Risk-averse Approaches for a Two-Stage Assembly-to-Order Problem

(Article begins on next page)

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Edoardo Fadda, Daniele Giovanni Gioia and Paolo Brandimarte

Abstract Assembly to order is a production strategy where components are manufactured under demand uncertainty and end items are assembled only after demand is realized. Risk-neutral approaches aim to maximize the expected profit. However, this approach may fail if heavy-tailed or multi-modal distributions are likely to generate significant disruptions or if the shrinking life of products is considered. Conversely, risk-averse models may tackle these problems. In the paper, we deal with an assembly-to-order problem, modeled as a two-stage stochastic linear programming problem considering the introduction of a classical risk measure from finance: the conditional value-at-risk. We examine the characteristics and the performance of the model by means of a large number of out-of-sample scenarios.

1 Introduction and Paper Positioning

Demand uncertainty is one of the main difficulties concerning the production planning problems. However, a wide array of buffering tools have been devised to ease the difficulty of demand forecasting. One example is delayed product differentiation, which aims at exploiting risk pooling by postponing product differentiation as late as possible along the supply chain (a well-known success story for this approach is the HP DeskJet case). This strategy is exploited in Assembly-to-Order (ATO) man-

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ufacturing environments and it can be naturally cast as a two-stage stochastic linear program with recourse [3], and possibly generalized to multiple stages. The standard models usually maximize expected profits. Nevertheless, it may not be enough to get a robust solution, as there could be high variability in profits across the different scenarios. This is a risk-neutral approach, whereas most decision-makers are risk-averse. In principle, risk aversion could be modeled by a concave utility function, but eliciting it is not very practical. So, one possibility is to modify the problem formulation in order to make it somewhat more robust. In this paper, the choice that we explore is the optimization of Conditional Value at Risk (CVaR). CVaR is defined as the conditional expected loss under the condition that it exceeds the Value-at-Risk (i.e., a given quantile) and has several desirable properties [1]. Moreover, it has been shown that the solution with (in-sample) optimal CVaR at a given level can be found by solving a linear programming model [5]. While in finance there is a huge number of applications considering this risk measure, concerning the ATO problem the authors are not aware of any risk-adverse application [2]. Thus, the aim of this paper is to fill this gap and start exploring the ATO problem by considering risk measures. Specifically, since considering the CVaR instead of the expected value increase the computational burden, we investigate the problem in terms of the number of scenarios to achieve stability. Furthermore, we test the solutions in order to assess its performance by means of several out-of-samples scenarios. The paper is organized as follows. In Section 2 we describe the mathematical models for the ATO problem. In Section 3 we present instance generation and in Section 4 we show the results of the computational experiment. Finally, in Section 5 we draw the conclusion of our work and we suggest the possible future lines of research.

2 Mathematical Models

In this section, we present the mathematical models that we are going to consider in the following. Let $\mathcal{I} = \{1,\dots,I\}$ be the set of components, $\mathcal{J} = \{1,\dots,J\}$ the set of end items, $\mathcal{M} = \{1,\dots,M\}$ the set of production resources (e.g., machines) and $\mathcal{S} = \{1,\dots,S\}$ the set of scenarios used to discretize the demand of the end items. The probability of each scenario is π^s. Moreover, let us define the following parameters:

- C_i cost of component $i \in \mathcal{I}$ and P_j price of the end item $j \in \mathcal{J}$.
- L_m production availability of machine $m \in \mathcal{M}$.
- T_{im} time required to produce $i \in \mathcal{I}$ in $m \in \mathcal{M}$.
- G_{ij} amount of components $i \in \mathcal{I}$ required for assembling end item $j \in \mathcal{J}$, commonly known as *gozinto factors*.
- d_j^s demand for end item $j \in \mathcal{J}$ in scenario $s \in \mathcal{S}$, sampled from a distribution D_j.

The decision variable of the model are x_i and y_j^s, the amount of component $i \in \mathcal{I}$ produced and the amount of end items $j \in \mathcal{J}$ assembled, respectively. The resulting sampling average approximation model formulation is:

$$\max_{x\in\mathbb{R}^I, y\in\mathbb{R}^J} \quad - \sum_{i\in\mathcal{I}} C_i x_i + \sum_{s\in\mathcal{S}} \pi^s \left(\sum_{j\in\mathcal{J}} P_j y_j^s \right) \tag{1}$$

$$\text{s.t.} \quad \sum_{i\in\mathcal{I}} T_{im} x_i \le L_m \qquad \forall m \in \mathcal{M} \tag{2}$$

$$y_j^s \le d_j^s \qquad \forall j \in \mathcal{J}\ \forall s \in \mathcal{S} \tag{3}$$

$$\sum_{j\in\mathcal{J}} G_{ij} y_j^s \le x_i \qquad \forall i \in \mathcal{I}\ \forall s \in \mathcal{S} \tag{4}$$

$$y_j^s, x_i \ge 0 \qquad \forall i \in \mathcal{I}\ \forall j \in \mathcal{J}\ \forall s \in \mathcal{S} \tag{5}$$

The objective function of the problem is the expected net profit, expressed in (1) as expected revenue at the second stage minus cost at the first stage. Constraints (2) limit the production resources; constraints (3) state that it is not possible to sell more than the demand, and constraints (4) preclude assembling items for which we lack the necessary components, thereby linking the two decision stages. Constraints (5) are non-negativity conditions since, as in [3], we experiment with a continuous linear program and not an integer one. We do so to ease the computational burden, without affecting the conclusions significantly since we consider settings characterized by a high number of produced components and end items demand. In the following we will call Model (1)-(5) recourse problem (RP).

Model (1)-(5) considers a risk neutral decision-maker which aims at maximizing the expected profit but does not care about the negative tails of the profit in some scenarios. Instead, these are taken into account if risk measures are considered. In the following, we consider the CVaR, which we optimize by minimizing an appropriate auxiliary function [5]. Specifically, it can be proved that the α percent CVaR of the negative profit can be expressed as:

$$CVaR_\alpha[\sum_{i\in\mathcal{I}} C_i x_i - \sum_{j\in\mathcal{J}} P_j y_j^s] = \min_{\zeta} \zeta + \frac{1}{1-\alpha} \sum_{s=1}^{S} \pi_s [\sum_{i\in\mathcal{I}} C_i x_i - \sum_{j\in\mathcal{J}} P_j y_j^s - \zeta]^+, \tag{6}$$

where $[\cdot]^+ = \max[\cdot, 0]$. The usage of the CVaR in the model leads to two alternative formulations: minimizing the CVaR but providing a minimum expected profit or maximizing expected profit and bounding the CVaR. We do not focus on this second formulation since it can be proved to be equivalent to the first one in terms of efficient frontier. Specifically, both models can be obtained by applying the ε-constraint method to the multi-objective model considering the maximization of expected profit and the minimization of the CVaR [4]. Thus, the considered model is:

$$\min_{x \in \mathbb{R}^I, y \in \mathbb{R}^J} \quad \zeta + \frac{1}{1-\alpha} \sum_{s=1}^{S} \pi_s z^s \tag{7}$$

$$\text{s.t.} \quad -\sum_{i \in \mathcal{I}} C_i x_i + \sum_{s \in \mathcal{S}} \pi^s \left(\sum_{j \in \mathcal{J}} P_j y_j^s \right) \geq \Psi \tag{8}$$

$$z^s \geq 0 \qquad \forall s \in \mathcal{S} \tag{9}$$

$$z^s \geq \sum_{i \in \mathcal{I}} C_i x_i - \sum_{j \in \mathcal{J}} P_j y_j^s - \zeta \qquad \forall s \in \mathcal{S} \tag{10}$$

$$(2), (3), (4), (5)$$

$$\zeta \in \mathbb{R}, \quad z^s \geq 0 \qquad \forall s \in \mathcal{S}$$

where Eq. (7) minimizes the CVaR and Contraint (8) enforces a minimum profit denoted by Ψ. Furthermore, constraints (9) and (10), are employed to linearize the CVaR expression in Eq. (6) by means of the z^s variables.

3 Instance Generation

In this section, we describe the generation procedure of the instances considered in the computational experiments. Due to space limitations, we focus on the gozinto matrix and on the demand generation procedure while we refer to [3] for the generation of the other parameters.

The gozinto matrix is randomly generated by setting a number of families and, for each of them a number of common and specific components. In Figure 1 we illustrate the two-dimensional heat map representation of the structure of the gozinto matrix that will be used in the experiments. Each color is associated with the number of components for each end item. We consider 35 end items and 60 components. The matrix looks block-diagonal, where blocks correspond to families. We call K the number of families, J_k the set of items belonging to the family k, and n_k the cardinality of J_k. The first columns of each block are the common components, while the others are the specific ones. At the bottom of the matrix, there are a few items that we call outcast items. Each one of them defines a family.

The only risk factor considered in the models is items' demand. It is generated with a process composed by two nested steps. Firstly, we sample the demand F_k for each family $k \in \{1, \dots, K\}$. We assume a bimodal normal distribution with expected values $n_k\mu_1, n_k\mu_2$, standard deviations $\sqrt{n_k}\sigma_1, \sqrt{n_k}\sigma_2$ and mixing parameter p. Namely,

$$F_k \sim \mathcal{N}(n_k\mu_1, n_k\mu_2, \sqrt{n_k}\sigma_1, \sqrt{n_k}\sigma_2, p), \qquad \forall k \in \{1, \dots, K\}. \tag{11}$$

We assume a multimodal demand distribution inspired by several real-world situations. Consider, for example, the demand of a company with a few large customers

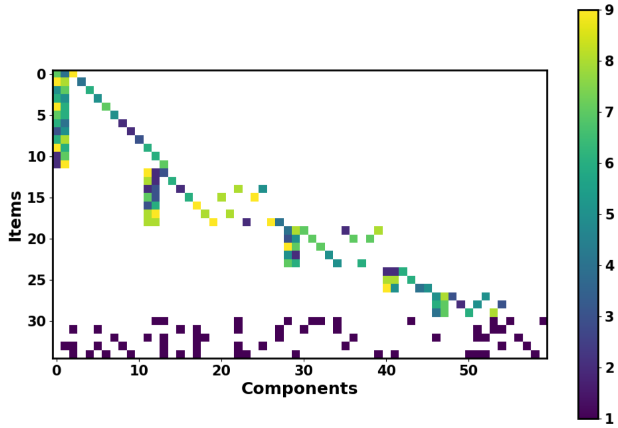

Fig. 1 Structure of gozinto matrix

with irregular bulk orders or the demand for new products, which can become either a shelf warmer or a top seller (e.g., fashion clothes).

In the second step, the overall demand for each family is divided into the demand for every single item. In the case of families composed of one item (as for the outcast items), the demand for the unique end item is equal to the demand for the family. Instead, if more items are present, the total demand is split and d_j is defined as:

$$d_j \sim w_{j,k} F_k, \qquad \forall j \in J_k, \quad \forall k \in \{1, \dots, K\}, \tag{12}$$

The weights $w_{j,k}$ are randomly sampled from a *Dirichlet distribution* with different parameters ζ_k for each family k, i.e.

$$w_{1,k}, \dots, w_{n_k,k} \sim \mathrm{Dirch}(\zeta_k), \qquad \forall k \in \{1, \dots, K\}. \tag{13}$$

This choice ensures that $\sum_{j \in J_k} w_{j,k} = 1, w_{j,k} > 0, \forall j \in J_k, k \in \{1, \dots, K\}$. A graphical representation of the demand generation is illustrated in Figure 2.

4 Computational Experiments

In this section, we study the properties of model (7)-(10). First, we consider in-sample and out-of-sample stability, then we study its properties.

While for model (1)-(5) stability has already been studied [3], in this section we consider in-sample and the out-of-sample stability for model (7)-(10). Due to space limitations, we cannot describe a full-fledged experimental design. Thus, for the

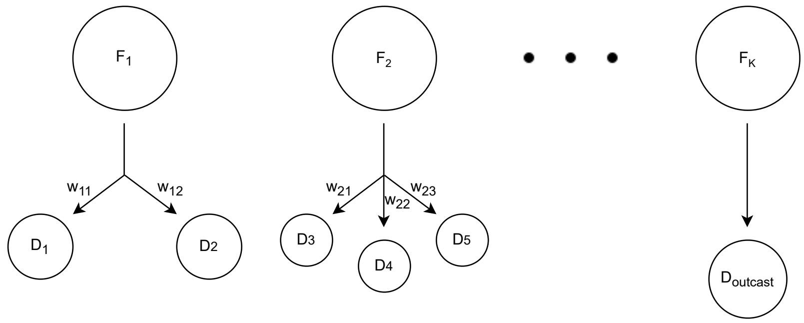

Fig. 2 Demand sampling schema. The first two demand for families 1 and 2 are divided. The last demand is related to a degenerative family.

experiments studying stability, we fix the Ψ parameter to be equal to the 80% of the in-sample expected profit obtained by solving model (1)-(5) with 2048 scenarios (this choice ensures the stability of the RP).

Concerning in-sample stability, in Figure 3, we report for $S = 16,\ 32, 64, 128, 256, 512, 1024$, and 2048 the average objective functions over 50 runs. The upper and lower points of the error bar represent the 0.05 and the 0.95 quantiles of the values collected, respectively. To increase the graph readability we present on the x-axis the logarithm of the number of scenarios.

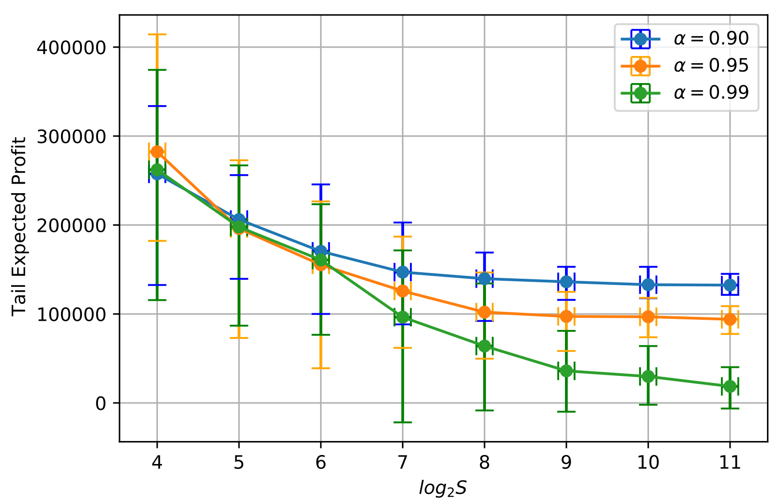

Fig. 3 Objective functions for different values of α

The size of the error bars decreases as more scenarios are considered and it increases as α increases since estimating the expected value of a smaller portion of the distribution requires more scenarios than the estimation of a greater one. For example, with $S = 100$ and $\alpha = 0.99$ just one scenario is considered for the CVaR while for $\alpha = 0.90$ the scenarios considered are 10. As the reader can notice, the average objective function value is decreasing from the left to the right. This is due to the fact that the greater is S, the more conservative is the solution since more bad scenarios are considered by the instance.

Furthermore, while for $\alpha = 0.90$, and $\alpha = 0.95$ the optimal values are always greater than zero, sometimes for the $\alpha = 0.99$ they have negative values meaning that in some cases, in order to maintain the given expected profit, the minimization of the in-sample *CVaR* for $\alpha = 0.99$ is at a loss.

The CPU time used to solve these problem instances on an *Intel(R) Core(TM)* i7-5500U CPU@2.40GHz computer with 16GB of RAM, running *Ubuntu* v20.04 and using Gurobi v9.1.1 as solver, goes from a few seconds $S = 32$ up to around 10 minutes $S = 2048$ and a few runs with $S = 4096$ required up to 40 minutes to be solved. Luckily, stability is achieved before that number of scenarios. Thus, the increment of the computational burden required to solve model (7)-(10) does not justify the development of ad-hoc heuristics for the considered setting. Nevertheless, we have considered small cardinality of the set $\mathcal{I}$ and $\mathcal{J}$ (i.e., 60 components and 35 items, respectively) hence the development of ad-hoc heuristics may be required for instances of greater dimension.

Concerning the out-sample stability, we consider the ratio:

$$\rho_S = \frac{CVaR_{out} - CVaR_{in}^S}{|CVaR_{in}^S|} \tag{14}$$

where $CVaR_{in}^S$ is the value of the objective function of model (7)-(10) computed with S scenarios (i.e., the in-sample CVaR) and $CVaR_{out}$ is the CVaR of the optimal solution computed out-of-sample. Since $CVaR_{out}$ is computed with more scenarios ($S = 10000$) than $CVaR_{in}^S$, we expect that to be greater (the objective function (7) is minimizing loss). Thus, in Eq.(14) we consider the absolute value just in the denominator since in some instances the lowest possible CVaR can be negative (i.e. it can be a profit). The average results over 50 runs are shown in Table 1.

	ρ_{16}	ρ_{32}	ρ_{64}	ρ_{128}	ρ_{256}	ρ_{512}	ρ_{1024}	ρ_{2048}
$\alpha = 0.90$	98 (50)	54 (20)	32 (12)	19 (7)	10 (5)	5 (3)	3(2)	2(1)
$\alpha = 0.95$	118(38)	79(24)	49(13)	31(11)	18(9)	10(6)	5(4)	4(3)
$\alpha = 0.99$	169(51)	107(31)	99(24)	82(12)	62(15)	44(14)	26(11)	18(10)

Table 1 Average and standard deviation (in brackets) of ρ_S for different values of α and S.

The value ρ_S increases as α increases while decreases as the number of considered scenarios increases. If we consider an out-of-sample stability of less than the

5%, we need to consider 512 scenarios for $\alpha = 0.90$, 2048 scenarios for $\alpha = 0.95$ and more than 2048 scenarios for $\alpha = 0.95$.

By considering both the results of Table (1) and Figure 3, in the following we consider $\alpha = 0.95$ and $S = 2048$. We decide to consider $\alpha = 0.95$ since it is a good trade-off for the production setting which does not require accounting for extreme conditions as in finance. In fact, in the ATO problem common components hedge against uncertainty. Moreover, for $\alpha = 0.95$, $S = 2048$ ensures both in-sample and out-of-sample stability.

By recalling that model (7)-(10) derives from a multi-objective problem, we can generate the in-sample efficient frontier by considering different minimum expected profits Ψ. In Figure 4 we show the results. On the y-axis, we represent the in-sample tail expected profit (i.e. the opposite of the optimal objective function) while, on the x-axis we report the minimum in-sample expected profit.

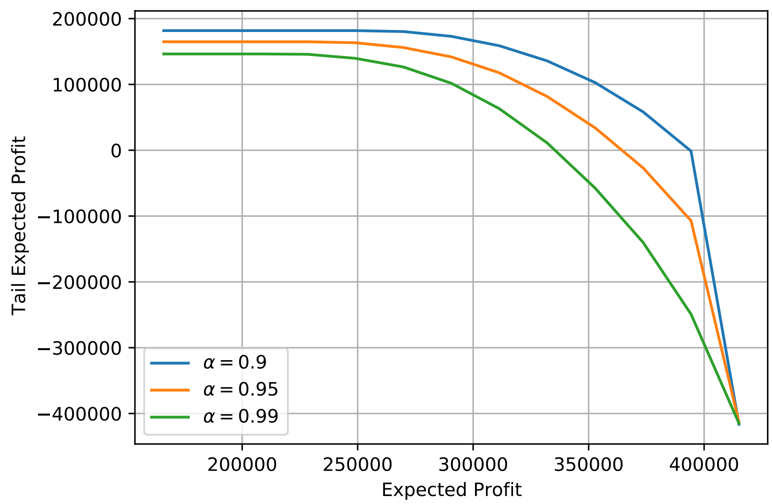

Fig. 4 In-sample efficient frontier for different values of α

Unfortunately, since the instance generation procedure does not use real data, the value on the axis are purely indicative. Nevertheless, they provide us with some interesting insight. First, for value of Ψ greater than the optimal value of the RP, model (7)-(10) becomes infeasible. Moreover, on the left-hand side of the graph, all the fronts are horizontals (defining a plateau) since the optimal solution is the same for different values of Ψ. Thus, for these values of Ψ constraint (8) is no more active. Moreover, it is interesting to notice that the length of the plateau increases as α decreases. In fact, for $\alpha = 0.9$ the plateau ends for an expected profit of near 275000, while for $\alpha = 0.9$ the plateau ends for a value of expected profit lower than 250000. Finally, the efficient front decreases for greater α, since more extreme quantiles are considered.

For the sake of completeness, we study the out-of-sample performance of the model. As noticed above, the instances have been randomly generated thus the optimal value of the objective function does not have any real meaning. Thus, we consider the following performance indicator:

$$EVPI_{\%} = \frac{P_{WS} - P_{CVaR}}{P_{WS}} \tag{15}$$

where P_{WS} and P_{CVaR} are the out-of-sample profit obtained by the wait and see problem and the one obtained by the model (7)-(10), respectively. Thus, $EVPI_{\%}$ is a measure of the expected value of perfect information in a given scenario. It is important to notice that if $EVPI_{\%} \geq 1$, then the profit is negative. The boxplots of the $EVPI_{\%}$ for several values of Ψ are shown in Figure 5. Each boxplot is computed on 10000 out-of-sample scenarios.

As the reader can notice, the minimum value of perfect information is around 0.5, meaning that in the best case, if we had known the real demand, the profit would have been double.

The leftmost boxplot represents the performance of the RP and in several scenarios, the profits are really low. Rather unsurprisingly, the smaller Ψ, the lower is the CVaR. Nevertheless, also the expected profit gets lower since as Ψ decreases the solution tends to be more conservative. Comparing the solutions of the problems for different Ψ, we can notice that the total quantity of components decreases, and the common components are preferred since they can be used to hedge against risk. While this strategy is effective in a two-stage setting (as in the fashion field), it may be less effective in the multistage one, since the overproduction in one stage can be used in the next ones. Thus accurate tests with multiple stages will be considered in future work.

As noticed above, there exists a value of Ψ such that the optimal solution is not yet influenced by constraint (8). For the considered setting ($\alpha = 0.95$, $S = 2000$) this value is $\Psi = 0.60P_{RP}$. It is worth noting that the worst-case profit of the solution obtained for $\Psi \leq 0.65P_{RP}$ is lower than one, therefore in all the 10000 out-of-sample scenarios the solution leads to a profit. Nevertheless, the expected profit from implementing these solutions is around 60% less than the one achieved by the RP solutions. Thus, a reasonable choice for Ψ in the practical field may range between $0.9P_{RP}$ and $0.8P_{RP}$.

5 Conclusions and Future Work

In this paper, we have discussed a simple model considering the minimization of CVaR for the two-stage production planning in an ATO environment. Clearly, the results from synthetic instances must be taken with great care, but it is clear that risk-neutral models can lead to huge losses in several scenarios.

The future lines of research will follow two directions. First, we will take into account that the knowledge of the exact distribution of the uncertain parameters af-

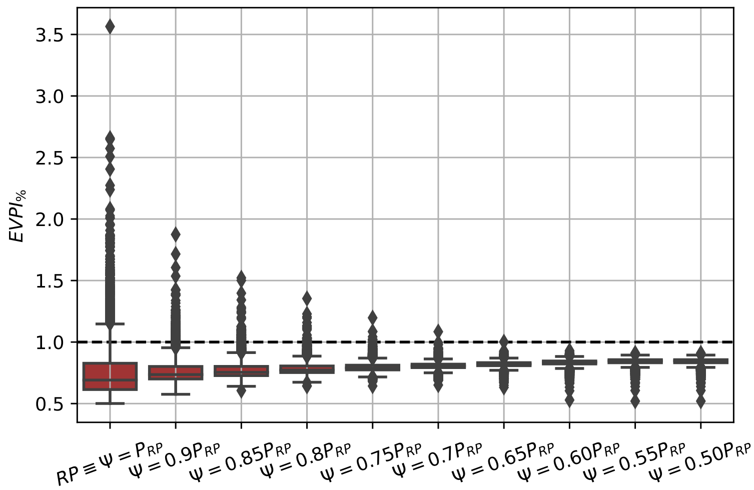

Fig. 5 Boxplot representing the $EVPI_{\%}$ on 10000 out-of-sample scenario. Below the horizontal dashed line the profit are positive, above are negative.

fecting a stochastic optimization problem must not be taken for granted. Thus, we will apply ad-hoc methodologies considering distribution ambiguity [6]. Second, we will analyse the multistage problem with both risk neutral and risk-averse models. In such a way it will be possible to better quantify the profitability of the two approaches in a wider and more realistic set of applications.

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