kind of problem. First, the problem is solved using an initial mesh, called coarse mesh. Once the initial solution is obtained, a process to detect the error over the body analyzed is performed, and those regions above a certain threshold are refined. The problem is solved again after getting the optimized mesh, obtaining an improved solution.

Despite its suitability for multiscale problems, the overall cost of this process is not efficient enough compared to other methods. In response, this paper proposes a framework that combines Domain Decomposition Methods (DDM) [4] that applies Huygens' equivalence principle to reduce the global cost of the mesh refinement process.

II. PROPOSED FRAMEWORK

The following framework is proposed to perform an efficient approach to this kind of problem. First, as in other existing mesh refinement approaches, the problem is solved using an initial coarse mesh. In this stage, DDM is applied along with Huygens' surfaces enclosing the domains to improve global performance. This is because part of the matrix system posed is also used in the refinement and final solving stages. These surfaces act as a proxy between the encapsulated domain and the rest of the geometry, isolating it from the part of the problem that is external to the HS and allowing us to tackle the local problem without re-computing the couplings of the rest of the problem with it when the mesh is refined.

Once the initial solution is obtained, the accuracy of the solution is evaluated locally, and mesh refinement is performed to obtain the adapted optimal mesh. The resulting adapted mesh integrates multibranch basis [5] functions to improve the transition from coarse mesh to fine mesh in those parts that require denser mesh.

At last, the solution of each domain is computed with the scheme represented in Fig. 1, expressed in terms of the local self-coupling matrix of the refined mesh (in grey), considering the global coupling with other domains only once on the right-hand side. This is done through a straightforward way for the part of the body intersected by the Huygens Surface (in red) and through the coupling matrix of the outer part of the body with the Huygens Surface (in blue) and the coupling of the Huygens Surface over the local domain (represented by the gradient arrow in blue/grey color). In addition to this, the excitation is also added to address the problem correctly.
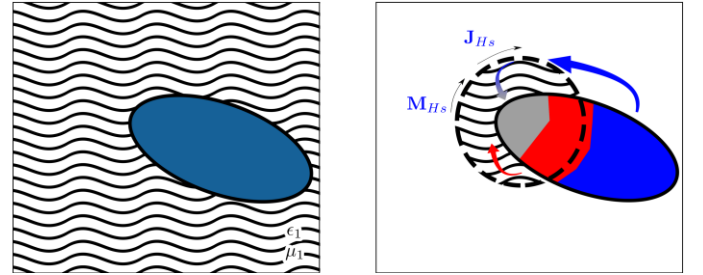

Figure 1. Example of an entire body in an environment with electromagnetic fields (left) and the isolation of one of its subdomains using Huygens' equivalence principle to describe the external fields (right).

III. NUMERICAL RESULTS

As a preliminary result of this framework's application, we present a PEC cube of 5λ of side divided into eight identical domains. The excitation of the problem is an obliquely impinging plane wave with a wavelength equal to 1. Figure 2 shows the initial (left) and adapted (right) meshes.
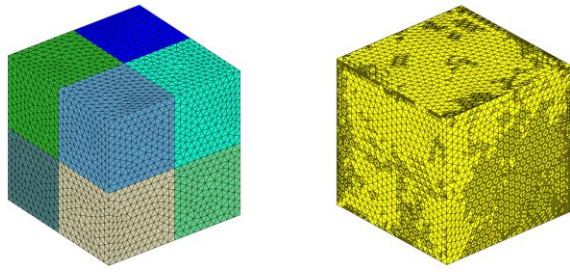
Figure 2. 5λ cube divided into eight subdomains with initial mesh (left) and its adapted mesh with local refinement scheme (right).

The superficial electric currents obtained for the adapted mesh are shown in Fig. 3. The figure reports the solution obtained for the proposed scheme using Huygens' surfaces to perform isolation (left) and the same adapted mesh solved globally using MLFMA (right). Both cases show good agreement, indicating that the introduction of isolation with this method has a limited impact on the accuracy.
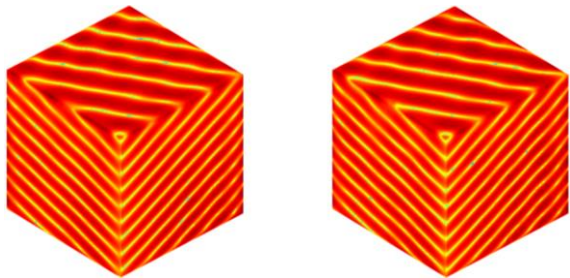

Figure 3. Electric currents were obtained for the adapted mesh using the local scheme with Huygens' surfaces (left) and computing the global solution with MLFMA (right).

The convergence times for this problem can be seen in Fig. 4. In this figure, we represent the solving times for different approaches: parallel solution of the refined local domains with adapted mesh using Huygens' surface isolation (first blue-dotted line), sequential solution of the refined local domains with adapted mesh using Huygens' surface isolation (second blue-dotted line), global solution of the adapted mesh with MLFMA (red-dotted line) and global solution of the fully refined mesh with MFLMA (solid black line, not fully represented).
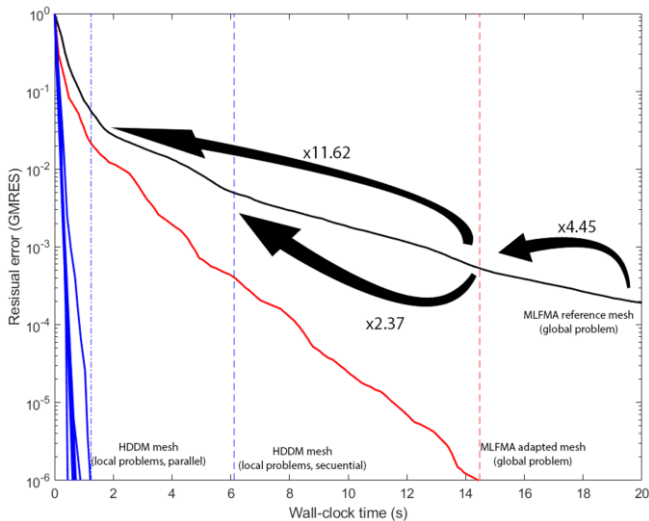

Figure 4. Total convergence times of the example with different approaches.

It can be appreciated that globally analyzing the adapted mesh, which provides an acceptable accuracy, provides an speed-up higher than 4 in comparison to the fully refined mesh. At the same time, the application in parallel of the isolation through the application of the Huygens' surfaces to perform the analysis locally provides a speed-up higher than 11 in comparison to the global analysis of the adapted mesh. The combination of both the local analysis with Huygens' surfaces and the adapted mesh achieves a total speed-up higher than 50 in comparison to the fully refined mesh analyzed globally.

IV.   CONCLUSION

The proposed h-refinement method provides adapted meshes that allow accurate solutions with high time-saving. The integration of the Huygens' equivalence principle into the proposed framework offers additional time saving, maintaining the accuracy of the solution, allowing a local solution of the problem. Finally, the integration of multibranch basis functions allows improving the transition between coarse and more refined meshes limiting the increment needed in the number of basis.

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