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Abstract. We analyze the finite-temperature scaling of the Lorenz ratio at the topological Kondo fixed point realized at a junction of three interacting quantum wires connected to a floating superconducting island. Using the Tomonaga-Luttinger liquid approach to the quantum wires, we derive the full functional dependence of the finite-temperature correction on the Luttinger parameter g.

1. Introduction and conclusions

Whenever transport through an electronic system can be described in terms of quasiparticles, which carry both energy and charge, the ratio between thermal and electric conductance is linear in the temperature, which is known as the Wiedemann-Franz law (WFL). The proportionality constant takes an universal value L_0, the so called Lorenz number, given by $L_0 = \frac{\pi^2 k_B^2}{3e^2} \approx 2.44 \times 10^{-8} W\Omega K^{-2}$ [1].

While the WFL is generally well-verified in three-dimensional metals and semiconductors at room temperature, pairing correlations, impurities and electron-phonon scattering can lead to important deviations from the predicted Lorenz ratio [2, 3]. In mesoscopic systems, it has been reported that the interplay of dissipation and approximate conservation laws, as well as selective transport phenomena also lead to the breakdown of this relation [4, 5, 6]. Finally, a most notorious violation takes place in one-dimensional Luttinger Liquids, in which the collective excitations carry a noninteger electric charge and the Lorenz ratio is therefore renormalized by the inverse Luttinger parameter [7]. As argued above, in order for the WFL to apply, charge and heat must be carried by the same excitations, hence, it is eventually not surprising that the WFL still holds in systems such as the overscreened Kondo model [8], which are known to exhibit non-Fermi liquid correlations at low temperatures/energies [9].

Systems exhibiting phases in which charge and heat transport are “disentangled” from each other have been identified as strongly coupled boundary fixed points (SFPs) in junctions of interacting effectively one-dimensional systems, either fermionic [10, 11, 12, 13], or bosonic [14, 15, 16, 17]. Nevertheless, in order to make an SFP stable at low temperatures/energies, a strong attractive bulk interaction in each wire is required. It is worth noting that probing charge and heat transport through the junction typically requires connecting the wires to large, noninteracting Fermi liquid like reservoirs, which ultimately neutralizes the effects of bulk interactions in the Lorenz ratio [18, 19].

A striking possibility of recovering a stable SFP with an expected robust, and universal, violation of the WFL in a weakly interacting, or even noninteracting (in the bulk) junction of quantum wires, has been recently put forward in [20, 21]. Specifically, it has been shown that the WFL breaks down in a robust and predictable way in a junction of $N = 3$ spinless quantum wires connected to a superconducting island in its charging regime hosting a set of Majorana modes (MMs). At low temperatures, such a system enters a non-trivial phase, associated to the topological Kondo fixed point (TKFP) [22, 23]. The stability of the TKFP is indeed due to the localized MMs, which eventually turn the residual boundary interaction at the SFP irrelevant [24, 25]. As a result, the predicted violation of the WFL at the TKFP can be regarded as a mean to detect the so far pretty elusive emerging MMs. Yet, real measurements are performed at finite temperature. This requires the knowledge of the leading, finite-T corrections to the conductances near by the TKFP. In order to compute them, we employ the formalism developed in [20, 21] and as a result, beside recovering the main temperature dependence, we provide the explicit corrections of the conductances at the TKFP on the bulk interaction in the leads.

The paper is organized as follows:

- In Section 2 we present our model Hamiltonian and review the results for the charge and the heat transport at the TKFP.
- In Section 3 we compute the finite-T leading corrections to the conductances and to the Lorenz ratio close to the TKFP.

2. Model Hamiltonian and charge and heat transport at the strongly coupled fixed point

Our system consists of three interacting fermionic quantum wires connected to a floating superconducting island, hosting a set of MMs as the only subgap degrees of freedom. The island is characterized by the charging energy E_c and by the applied gate voltage V_g. To model the wires, we use the spinless Luttinger liquid Hamiltonian

$$H_0 = \sum_{a=1}^{3} \frac{u}{2} \int dx \, \left\{ g(\partial_x \phi_a(x))^2 + g^{-1}(\partial_x \theta_a(x))^2 \right\} \quad , \tag{1}$$

with $\{\phi_a\}$ being the bosonic Luttinger liquid plasmon fields and $\{\theta_a\}$ being their dual fields. The parameters u and g respectively denote the plasmon velocity and the Luttinger parameter of the leads, $g > 1(< 1)$ corresponding to an attractive (repulsive) bulk interaction. The ends of the quantum wires are proximitized to the MMs with tunneling strength t. Calling ρ_0 the density of states around the Fermi energy, we focus on the limit $E_c \gg T, t^2\rho_0$ (Coulomb blockade) and tune V_g in such a way that the total charge at the island is integer. One can then sum over the leading cotunneling processes through the junction and recover an effective, purely bosonic tunneling Hamiltonian H_K

$$H_K = -2J_K \sum_{a=1}^{3} \cos\left[\sqrt{\pi}(\phi_a(0) - \phi_{a+1}(0))\right] \;\; , \; (\phi_{a+3} \equiv \phi_a, J_K \sim t^2/E_c) \quad . \tag{2}$$

The total Hamiltonian $H = H_0 + H_K$ describes a purely bosonic junction [16, 17] in which, for $g \leq \frac{3}{4}$ the stable phase corresponds to the “disconnected” fixed point (DFP) while, for $g \geq 1$, it is described by a strongly coupled fixed point, the TKFP. Finally, for $\frac{3}{4} < g < 1$, both the DFP and the TKFP are stable, so that there is a finite-coupling, repulsive fixed point, encoding the quantum phase transition between the two stable phases [22, 23]. To compute the fixed point charge and the heat conductances, we resort to the splitting matrix approach put forward in [26]. The splitting matrix ρ is defined so that, denoting respectively with $\phi_{R,a}(x)$ and with $\phi_{L,a}(x)$ the right- and the left-handed components of the field $\phi_a(x)$, one has $\phi_{R,a}(0) = \sum_{b=1}^{3} \rho_{a,b}\phi_{L,b}(0)$. Given ρ and letting $G_{a,b}$ and $K_{a,b}$ respectively being the charge and the heat conductance tensor elements at the junction, one finds [20, 21]

$$G_{a,b} = \frac{e^2 g}{2\pi}\left\{\rho_{a,b} - \delta_{a,b}\right\}, \quad K_{a,b} = \frac{\pi k_B^2 T}{6}\{\rho_{a,b}^2 - \delta_{a,b}\} \quad . \tag{3}$$

At the DFP one has $\rho_{a,b} = \delta_{a,b}$, which implies $G_{a,b} = K_{a,b} = 0\ \forall a, b$. At variance, at the TKFP one finds $\rho_{a,b} = \frac{2}{3} - \delta_{a,b}$, which implies $G_{a,b} = \frac{e^2 g}{\pi}\left\{\frac{1}{3} - \delta_{a,b}\right\}$ and $K_{a,b} = \frac{2\pi k_B^2 T}{9}\left\{\frac{1}{3} - \delta_{a,b}\right\}$. As a result, taking the (Lorenz) ratio $\mathcal{L}$ between the heat and the charge conductance tensor matrix elements with the same indices, we find $\mathcal{L} = \frac{K_{a,b}}{TG_{a,b}} = \frac{2}{3g}L_0$. We therefore identify a "bulk" renormalization of L_0 by the nonuniversal factor g^{-1}, which is typical of a the WFL in a spinless Luttinger liquid [27, 28]. As stated above, such an effect disappears once the junction is connected to Fermi liquid reservoirs. In addition, we see the universal (that is, independent of the bulk interaction) factor $\frac{2}{3}$. This is purely determined by the junction dynamics and, in particular, by the onset of multi-particle scattering processes at the TKFP. Probing this renormalization should provide a remarkable piece of evidence of the presence of localized MMs at the junction, which are deemed to be the main mechanism stabilizing the TKFP [20, 21, 17].

Typical measurements are performed at low, yet finite, temperature. For this reason, it is important to derive how the TKFP result for $\mathcal{L}$ is modified at finite T, which is the topic we address next.

3. Finite-T corrections to the Lorenz ratio close to the strongly coupled fixed point

As a reference calculation, we briefly review the derivation of the finite-T corrections at the DFP in a perturbative expansion in J_K [20, 21]. At the DFP, the boundary interaction in Eq.(2) takes a scaling dimension g^{-1}. Therefore, the corrections will exhibit a nontrivial scaling behavior, as a function of the running scale T. Specifically, one finds

$$G_{a,b}(T) = (3\delta_{a,b} - 1)\frac{2\pi e^2 \Gamma^2(1/g)}{\Gamma(2/g)}\tilde{J}_K^2(T)\ ,\ K_{a,b}(T) = T\frac{\Gamma(2/g)}{\pi g \Gamma^4(1/g)}\Phi(g^{-1}, g^{-1})L_0 G_{a,b} \quad , \tag{4}$$

with $\Gamma(z)$ being Euler Γ function and the running coupling $\tilde{J}_K(T) = J_K(2\pi k_B T)^{-1+g^{-1}}$, while the function $\Phi(x, y)$ is

$$\Phi(x, y) = \int dzdw\, \frac{w}{\sinh(\pi z)}|\Gamma(y + iw)|^2|\Gamma(x + i(z - w))|^2 \quad . \tag{5}$$

In the non-interacting case $g = 1$, one finds no violation of the WFL even at finite-T.

The TKFP is accessed in the strongly coupled limit $J_K \to \infty$ in which, in order to minimize H_K, one has to "pin" the field operators entering H_K. To do so, we first note that, once resorting to the center-of-mass and to the relative-field basis, $X = \frac{1}{\sqrt{3}}\sum_{a=1}^{3}\phi_a$, $\varphi_1 = \frac{1}{\sqrt{2}}\{\phi_1 - \phi_2\}$, $\varphi_2 = \frac{1}{\sqrt{6}}\{\phi_1 + \phi_2 - 2\phi_3\}$, H_K only depends on $\varphi_1(0)$ and $\varphi_2(0)$. Thus, in order to minimize H_K, one has to pin $\varphi_1(0), \varphi_2(0)$. Accordingly, the leading boundary operator (LBO) at the TKFP must necessarely depend only on the relative dual fields, $\vartheta_1(0) = \frac{1}{\sqrt{2}}\{\theta_1(0) - \theta_2(0)\}$ and on $\vartheta_2(0) = \frac{1}{\sqrt{6}}\{\theta_1(0) + \theta_2(0) - 2\theta_3(0)\}$ [11, 16]. Specifically, the LBO $\tilde{H}_K$ is a linear combination of boundary operators describing instanton "jumps" between the minima of H_K [29, 30]

$$\tilde{H}_K = -2h\cos\left[\frac{4\sqrt{2\pi}}{3}\vartheta_2(0)\right] - 2h\sum_{\alpha=\pm1}\cos\left[\frac{4\sqrt{2\pi}}{3}\left(-\frac{\vartheta_2(0)}{2} + \alpha\frac{\sqrt{3}\vartheta_1(0)}{2}\right)\right] \quad , \tag{6}$$

h being the corresponding coupling strength. $\tilde{H}_K$ has scaling dimension $\tilde{d}_K = \frac{4g}{3}$, which implies the stability of the TKFP as soon as $g > \frac{3}{4}$. Applying the analysis of [21] to the perturbation Eq.(6), we obtain

$$G_{a,b}(T) = \frac{e^2 g}{\pi}\left\{1 - \mathcal{A}_G(g)\tilde{h}^2\right\}\left(\frac{1}{3} - \delta_{a,b}\right) \quad , \quad K_{a,b}(T) = \frac{2\pi k_B^2 T}{9}\left\{1 - \mathcal{A}_K(g)\tilde{h}^2\right\}\left(\frac{1}{3} - \delta_{a,b}\right) \quad , \tag{7}$$

with $\mathcal{A}_G(g) = 8\pi^2 g\Gamma^2\left[\frac{4g}{3}\right] / \left(3\Gamma\left[\frac{8g}{3}\right]\right)$, $\mathcal{A}_K(g) = 16\pi g\left\{5\frac{\Phi\left(\frac{2g}{9},\frac{10g}{9}\right)}{\Gamma\left[\frac{4g}{9}\right]\Gamma\left[\frac{20g}{9}\right]} + \frac{\Phi\left(\frac{8g}{9},\frac{4g}{9}\right)}{\Gamma\left[\frac{8g}{9}\right]\Gamma\left[\frac{4g}{9}\right]}\right\}$ and the running coupling $\tilde{h} = \tilde{h}(2\pi k_B T) = h(2\pi k_B T)^{-1+\frac{3g}{4}}$. Eq.(7) is the ultimate result of this paper. When considering the Lorenz ratio at finite T, $\mathcal{L}(T)$, from Eqs.(7) we obtain

$$\mathcal{L}(T) = \frac{2}{3g}L_0\left\{1 - \mathcal{A}_{\mathcal{L}}(g)\tilde{h}^2(2\pi k_B T)\right\} \quad , \tag{8}$$

with $\mathcal{A}_{\mathcal{L}}(g) = \mathcal{A}_K(g) - \mathcal{A}_G(g)$. We conclude that, in proximity of the TKFP, $\mathcal{L}(T)$ takes a nontrivial dependence on T even in the absence of a bulk interaction in the leads.

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