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Abstract: Isofrequency topologies are studied for a gyroelectromagnetic medium obtained by stacking optically thin magnetized ferrite and semiconductor layers into a unified structure. In such a structure, both bi-hyperbolic and tetra-hyperbolic isofrequency contours appear as a simultaneous effect of both periodic arrangement of constitutive layers and external magnetic field influence. It is proposed to consider the obtained bi-hyperbolic and tetra-hyperbolic isofrequency contours as new topology classes of the wave dispersion.

A hyperbolic metamaterial (HMM) is a special type of artificial anisotropic structures whose isofrequency contours take the form of an open hyperboloid since the principal components of their constitutive electric or magnetic tensors have opposite signs. By tuning the shape of the hyperbolic dispersion, the propagation of light in HMMs may be flexibly controlled [1]. The hyperbolic dispersion exists also in natural media featuring gyroelectric (e.g., semiconductor) or gyromagnetic (e.g., ferrite) properties under the influence of an external static magnetic field. Under saturated magnetization, aforementioned media become extremely anisotropic in a specific frequency band due to the plasma or ferromagnetic resonance. This leads to the appearance of the hyperbolic topology of isofrequency contours related to particular waves.

In this Report, we demonstrate that the utilization of natural hyperbolicity inherent to magnetized ferrite and semiconductor materials in an artificial finely-stratified structure (superlattice) can result in arising new forms of isofrequency topology. We consider a gyroelectromagnetic medium as a HMM which is described by the constitutive relations $\vec{D} = \hat{\varepsilon}\vec{E}$ and $\vec{B} = \hat{\mu}\vec{H}$, where relative permittivity and permeability tensors are given by $\hat{\eta} = \{\eta_{xx}, \eta_{xy}, 0; -\eta_{xy}, \eta_{yy}, 0; 0, 0, \eta_{zz}\}$ $(\eta = \varepsilon, \mu)$. These tensors are the effective parameters obtained by applying the homogenization procedure (for details, see Ref. [2]) to a finely-stratified structure composed of magnetized ferrite and semiconductor layers [see Fig. 1(a)]. From Maxwell's equations, the dispersion equation, which describes propagation of electromagnetic waves through an unbounded gyroelectromagnetic medium, is derived as follows:

$$\begin{gathered}(\varepsilon_{zz}\mu_{zz})^{-1}\{k_x^4\varepsilon_{xx}\mu_{xx} + k_y^4\varepsilon_{yy}\mu_{yy} + k_z^4\varepsilon_{zz}\mu_{zz} + k_x^2k_y^2(\varepsilon_{xx}\mu_{yy} + \varepsilon_{yy}\mu_{xx}) \\ +k_x^2k_z^2(\varepsilon_{xx}\mu_{zz} + \varepsilon_{zz}\mu_{xx}) + k_y^2k_z^2(\varepsilon_{yy}\mu_{zz} + \varepsilon_{zz}\mu_{yy}) - k_0[k_x^2(\varepsilon_{xx}\varepsilon_{zz}\mu_\perp + \mu_{xx}\mu_{zz}\varepsilon_\perp) \\ +k_y^2(\varepsilon_{yy}\varepsilon_{zz}\mu_\perp + \mu_{yy}\mu_{zz}\varepsilon_\perp) + k_z^2\varepsilon_{zz}\mu_{zz}(\varepsilon_{xx}\mu_{yy} + \varepsilon_{yy}\mu_{xx} - 2\varepsilon_{xy}\mu_{xy})]\} + k_0^4\varepsilon_\perp\mu_\perp = 0,\end{gathered} \tag{1}$$

where $\varepsilon_\perp = \varepsilon_{xx}\varepsilon_{yy} - \varepsilon_{xy}^2$; $\mu_\perp = \mu_{xx}\mu_{yy} - \mu_{xy}^2$; k_x, k_y and k_z are the components of the wavevector.

We assume that the superlattice is composed of BaCo and doped Si layers (see material parameters in Ref. [3]). In the studied case, isofrequency topologies are mostly conditioned by values and signs of the three principal components μ_{xx}, μ_{yy} and ε_{zz} of the constitutive tensors [4]. Several representative regions on the k_0 scale are distinguished in Fig 1(b), namely: Region I where $\mu_{xx} > 0$ and $\mu_{yy} > 0$; Region II where $\mu_{xx} < 0$ and $\mu_{yy} < 0$; Region III where $\mu_{xx} < 0$ and $\mu_{yy} > 0$. In all cases $\varepsilon_{zz} < 0$.

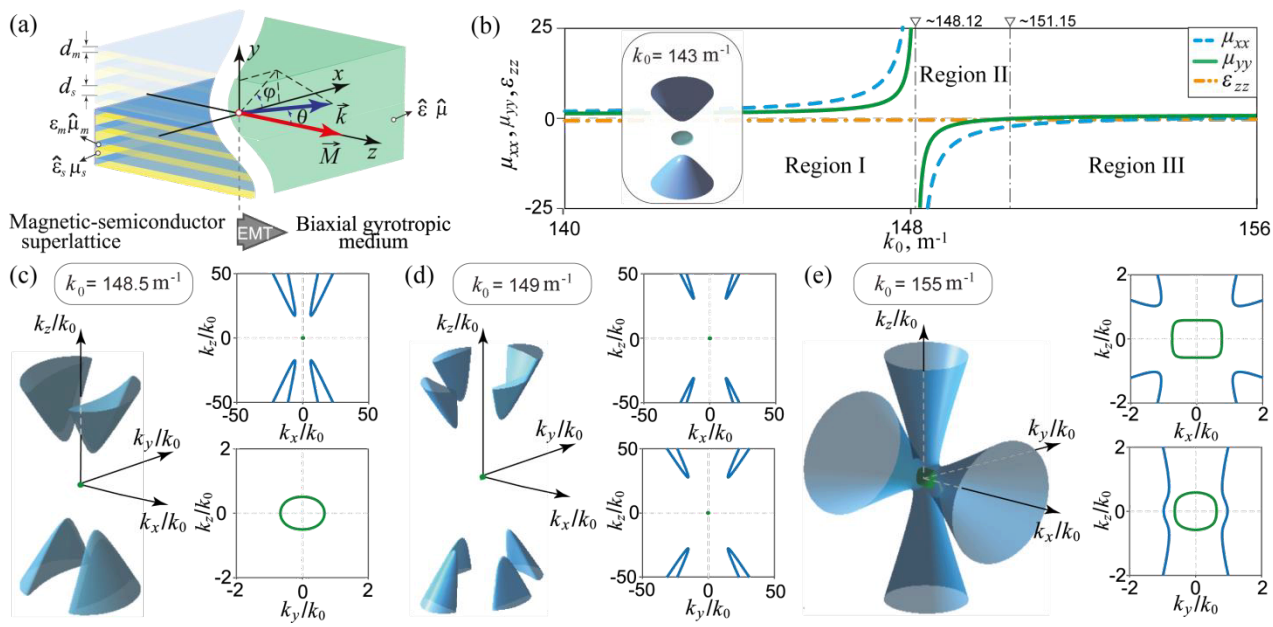

Figure 1: (a) The magnetic-semiconductor superlattice influenced by an external static magnetic field $\vec{M}$ and resulting homogenized biaxial gyrotropic medium. (b) Principal values of the components of relative permittivity ε_{zz} and relative permeability μ_{xx} and μ_{yy} versus k_0 at fixed values $d_m/d = 0.06$ and $d_s/d = 0.94$, where d is the period. (c-e) Topological forms of isofrequency surface, and their cross-sections, related to both extraordinary (blue surfaces) and ordinary (green surfaces) bulk waves propagating through the studied medium for (c, d) Region II and (e) Region III at the fixed values of k_0. The inset of panel (b) shows typical topological forms of isofrequency surfaces in Region I.

In Region I, the isofrequency surface for the extraordinary waves arises in the form of a two-fold Type I uniaxial-hyperboloid [Fig. 1(b)], which is caused by the hyperbolicity of semiconductor subsystem along the z-axis. In Region II, due to the influence of the external static magnetic field, there is a topological transition of the isofrequency surface to the form of cones cut into either two or four parts which are oriented along the z-axis. Obtained forms of isofrequency surface are attributed to the bi-hyperbolic topology [Fig. 1(c)] and tetra-hyperbolic topology [Fig. 1(d)], respectively. In Region III, for the extraordinary waves, the isofrequency contour is in the form of two one-fold Type II hyperboloid with orthogonal revolution axes [Fig. 1(e)]. This isofrequency contour is attributed to the bi-hyperbolic topology. The revealed novel types of topology appear due to the simultaneous effect of both the structure periodicity and an external magnetic field influence. They exist when both relative permittivity and permeability are tensor values, whose principal components have different combinations of signs.

References

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