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In the absence of kinetic energy production, the influence of the initial conditions on turbulent transport can be characterized by the presence of an energy gradient or by the concurrency of an energy and a macroscale gradient. In this work, we present a similarity analysis that interpret a new result on the subject recently obtained by means of numerical experiments on shearless mixing (Tordella & Iovieno, 2005, a–b). In short, the absence of a macroscale gradient is not a sufficient condition for the setting of the asymptotic Gaussian state hypothesized by Veeravalli and Warhaft (1989), where, regardless of the existence of velocity variance distributions, turbulent transport is mainly diffusive and the intermittency is nearly zero up to moments of order four. In fact, we observed that the intermittency increases with the energy gradient, with a scaling exponent of about 0.29. The similarity analysis, which is in fair agreement with the previous experiments, is based on the use of the kinetic energy equation, which contains information concerning the third order moments of the velocity fluctuations. The analysis is based on two simplifying hypotheses: first, that the decays of the turbulences being mixed are almost nearly equal (as suggested by the experiments), second, that the pressure-velocity correlation is almost proportional to the convective transport associated to the fluctuations (Yoshizawa, 1982, 2002).

1 Numerical results on the energy mixing. Second and higher order velocity moments

A few aspects of the interaction of different decaying homogeneous and isotropic turbulences in absence of mean shear are described in the laboratory experiments by Gilbert (1980) and Veeravalli and Warhaft (1989). In this particular flow configuration the two turbulences external to the mixing have the same mean velocity but different turbulent kinetic energy and have

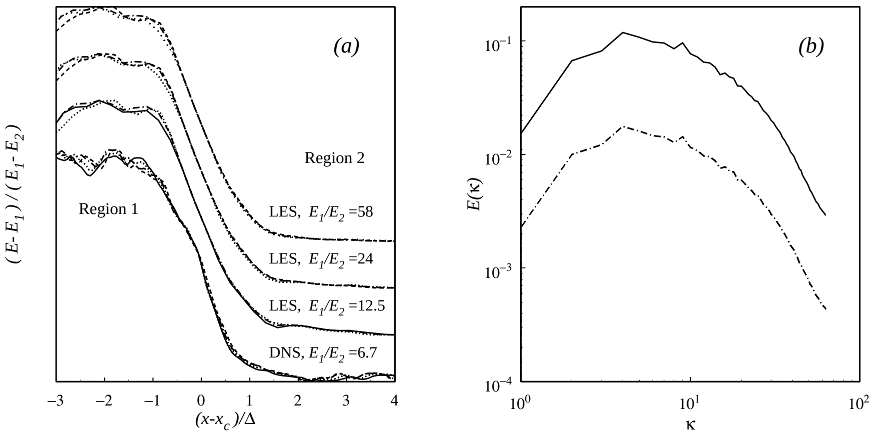

Fig. 1. (a) Normalized kinetic energy during decay: x_c is the mixing centre, the position where normalized energy is equal to $1/2$, and $\varDelta$ is the mixing layer thickness, suffices $1, 2$ correspond to high and low turbulent kinetic energies respectively. (b) Initial three-dimensional energy spectra for the DNS simulation. Continuous line: high energy region; dashed line: low energy region.

been experimentally investigated by means of turbulence generated by grids with different size, but with the same solidity. This produces two homogeneous turbulences with the same mean velocity but with different energy and scale.

In this paper, we consider the influence on turbulent transport of initial conditions characterized by the presence of an energy gradient and the absence of a macroscale gradient, see fig.1(a)-(b) and fig.2. The simulations were carried out by means of either direct numerical or large eddy simulations. The direct numerical simulations here presented were carried out by means of a new technique for the parallel dealised pseudospectral integration of the Navier-Stokes equations (Iovieno et al., 2001). The boundary conditions are periodic in all directions. Two computational domains, a $(2\pi)^3$ cube with 128^3 points, and a $4\pi(2\pi)^2$ parallelepiped with 256×128^2 points, were used to obtain an estimate of the numerical accurary. In the initial condition, the two turbulence fields are matched by using the Briggs et al.(1996) technique.

The same numerical method was used to implement the large eddy simulations, which were carried out by using the Intrinsic Angular Momentum (IAM) subgrid scale model (Iovieno & Tordella, 2002). This model is based on the proportionality of the turbulent diffusivity to the intrinsic moment of momentum of the finite element of a fluid. The IAM correctly scales the eddy diffusivity ν_δ, with respect to both the filtering length and the dissipation rate, and introduces a differential equation – the intrinsic angular momentum equation – to follow the evolution of ν_δ. This is particularly advantageous in the case of nonequilibrium turbulence fields, since it adds a degree of freedom to the subgrid modelling.

It is recalled that the basic definition of a longitudinal integral scale that permits a direct measure and does not depend on the flow global Reynolds number is

$$\ell(t) = \frac{1}{3} \sum_i \frac{\int_0^\infty R_{ii}(r,t)dr}{R_{ii}(0,t)}, \qquad (1)$$

where R_{ii} is the longitudinal velocity correlation, see Batchelor (1953). The integral length approximation deduced from the hypothesis of statistical equilibrium, i.e. $\ell = E^{3/2}/\epsilon$, should be applied with caution whenever the Re does not allow the great divergence of scales to be obtained that the universal equilibrium theory requires. In fact, at the relatively low Reynolds numbers, typical of the current literature,

$$\ell_\epsilon / E^{3/2} = f(Re). \qquad (2)$$

Function f is of order 1, but is not yet completely known. Simulations of homogeneous and isotropic turbulence in the periodic box and laboratory experience, see Batchelor and Towsend (1948) and the collection of experimental data in Sreenivasan (1998), show that, in the low Re number range, its value almost halfs when the Re quadruple. In this paper, we use definition (1), principally because it is not affected by the actual value of Re and because it evidences that the integral scale does not depend on the level of kinetic energy but on the spectral distribution of energy over the wave numbers. Furthermore, definition (1) implies that turbulences which have similar spectra, but a different overall kinetic energy, see fig.1 b, have the same spatial macroscale.

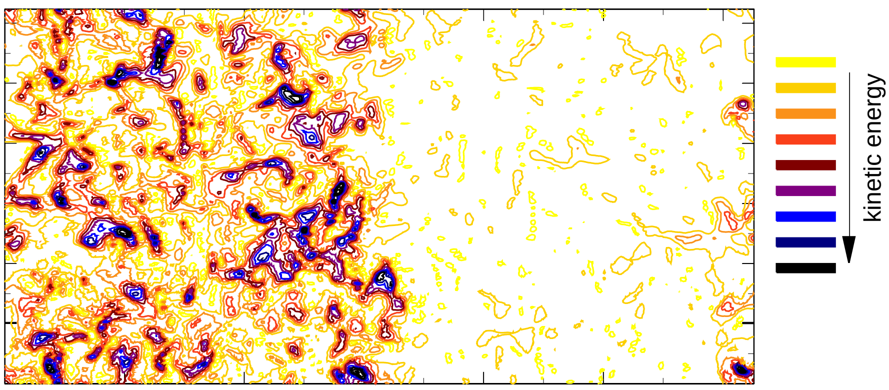

Fig. 2. Kinetic energy contours. Plots at $3 < t/\tau < 4$.

The dependence of the turbulence mixings with a macroscale ratio $\mathcal{L} = \frac{\ell_2}{\ell_1} = 1$ on the initial conditions has been considered and documented through single-point statistics (Tordella & Iovieno, 2005, Iovieno & Tordella, 2002

– here and in the following subscript 1 and 2 refer to the high/low energy regions, respectively). It is seen that the statistical distributions of orders higher than the second highly depend on the initial values of the ratio of energy, $\mathcal{E} = E_1/E_2$. If the energy ratio is far from unity, no Gaussian behaviour is observed up to order four. The asymptotic state for the shearless turbulence, where the velocity variance follows the form of an error function and the velocity fluctuations are Gaussian, which was attributed by Veeravalli and Warhaft(1989) to the $\mathcal{L} \approx 1$ type of mixing and, in particular, to Gilbert's experiment (where, because of the very low energy gradient exploited, it was very difficult to show the weak, eventual removing of the velocity statistics from the Gaussian behaviour) was not observed. On the contrary, the mixing is very intermittent. If the lateral penetration is considered in terms of the position of the maximum of skewness and kurtosis distributions, it is observed, that, when $\mathcal{L} = 1$, the intermittency increases with the energy ratio with a scaling exponent that is almost equal to 0.29, see fig.s 3(a,b). Independently of the values of the control parameters a set of common properties exists for all the studied mixings. First, the statistical distributions become self-similar after nearly a decay of three time units. Second, in the self-similarity region of the decay, the lateral spreading rate is on average close to 0.15. Third, the kinetic energy distribution has a common shape (see, (13) below). Fourth, all the mixings are very intermittent, as the skewnes S and kurtosis K distributions show, see fig.s 3(a) and 3(b).

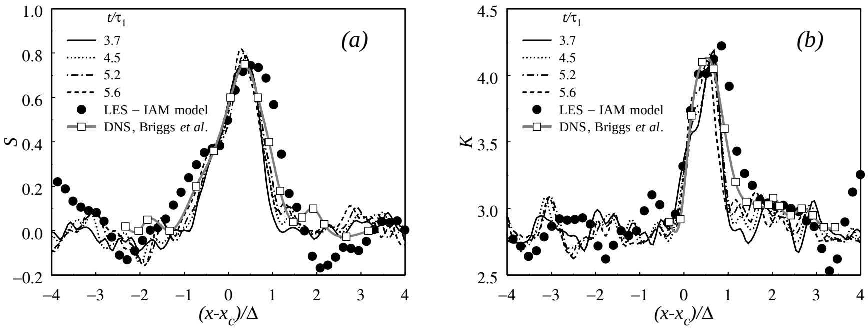

Fig. 3. Skewness (a) and kurtosis (b) of the velocity component in the inhomogeneous direction x ($\mathcal{E} = 6.7$, $\mathcal{L} = 1.0$): x_c is the mixing centre, Δ the mixing layer thickness and $\tau_1 = \ell(0)/E_1(0)^{\frac{1}{2}}$. Data from [3] and [6] are also shown.

2 Similarity analysis

We analyze here the consequences of the observation that in all the numerical mixing experiments a self-similar state appears to exist. In the time interval where the near–similarity is reached ($t/\tau > 3$), to carry out the similarity analysis, we considered the second moment equations for the velocity fluctuations (u, in the inhomogeneous direction x, v_1, v_2 in the plane normal to x),

$$\partial_t \overline{u^2} + \partial_x \overline{u^3} = -2\rho^{-1}\partial_x \overline{pu} + 2\rho^{-1}\overline{p\partial_x u} - 2\varepsilon_u + \nu\partial_x^2 \overline{u^2} \quad (3)$$
$$\partial_t \overline{v_1^2} + \partial_x \overline{v_1^2 u} = 2\rho^{-1}\overline{p\partial_{y_1} v_1} - 2\varepsilon_{v_1} + \nu\partial_x^2 \overline{v_1^2} \quad (4)$$
$$\partial_t \overline{v_2^2} + \partial_x \overline{v_2^2 u} = 2\rho^{-1}\overline{p\partial_{y_2} v_2} - 2\varepsilon_{v_2} + \nu\partial_x^2 \overline{v_2^2} \quad (5)$$

The two mixed turbulences decay in a similar way, as shown by the numerical simulations (Tordella & Iovieno, 2005). In the decay laws

$$E_1(t) = A_1(t+t_0)^{-n_1}, \quad E_2(t) = A_2(t+t_0)^{-n_2} \quad (6)$$

the exponents n_1, n_2 are close each other, which assures the constancy of $\mathcal{E}$ with respect to the time variable. In this analysis, we suppose $n_1 = n_2 = n = 1$, a value which corresponds to $R_\lambda \gg 1$ (Batchelor & Townsend, 1948).

In the absence of energy production, the pressure-velocity correlation has been shown to be approximately proportional to the convective fluctuation transport (Yoshizawa, 1982, 2002)

$$-\overline{pu} = a\rho\frac{\overline{u^3} + 2\overline{v_1^2 u}}{2}, \quad a \approx 0.10, \quad (7)$$

moreover all experiments show no appreciable difference in the second order moments in the mixing, i.e. $\overline{u^2} \simeq \overline{v_i^2}$, so that $\overline{u^3} - \overline{v_1^2 u} \simeq 2\rho^{-1}\overline{p\partial_x u}$ and consequently

$$-\rho^{-1}\overline{pu} = \alpha\overline{u^3}, \quad \alpha = \frac{3a}{1+2a} \approx 0.25. \quad (8)$$

In the initial value problem here considered, the moment distributions are determined by the coordinates x, t, and by the energy E and the macroscale ℓ of the two mixing turbulences. Thus, through dimensional analysis

$$\overline{u^k} = E_1^{\frac{k}{2}}\varphi_{u^k}(\eta, R_{\ell_1}, \vartheta_1, \mathcal{E}, \mathcal{L}) \;\; \forall k, \;\; \varepsilon_u = E_1^{\frac{3}{2}}\ell^{-1}\varphi_{\varepsilon_u}(\eta, R_{\ell_1}, \vartheta_1, \mathcal{E}, \mathcal{L}), \quad (9)$$

where $\eta = x/\Delta(t)$, $\Delta(t)$ is the mixing layer thickness, $R_{\ell_1} = E_1^{\frac{1}{2}}(t)\ell_1(t)/\nu$ is the Reynolds number relevant to the high energy turbulence, $\vartheta_1 = tE_1^{\frac{1}{2}}(t)/\ell_1(t)$ is the dimensionless time scale of the flow and $\mathcal{E} = E_1(t)/E_2(t)$, $\mathcal{L} = \ell_1(t)/\ell_2(t)$. It should be noticed that, if $n = 1$, $\mathcal{E}$, $\mathcal{L}$, $\vartheta_1 = n/f(R_{\lambda_1})$ and $R_{\ell_1} \propto t^{1-n}$ are constant (Batchelor 1953). By inserting relation (9) in (3), it is possible to deduce that $\Delta(t) \propto \ell_1(t)$. By putting $\Delta(t) = \ell(t)$, one obtains:

$$-\frac{1}{2}\eta\frac{\partial\varphi_{uu}}{\partial\eta}+\frac{1}{f(R_{\lambda_1})}(1-2\alpha)\frac{\partial\varphi_{uuu}}{\partial\eta}-\frac{\nu}{Af(R_{\lambda_1})^2}\frac{\partial^2\varphi_{uu}}{\partial\eta^2}= \\ =\varphi_{uu}-\frac{2}{f(R_{\lambda_1})}\varphi_{\varepsilon_u} \qquad (10)$$

Given the lateral boundaries of the mixing, which correspond to homogeneous conditions for the turbulence, one can observe that the **rhs** of (10) must be an odd function of η. It is zero in homogeneous (equilibrium) turbulence, while the previously mentioned experiments (Tordella & Iovieno, 2005) suggest that this **rhs** could be modelled by means of a diffusive term, so that

$$\frac{2}{f(R_{\lambda_1})}\varphi_{\varepsilon_u}-\varphi_{uu}=\beta\frac{\partial^2\varphi_{uu}}{\partial\eta^2} \qquad (11)$$

where β is a constant of proportionality; $\beta=0$ corresponds to the hypothesis of local equilibrium. In the following, by simply writing f instead of $f(R_{\lambda_1})$, the skewness, $S=\varphi_{uuu}/\varphi_{uu}^{3/2}$, reads

$$S=\frac{\varphi_{uu}^{-\frac{3}{2}}}{(1-2\alpha)}\left[\frac{f}{2}\int_{-\infty}^{\eta}\eta\frac{\partial\varphi_{uu}}{\partial\eta}\mathrm{d}\eta+\left(\frac{\nu}{A_1 f}-\beta f\right)\frac{\partial\varphi_{uu}}{\partial\eta}\right] \qquad (12)$$

By representing the second moments with the fitting curve given by the experimental distributions (Veeravalli-Warhaft, 1989 and Tordella & Iovieno, 2005)

$$\frac{3}{2}\varphi_{uu}=\frac{1+\mathcal{E}^{-1}}{2}-\frac{1-\mathcal{E}^{-1}}{2}\mathrm{erf}(\eta) \qquad (13)$$

one obtains

$$S=\frac{1-\mathcal{E}^{-1}}{\sqrt{\pi}}\frac{f}{4(1-2\alpha)}\left(\frac{3}{2}\right)^{\frac{1}{2}}\left(1-\frac{4\nu}{A_1 f^2}+4\beta\right)\mathrm{e}^{-\eta^2}\times \\ \left[\frac{1+\mathcal{E}^{-1}}{2}-\frac{1-\mathcal{E}^{-1}}{2}\mathrm{erf}(\eta)\right]^{-\frac{3}{2}} \qquad (14)$$

Figure 4 shows the good agreement of the modelled variance and skewness distributions (relations 13 and 14) with the experimental data. The intermittency parameter associated to the lateral penetration of the mixing is compared in fig.5 with the values given by the present similarity law. It can be observed that the scaling exponent deduced from the experiment (Tordella & Iovieno, 2005), which is approximately equal to 0.29, is correctly represented. It should be noticed that such scaling is independent from the energy-dissipation model (11), because the model coefficient β does not influence the shape of the skewness distribution (14) and does not modify the position of the skewness maximum, which appears to be a function of the energy ratio $\mathcal{E}$ only. However, β determines the value of the maximum of the skewness distribution, and $\beta\approx 0.08$ gives the best fit with experimental data by Tordella & Iovieno (2005). The other parameters that appear in figures 4 and 5 are $\alpha=0.25$ (see equation 8) and $f(R_{\lambda_1})=0.65$. This value has been obtained for $Re_{\lambda_1}=45$ from Sreenivasan (1998).

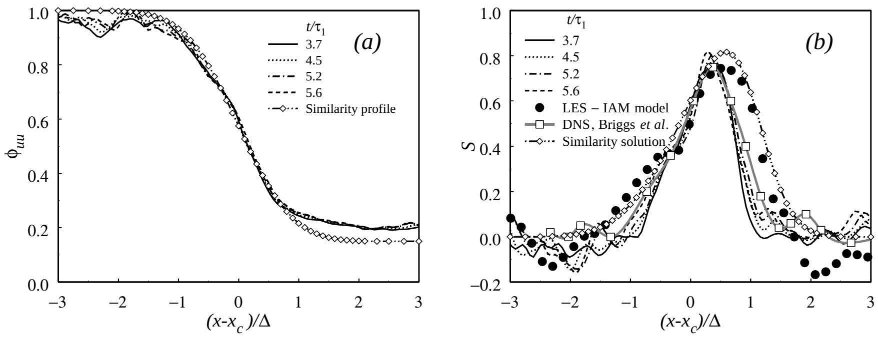

Fig. 4. Normalized energy and skewness distributions; $\mathcal{E} = 6.7$ and $\mathcal{L} = 1$.

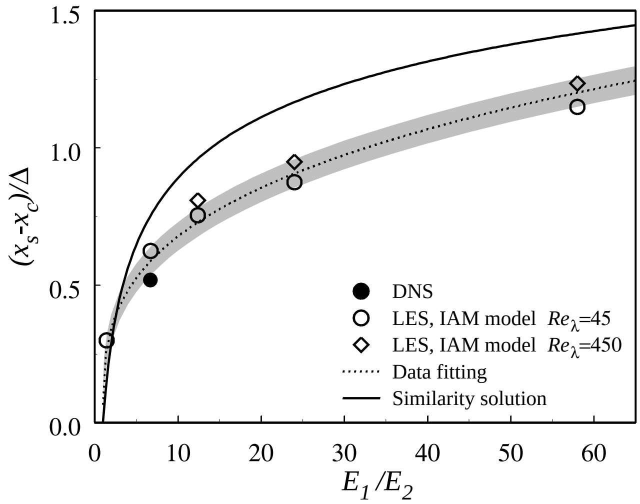

Fig. 5. Position x_s of the maximum of the skewness S distribution as a function of the initial ratio of energy $\mathcal{E} = E_1/E_2$ with $\mathcal{L} = 1$.

3 Conclusions

The present similarity analysis confirms our numerical experiment result where the turbulent transport is highly intermittent for shear-free decaying homogeneous isotropic interacting flows with kinetic-energy ratios far from unity in contrast to a Gaussian asymptotic state.

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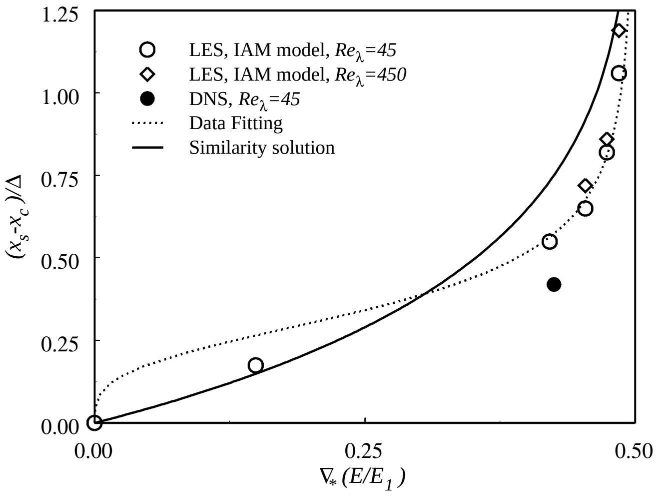

Fig. 6. Position x_s of the maximum of the skewness distribution as a function of the normalized initial gradient of energy $\nabla_* = \partial/\partial(x/\Delta)$, $\nabla_*(E/E_1) \simeq (1-\mathcal{E}^{-1})/2$.

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