Control-Oriented Wave Surface Elevation Forecasting Strategies: Experimental Validation and Comparison

Control-oriented wave surface elevation forecasting strategies: Experimental validation and comparison ⋆

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Abstract: Optimal control strategies are a key development step towards commercialization of wave energy converters (WECs). Most of these rely on optimization routines to find a suitable control action to maximize WEC power production. Nevertheless, most of these solutions make use of device dynamical models, with the free-surface elevation as the external (uncontrollable) input, effectively representing the incoming wave field. Consequently, predictive strategies, such as model predictive control, strongly depend on the availability of future wave information, and hence suitable forecasters are commonly used to 'restore' the causality of the optimization problem. Motivated by the intrinsic requirement of suitable forecasting strategies within optimal WEC control, this study provides a validation and comparison of different algorithms, including adaptive and non-adaptive techniques, based on experimental data. The paper focuses on the adaptability of each algorithm, which must be capable to fit properly each wave surface elevation signal, thus not affecting the optimality condition by providing poor prediction results.

Keywords: Forecast, wave spectrum, AR model, recursive least squares, wave energy.

1. INTRODUCTION

The recent necessity of finding CO_2 emission-free power production systems has significantly pushed research towards efficient renewable energy sources. Among them, solar and wind have reached a technological maturity, and are now facing the commercial stage. For what concerns wave energy, which is widely considered to be promising due to the vast potential of the wave source (Mattiazzo (2019)), current efforts are mostly focused on lowering the energy cost, and converging to a 'restricted' set of technology concepts (Guo and Ringwood (2021)). In this regard, optimal control algorithms have a paramount importance towards effective commercialization of wave energy converters (WECs) (Ringwood et al. (2014)).

Among state-of-the-art algorithms, model-based techniques (Faedo and Ringwood (2018), Li and Belmont (2014), Scruggs et al. (2013)) have proven their efficiency in optimizing power production, while respecting typical constraints of WEC systems. Nevertheless, such controllers often rely on knowledge of an explicit model of the device to control the system in a predictive fashion (Faedo et al. (2017)). Consequently, the optimal solution of this problem requires knowledge of the future wave elevation signal exciting the system (which is not known a-priori, see *e.g.* Scruggs et al. (2013)). Motivated by these considerations, a direct approach to the problem consists in providing such optimization routines with accurate, short-term prediction of the time series associated with wave elevation, in such a way that the control problem can be solved, without significantly departing from the (non-causal) theoretically optimal solution.

In the light of the role that forecasting techniques play in optimal control of WEC systems, this paper presents a validation using experimental data, and a subsequent critical comparison, of five different strategies (Peña-Sanchez et al. (2020)) to highlight their strengths and weaknesses. In this view, static forecasters are expected to perform more effectively, compared to the adaptive algorithms, when the wave spectrum is similar to that used for training. In contrast, adaptive strategies are expected to have a more constant performance over the validation set.

The reminder of this paper is structured as follows. In Section 2, the different algorithms are presented. Section 3 describes the experimental data and its main characteristics. In Section 4, the main results of this work are presented and subsequently discussed, and finally, in Section 5, the main conclusions are elucidated.

2. FORECASTING MODELS

In this section, a complete description of each wave surface elevation forecasting strategy is provided, to clearly expose the main characteristics associated with each technique, and the advantages and limitations of the different static and adaptive approaches considered.

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From now on, the term *static* for any predictor implies that the model parameters are computed offline usen a given data set, different from the one used for assessment and validation. In contrast, an *adaptive* approach trains dynamically its parameters on the simulation (validation) data set. The 5 strategies considered are a spectral optimal predictor, a one-step ahead auto regressive model (AR), and a direct multi-step ahead AR model, which belongs to the first subclass (static), while an adaptive one-step ahead AR model, and the recursive least squares AR model, are included within the adaptive family.

2.1 Static - spectral optimal predictor (OP)

Modeling the wave surface elevation in irregular sea states as a zero-mean, stationary stochastic process (within a time window of approximately 30 min) with slow varying power spectral density distribution function (PSD), is a well-known methodology among the wave research community (Peña-Sanchez et al. (2020)).

Such PSD can assume different shapes (Ryabkova et al. (2019)) according to the specific location of the wave resource, wind speed, wave length, wave height, and other parameters. As a consequence, each sea state can be modeled directly according to its spectral density function (SDF). For a given elevation signal, the auto covariance function can be defined as:

$$R_{\eta\eta}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \eta(t)\eta(t+\tau)\mathrm{d}t, \tag{1}$$

with $\eta(t) \in \mathbb{R}$ the wave surface elevation at a specific spatial location, and $t \in \mathbb{R}^+$ the time. Such distribution is unequivocally defined by the SDF function, since they constitute a Fourier transform pair (Kullback (1965)).

Following (Peña-Sanchez et al. (2020)), the theoretical optimal prediction strategy (OP) can be derived from the SDF, since the forecast for each sampling instant is defined directly by the frequency domain characterization of the time trace itself.

Defining

$$\eta_{k-h|k} = [\eta(k)\ \eta(k-1)\ \dots\ \eta(k-h+1)]^{\mathsf{T}}, \tag{2}$$

where $k = t/T_s \in \mathbb{N}$ is a given signal sample, $T_s \in \mathbb{R}$ is the sampling time, $h \in \mathbb{N}$ is the model order, and $\eta_{k-h|k} \in \mathbb{R}^h$ is the vector of past wave measured values. Defining $p \in \mathbb{N}$ as the prediction horizon, the p multi-step ahead forecast, namely $\hat{\eta}_{k+1|k+p}$, can be written as

$$\hat{\eta}_{k+1|k+p} = \theta_{\mathrm{OP}}\eta_{k-h|k}, \tag{3}$$

with $\theta_{\mathrm{OP}} \in \mathbb{R}^{p\times h}$ a matrix of parameters obtained starting from the SDF, where the superscript $\{\hat{\cdot}\}$ denotes the signal forecast. The reader can refer to Peña-Sanchez et al. (2020) for a discussion on the mathematical derivation of (3).

Remark 1. The structure in (3) defines a multi-step ahead prediction strategy, meaning that the error generated in the first wave forecast does not propagate in the subsequent prediction element. In contrast, single-step ahead minimization strategies do propagate such error. In particular, if a stable model is not obtained accordingly within the training stage, the error between prediction and actual signal can diverge.

2.2 Static - one-step ahead AR model (OSA)

An AR model defines the future value of the considered signal, with respect to the current discrete time instant k, as a linear combination of its previous measurements, *i.e.*

$$\hat{\eta}(k+1) = \sum_{i=0}^{h-1} \theta_{\mathrm{AR}}(i)\eta(k-i) + \varepsilon(i), \tag{4}$$

where $\theta_{\mathrm{AR}} \in \mathbb{R}^h$ are the prediction coefficients, and $\varepsilon(k) \in \mathbb{R}$ is a Gaussian distributed, zero mean, white noise.

The strategy consists in finding a suitable set of prediction parameters which, as described in (Peña-Sanchez et al. (2020)), can be obtained by minimizing the single-step ahead standard deviation of the prediction with respect to the training set (one-step ahead (OSA) strategy), which is essentially a least squares (LS) problem. In particular, the objective function can be written as

$$J_{\mathrm{OSA}} = \sum_{i=h+1}^{l} (\eta(i) - \hat{\eta}(i))^2. \tag{5}$$

Let the matrix $\Phi_k \in \mathbb{R}^{l\times h}$ be

$$\Phi_k = \left[\eta_{k-l-1|k-1}\ \eta_{k-l-2|k-2}\ \dots\ \eta_{k-l-h|k-h}\right]. \tag{6}$$

Then, the minimization in Eq. (5) can be expressed as

$$\theta_{\mathrm{OSA}} = \eta_{k-l|k}\Phi_k^{\dagger}. \tag{7}$$

where the symbol $\{\cdot\}^{\dagger}$ denotes the Moore-Penrose (pseudo) inverse operation.

Remark 2. If the forecaster is not able to ensure a bounded output signal, existence of a solution for the corresponding WEC optimal control problem may not be guaranteed. Actually, note that the optimization in Eq. (5) does not include, as part of its formulation, a stability condition (*i.e.* constraint) for the corresponding AR dynamical system, hence compromising the forecaster-controller response.

To prevent the condition discussed in Remark 2, it is customary to 'inject' an additional white noise signal in the AR model training dataset, in accordance with the corresponding signal energy, to ensure model stability.

Finally, note that, the corresponding forecast for the OSA strategy, up until the defined prediction horizon p, is obtained by repeating Eq. (7) recursively, taking as input the previous prediction value.

2.3 Static - direct multi-step ahead AR model (DMS)

The AR model can be alternatively obtained by minimizing each prediction error within the forecast horizon with a different set of parameters (direct multi-step (DMS) approach). In particular, the cost function is:

$$J_{\mathrm{DMS}} = \sum_{i=h}^{l-p} \sum_{j=1}^{p} (\eta(i+j) - \hat{\eta}(i+j|i))^2, \tag{8}$$

where the notation $\hat{\eta}(k+1|k)$ stands for the signal forecast obtained at time k. The minimization of (8) gives rise to a corresponding set of DMS parameters $\theta_{\mathrm{DMS}} \in \mathbb{R}^{p\times h}$.

As highlighted in Section 2.1, the wave free-surface elevation forecast can be equivalently written as in Eq. (3), using, in this case, θ_{DMS} as parameter matrix. As in the optimal predictor, this strategy has the advantage of avoiding a direct error propagation in each subsequent forecast step. Consequently, its set of values θ_{DMS} does not necessarily have to be associated to a stable system.

2.4 Adaptive - one-step ahead AR model (AOSA)

The adaptive version of the one step ahead AR model (AOSA) differs from that described in Section 2.2 in data set used to retrieve the model parameters. In particular, at each sampling time, a new wave sample is stored, and a LS problem is solved over past wave surface elevation values to obtain the corresponding (updated) model parameters.

The concept behind this approach is based on the assumption that recent (earlier) waves have a similar behavior to those following, so that a continue adjustment of the parameters should be able to track the underlying process of the current sea state, and hence adapt to varying conditions.

Note that the particular solution of this algorithm can be retrieved analogously to Eq. (7), with the adaptive feature obtained by updating $\eta_{k-l|k}$ and Φ_k at each time step, *i.e.*

$$\Phi_k \rightarrow \Phi_{k+1} = \begin{bmatrix} \eta_{k-l|k} \\ \eta_{k-l-1|k-1} \\ \vdots \\ \eta_{k-l-h+1|k-h+1} \end{bmatrix}^{\mathsf{T}}, \quad (9)$$

$$\eta_{k-l|k} \rightarrow \eta_{k-l+1|k+1}.$$

A critical aspect for this adaptive strategy is the lack of a methodology to guarantee model stability. However, as previously mentioned within this section, a suitable white noise signal can be applied to the model training set to help enforcing such property. Additionally, note that including noise can help towards a good numerical conditioning of the LS problem.

Another issue with the implementation of this strategy resides in the necessity for efficient real-time performance: since, as previously mentioned, forecast strategies in wave energy are vastly used for optimal control purposes, it is mandatory that their execution does not compromise real-time limits (often by keeping a small data set length l and model order h). However, it is well-known that, to obtain an accurate model, the dataset must be sufficiently representative (Golub (1965)), and the forecaster should have a sufficiently large order to capture any relevant dynamics (both improved by enlarging l and h). These principles are clearly in contrast, and a tuning phase is necessary. A common procedure consists in fixing the model order to reasonably reduce the fitting error with a pre-defined sufficiently large l, and then adjust the dataset length to preserve real-time performances.

2.5 Adaptive - recursive least squares AR model (RLS)

As an alternative adaptive strategy, a recursive least squares filter (RLS) (Diniz (2020)) estimates the AR model parameters by minimizing the one-step ahead prediction error, using these to forecast the corresponding wave elevation.

To estimate the AR model parameters, an approach similar to (Schlögl et al. (1997)) is adopted:

Algorithm 1 Recursive Least Squares

Initialization:
$\Theta^{h-1} = 0$;
while $k \geq h$ **do**
$e(k) \;\; = \eta(k) - \hat{\theta}^{k-1}_{\text{RLS}} \eta_{k-h-1|k-1}$;
$r(k) \;\; = \lambda^{-1} \Theta^{k-1} \eta_{k-h-1|k-1}$;
$K(k) = r(k) / (\eta^T_{k-h-1|k-1} r(k) + 1)$;
$\hat{\theta}^{k}_{\text{RLS}} = \hat{\theta}^{k-1}_{\text{RLS}} + K(k) e(k)$;
$\Theta^k \;\; = \lambda^{-1} \Theta^{k-1} - K(k) r(k)^{\mathsf{T}}$;
$k \;\; = k + 1$;
end while
Compute the prediction $\hat{\eta}_{k+1|k+p}$,

where $\Theta^k \in \mathbb{R}^{h \times h}$

$$\Theta^k = \left[\hat{\theta}^{k}_{\text{RLS}} \; \hat{\theta}^{k-1}_{\text{RLS}} \; \ldots \; \hat{\theta}^{k-h+1}_{\text{RLS}} \right], \quad (10)$$

is the matrix of previous estimated model parameters, and $\lambda \in \mathbb{R}^+$ is the so-called forgetting factor.

The main tuning parameter for this algorithm is the forgetting factor λ, constituting an essential step to guarantee convergence of the parameters in a reasonable time.

One of the main advantages of the RLS algorithm consists in the parameter speed computation: as a matter of fact, matrix inversion, which is a burdening operation for large datasets (Bellman (1965)), is not required by this algorithm, reducing the computational effort within the target machine. On the other hand, recursion inherently requires a convergence time to provide an accurate estimate of the corresponding parameters, since the initial guess can strongly influence the forecast performance. As such, an analysis based on the specific application case must be carried out to trade off convergence time and accuracy.

3. WAVE DATA SET

The different forecasting strategies have been evaluated on a retrieved data set from the sea-states considered within the first edition of the wave energy converter control competition (WECCCOMP) (Ringwood et al. (2019)).

Three different irregular sea states, generated in the wave tank facilities available at Aalborg University, Denmark, characterised in terms of three different JONSWAP spectra (Ryabkova et al. (2019)), are considered, with two realisations per sea state (indicated with a corresponding subscript). All the sea states share a peak shape factor of $\gamma = 3.3$. Table 1 reports the different wave scenarios in terms of significant wave height H_s and peak period T_p. Note that the raw data consists in a measure of the wave surface elevation, via wave probes placed within the tank.

Sea State	H_s [m]	T_p [s]
$SS1_1$	0.0625	1.412
$SS1_2$	0.0625	1.412
$SS2_1$	0.1042	1.936
$SS2_2$	0.1042	1.936
$SS3_1$	0.0208	0.988
$SS3_2$	0.0208	0.988

Table 1. Experimental data generation.

4. RESULTS

To provide a fair comparison between static and adaptive strategies, two main scenarios can be distinguished. The first scenario consists in performing the training of the static models on the same simulation (validation) data set, while the second one focuses on the performance when the validation set is not coincident with the training set. While it is appreciated that the former scenario (test) is not realistic, *i.e.* the same waves used for model identification never repeat again in an exact form during the prediction process, it gives an appraisal of the behavior associated with each predictor in 'idealised' conditions.

The main parameter chosen for performance assessment is the goodness of fit (GoF) of the prediction with respect to the original signal, *i.e.* the normalized root mean squared error (NRMSE) between the prediction at each time step, and the actual wave signal. Furthermore, an additional evaluation factor is included, in terms of the computational time employed by each strategy. The dataset is sampled at $f_s = 100$ Hz, with a time length of 300 [s] for each wave elevation signal.

The orders associated with each model, and prediction horizon, $T_{hor} = p/f_s$, are fixed, together with the noise characterization employed for model training. All relevant tuning parameters are reported in Table 2, where with SNR it is intended the signal-to-noise ratio. Additionally,

	OP	DMS	OSA	AOSA	RLS
Model order (h)	100	100	100	100	100
T_{hor} [s]	5	5	5	5	5
Noise SNR [dB]	70	70	70	70	70
Forgetting factor (λ^{-1})	/	/	/	/	10^{-3}
Online dataset Training length [s]	/	/	/	20	20

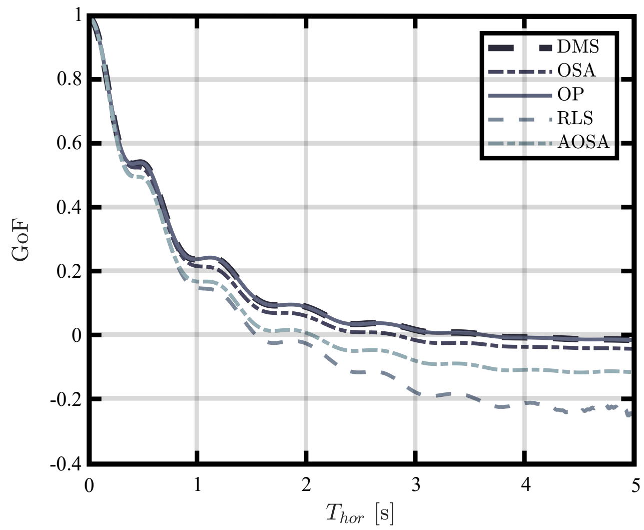
Table 2. Forecasting tuning parameters.

the prediction data considered to compute the evaluation parameters is collected starting from $t = 20$ [s], to 'fill' the dataset used by the adaptive strategies before effective evaluation.

Remark 3. This study does not focus on obtaining a 'best set' of parameters for each strategy, but aims at comparing the different forecasting alternatives over a set of *equivalent* conditions. Nonetheless, note that the model order h and the training noise have been chosen sufficiently large to guarantee convergence of each of the strategies considered, and to represent, with sufficient accuracy, the wave elevation surface process itself. The reminder of the tuning parameters have been chosen following the same considerations.

The first scenario, referred to as Test 1, is performed by training the static strategies on $SS1_1$, and using the same wave set for testing the effectiveness of each algorithm. The optimal predictor (OP) has been trained using the exact spectrum retrieved from the experimental free surface elevation data. Test 1, as illustrated in Fig. 1, shows the superior accuracy of the (OP), which maintains the highest GoF value for all considered prediction steps. Nonetheless,

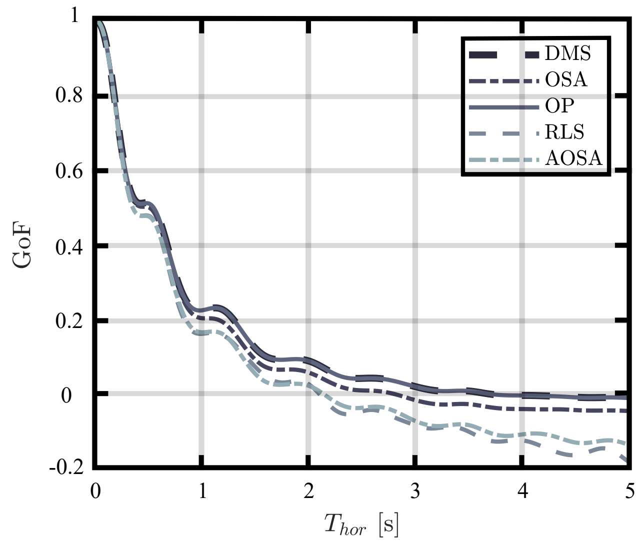
Fig. 1. Test 1 - GoF model comparison. Training set: $SS1_1$, Validation set: $SS1_1$.



it can be seen that the DMS strategy performs remarkably similar, almost matching the GoF obtained with the OP algorithm. In contrast, adaptive algorithms show a faster degradation of the overall performance (their training set must be reduced in dimension to fulfill real-time requirements and this implies a lower quality in the phenomenon characterization).

As a (more realistic) second evaluation test, referred to as Test 2, the static coefficients are trained on the $SS1_1$ data set, while the simulation is taken over the $SS1_2$ time series. Even within Test 2 (Fig. 2), the static strategies prove

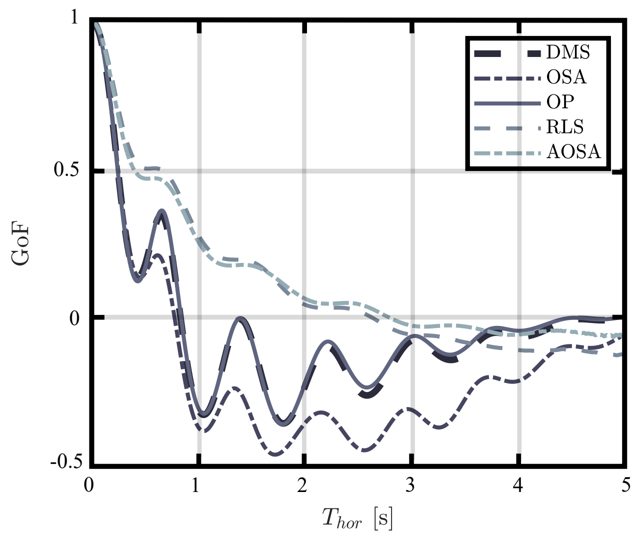
Fig. 2. Test 2 - GoF model comparison. Training set: $SS1_1$, Validation set: $SS1_2$.



their effectiveness in predicting sea states which present a similar SDF compared to the training data set, while adaptive strategies degrade faster in performance as the prediction horizon increases.

The third evaluation test considered (Fig. 3), termed Test 3, aims at reproducing the most common situation in real sea operation: the static forecasting models are computed on a given data set, while the validation data is taken from a completely different sea state. As a matter of fact, note that the sea spectrum condition changes realistically every 30 minutes (Ochi, 1998) and the AR static model cannot be re-trained using future (*i.e.* unknown) conditions, so their robustness to sea-state changes can be a fundamental issue, and must be analyzed. The validation data for Test 3 refers to $SS2_1$, having a different spectral representation with respect to that characterizing the training dataset $SS1_1$ (see Table 1).

Fig. 3. Test 3 - GoF model comparison. Training set: $SS1_1$, Validation set: $SS2_1$.



It is evident how a different wave sea state leads to performance degradation for the static strategies (due to the underlying spectral difference between sea conditions), while adaptive forecasters keep an almost constant performance across the different sea states, and manage to ensure a robust implementation independently from the operating condition.

As a final numerical appraisal of the performance for the considered forecasting strategies, Table 3 performance for each case according to the following. In particular, the parameter set computation for the static forecasters is fixed (computed over $SS2_1$), while the validation set changes accordingly. For this comparison, the GoF characterizing each strategy is compared with the best predictor available, *i.e.* the OP forecaster trained on the simulation signal itself (from now on termed GoF^{opt}). The evaluation is performed with a prediction horizon of 2 [s], using the following indicator:

$$\overline{\text{GoF}}_{\text{SS}i_j} = \text{GoF}^{\text{opt}}_{\text{SS}i_j} - \text{GoF}_{\text{SS}i_j}, \tag{11}$$

where the notation $\text{GoF}^{\text{opt}}_{\text{SS}i_j}$ refers to the performance of the corresponding OP forecaster trained on $\text{SS}i_j$, while $\text{GoF}_{\text{SS}i_j}$ indicates the actual predictor performance evaluated on $\text{SS}i_j$, and trained, for the case of static forecasters, using $SS2_1$. Note that, considering all the evaluation scenarios, it is evident that the performance of the static strategies degrade when changing sea spectrum, while adaptive algorithms are able to maintain a relatively constant fitting performance.

Sea State	DMS	OSA	OP	RLS	AOSA
$SS1_1$	0.215	0.268	0.209	0.131	0.103
$SS1_2$	0.215	0.266	0.208	0.065	0.066
$SS2_1$ (training)	10^{-5}	0.021	0	0.099	0.079
$SS2_2$	10^{-5}	0.021	0	0.116	0.081
$SS3_1$	0.132	0.095	0.135	0.112	0.053
$SS3_2$	0.129	0.095	0.133	0.030	0.059

Table 3. $\overline{\text{GoF}}$ as in (11).

Remark 4. Even if adaptive models do not rely on predefined data collections, from the results, it is evident how the models fit more accurately certain wave datasets than others. Such behavior can be reconducted to the necessity of finding a more suitable model order capable to better capture the process dynamics. Alternatively, the real-time dataset training length can be additionally tuned, allowing the parameter computation to adequately describe the current sea state spectrum. Note, although, that real-time requirements must be considered while performing such re-tuning.

The second evaluation parameter considered, as discussed earlier in this section, is the worst-case execution time [2] (WCET) over the simulation set, reported in Table 4.

	DMS	OSA	OP	RLS	AOSA
WCET	$42\mu s$	$151\mu s$	$45\mu s$	$286\ \mu s$	$8.86{\cdot}10^3\mu s$

Table 4. WCET for each algorithm step.

In particular, Table 4 clarifies one main advantage pertaining the set of static strategies: since the parameters are computed offline, the prediction is effectively obtained by simple matrix multiplications, having a mild impact on the overall execution time.

In contrast, despite the fact that both adaptive algorithms require a greater computation effort, it can be noticed that the recursive implementation of the RLS forecaster is still within real-time requirements (equal to $T_s = \frac{1}{f_s} = 10$ [ms] since the prediction is computed each sample time). Differently, the AOSA predictor, which requires matrix inversion at each step, is closer to the real-time limit.

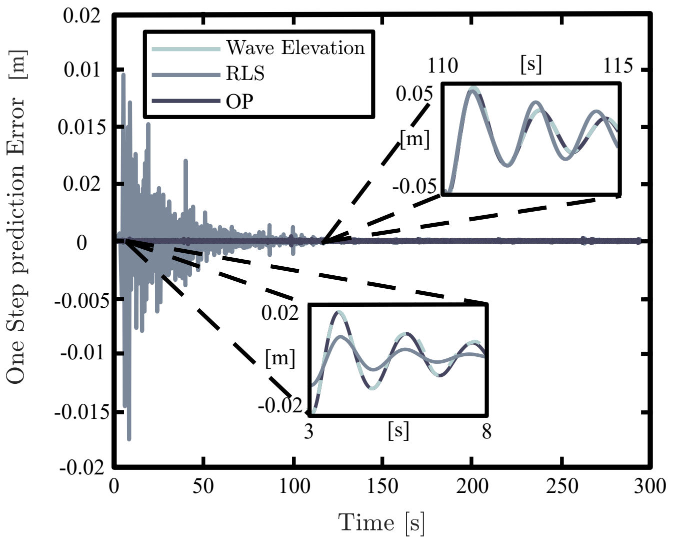
4.1 A note on the RLS algorithm

The RLS filter requires a given convergence time (Diniz (2020)), to achieve a one-step prediction error within acceptable limits. However, note that such error decreases over time until achieving a convergence value similar to the forecast error characterizing the optimal predictor.

Fig. 4 shows the deviation characterizing the RLS prediction as the time moves forward, compared to the OP

[2] Simulations are carried out on a laptop PC with a 12th Gen Intel(R) Core(TM) i7-12700H, 2700 Mhz CPU.

Fig. 4. One-step prediction error evolution over time: RLS and OP algorithm comparison.



one-step prediction error. Note that the RLS algorithm converges at around 100 [s], while the OP reaches low error values from the very beginning. As a direct consequence, the large one-step ahead prediction error may lead to poor forecast results, as remarked in the two sub-figures included within Fig. 4. Nonetheless, this fact does not necessarily influence operating performance, since, within a realistic deployment time in operative conditions, is sufficient to wait for the algorithm to converge before its inclusion in any WEC control loop.

5. CONCLUSIONS

This study presents a validation and comparison of different short-term wave elevation signal forecasters over a set of collected data from the sea-states considered within the first edition of the wave energy control competition (WECCCOMP). For such purposes, five different strategies, constituted by static and adaptive algorithms, are considered and analyzed in detail.

To evaluate each method, two indexes are chosen: the GoF, which provides a measure of the approximation quality associated with each forecaster, and the computational time characterizing each technique. It is found that the static strategies computed with exact knowledge of the operating sea-state, especially the optimal predictor (OP), showed their effectiveness in matching properly the future wave behavior. Nevertheless, if the validation (operational) scenario differs significantly from the training signal main characteristics (especially in terms of spectral representation), the adaptive approaches are able to consistently outperform the static models.

Concerning the computational burden, static strategies demonstrate mild requirements. Among adaptive predictors, RLS shows a more efficient implementation than the AOSA, despite the necessity of a minimum time for prediction error convergence.

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