

Foreword to the Special Issue in honour of Prof. Luigi Preziosi “Nonlinear mechanics: The driving force of modern applied and industrial mathematics”

*Original*

Foreword to the Special Issue in honour of Prof. Luigi Preziosi “Nonlinear mechanics: The driving force of modern applied and industrial mathematics” / Giverso, C.; Grillo, A.; Saccomandi, G.. - In: INTERNATIONAL JOURNAL OF NON-LINEAR MECHANICS. - ISSN 0020-7462. - 145:(2022), p. 104090. [10.1016/j.ijnonlinmec.2022.104090]

*Availability:*

This version is available at: 11583/2979639 since: 2023-06-27T15:34:35Z

*Publisher:*

Elsevier

*Published*

DOI:10.1016/j.ijnonlinmec.2022.104090

*Terms of use:*

This article is made available under terms and conditions as specified in the corresponding bibliographic description in the repository

*Publisher copyright*

(Article begins on next page)

# Foreword to the Special Issue in honor of Prof. Luigi Preziosi “Nonlinear mechanics: the driving force of modern applied and industrial mathematics”

Giuseppe Saccomandi<sup>a,b,\*</sup>, Chiara Giverso<sup>c</sup>, Alfio Grillo<sup>c</sup>

<sup>a</sup>*Dipartimento di Ingegneria, Università di Perugia, Via Goffredo Duranti, 93, 06125 Perugia PG, Italy*

<sup>b</sup>*NUI Galway, University Road, Galway, Ireland H91 TK33*

<sup>c</sup>*Dipartimento di Scienze Matematiche (DISMA) “G. L. Lagrange”, Politecnico di Torino, C.so Duca degli Abruzzi, 24, 10129 Torino TO, Italy*



Figure 1: Prof. Luigi Preziosi, in the occasion of his 60<sup>th</sup> birthday.

Mathematical modelling is a discipline pledged to identify problems, which may arise from virtually any branch of the human knowledge, and formalise them in the language of mathematics by developing methodologies of investigation framed within the appropriate rational framework. The problems addressed  
5 by mathematical modelling may stem from biology or physics, or from the study of anthropic phenomena, such as those pertaining economics, medicine, pharmaceuticals, social sciences, industry as well as the more recent contexts of big

---

\*

\*Corresponding author

*Email address:* [support@elsevier.com](mailto:support@elsevier.com) (Giuseppe Saccomandi)

data and artificial intelligence.

To pursue its goals, modelling must build connections with other mathematical sciences and, in the last decades, the accessibility to always richer computational resources has given the thrust for a tight combination of modelling and numerics. Indeed, on the one hand, mathematical models have become more descriptive, since they can rely on more efficient and robust numerical methods, which permit to account for several effects simultaneously. On the other hand, numerics has advanced under the stimulus of increasingly involved models, and has led to a better understanding of the investigated problems by improving the precision and the accuracy of simulations, and granting more realistic visualisations of the simulated problems.

While all this has applied to almost all fields of scientific and technical interest, the root of this intellectual viewpoint is to be considered in the framework of mechanics. Leonardo da Vinci (1452-1519) says (Codice E, f. 8 v)

*“La Meccanica è il paradiso delle scienze matematiche, perché in quella si viene al frutto matematico.”*<sup>1</sup>

Two major examples of the efficiency of the combination of modelling and simulations and of their tight relationship with nonlinear mechanics are industrial mathematics and the mathematics in biology and biomechanics.

At first sight, industrial mathematics is just a branch of applied mathematics focusing on problems that come from industry and it aims at determining solutions relevant to manufacturing, including the determination of the most efficient ways for solving those problems. However, it is clear that the increasing complexity and sophistication of modern industry involve technical issues that cannot be solved by just using simulation models. Indeed, they need true mathematical models grounded on well established *general laws*. Some relevant examples are petroleum engineering and hydrogeology, in which no real ad-

---

<sup>1</sup>“Mechanics is the paradise of the mathematical sciences, because by means of it one comes to the fruits of mathematics.”

35 vances are possible without a detailed knowledge of the constitutive modelling  
of non-Newtonian fluids, or environmental sciences, in which one has to handle  
multi-phase flow problems in the presence of phase transitions and/or coupled  
phenomena, or the detailed description of sand dynamics in the neighbourhood  
of railways in desert zones.

40 Although sprouting in the completely different context of biology, the adop-  
tion of mathematics to formalise problems of biological relevance has attracted  
many scientists all through the years. Besides the models typically encountered  
in biomathematics, such as those addressing population dynamics, epidemiology  
and related fields, a strong impact on the scientific community has been given  
45 by the combination of mathematical modelling with the mechanics of biolog-  
ical tissues, thereby giving rise to biomechanics. This branch of science grew  
considerably in the nineteenth and in the twentieth centuries, and especially  
in the second half of the latter one, when it gathered together a considerably  
big number of scientists coming from continuum mechanics. In the last thirty  
50 years, particular attention has been drawn on the problems of tissue growth  
and remodelling, especially in the context of the mechanics of bone and of “soft  
tissues”, such as tendons, ligaments, articular cartilage, blood vessels, organ  
tissues, cell cultures, etc. A huge step forward, in addition, was provided by  
the introduction of the mechanics of porous media and the theory of mixtures  
55 and multi-phasic materials in the realm of tissue biomechanics, thereby provid-  
ing the ground for formulating new models in which the mechanics of tissues  
and cellular aggregates could be studied in connection with fluid dynamics and  
chemistry. All this, in turn, suggested to develop new branches of research in  
which once again the driving force of the innovation is mechanics. Few examples  
60 in this respect are the mechanics of cell motion and migration, which necessar-  
ily relate to kinetic theories, the mechanics of the interactions of cells with the  
extracellular matrix, the conversion of mechanical signals into chemical stimuli,  
and the vast field of research referred to as mathematical oncology.

The generality and complexity of the above scenario are today under an in-  
65 tellectual “onslaught”. As pointed out by Coveney et al. [1], the growing appeal

exerted by disciplines like machine learning or the relatively new subjects pertaining to data sciences, like data analytics and big data, seems to suggest that the classical scientific method of investigation is no longer necessary. For this reason we need to emphasise once again the relationship between mathematical  
70 modelling and nonlinear mechanics. To understand this relationship, we first need a deep reflection about the “complementary” difference between a *model* and a *simulation*. The simulation of a real world phenomenon increases in usefulness with the quantity of specific data incorporated. For this reason, big data revolution and statistical methods may be very useful in this framework. On the  
75 other hand, the mathematical models should find the correct balance between the maximal level of complexity and the minimal level of detail that are necessary to preserve the essential outline of a given phenomenon. In this respect, modelling and situation are complementary. Whereas a simulation is a concretely descriptive approach that aims at the visualisation of one case at a time,  
80 a mathematical model is abstract and universal and it allows our intellectual activity to go over the actual set of observations and realisations. Mathematical models and simulations are fundamental tools in our system of knowledge for producing true innovation, because they can be used to *dissecare naturam* in the Baconian sense. However, in doing this, one should bear in mind that  
85 mathematical models are grounded on *laws* and not on *correlations*. Clearly, nowadays, we are far away from the reductionism paradigm, but, because we are far away from this approach, we are aware that it is not possible to *dissecare naturam* if we are not trained in a concrete way in mathematical modelling. It is well-known, in this respect, that the model *par excellence* in the humankind  
90 system is mechanics.

To date, we are still far away from an axiomatic *metatheory of models*<sup>2</sup>, although it is possible to be trained implicitly by operating in a tangible way in this activity. By slightly paraphrasing Piero Villaggio’s view of elastic structures

---

<sup>2</sup>See also <https://writings.stephenwolfram.com/2021/09/charting-a-course-for-complexity-metamodeling-ruliology-and-more/>.

[2], nonlinear mechanics “*is, by definition, the collection of all reasonable models*  
95 *[and mathematical methods], proposed during almost three centuries, concerned*  
*with simplifying the solution of problems involving [material] bodies*”, regardless  
of whether one considers the “*vibration of bars or plates, and the stability of*  
*columns*”, the motion of a fluid or the dynamics of a rigid body. The progress  
of nonlinear mechanics has been possible by the complete blend of experimental  
100 and theoretical ideas adopted by the human kind. nonlinear mechanics is a the-  
oretical discipline where your testing activity is the rule. Nonlinear mechanics  
is concrete because it addresses problems of interest in everyday life, with a  
widespread range of applications, at your home, in a factory or in a scientific  
laboratory. These are the reasons why nonlinear mechanics is the driving force  
105 of mathematical modelling.

Since it is not possible to present a theoretical corpus of all that, our idea is  
to present a special issue in which remarkable examples of the cited relationships  
are given by the solution of explicit problems. The aim of the present special  
issue is to put together a list of outstanding scientific papers giving clear con-  
110 nections among nonlinear mechanics, industrial mathematics, biomathematics,  
biomechanics and kinetic theories, in different fields of interest.

With reference to the general and industrial aspects of mathematical physics,  
this Special Issue presents five contributions, whose topics range from the char-  
acterisation of non-Newtonian fluids and viscoelastic materials to the thermo-  
115 dynamics of gases, from the modeling of swelling as the origin of “morphing”  
to traffic dynamics. In particular, the work by Fusi et al. [3] addresses the  
flow of “*shear-thinning fluids*” exhibiting Bingham-type behavior. Farina et al.  
[4] study the phenomena of “*creep, recovery and vibration of an incompressible*  
*viscoelastic material*” and consider, in particular, the response of the material  
120 in simple tension. Arima et al. [5] study the “*gases with internal structure*”  
in the context of Rational Extended Thermodynamics and investigate the mo-  
ments that are more suitable for their characterisation. The contribution by  
Colorado Cervantes et al. [6] deals with the “*morphing of soft structures*”, as-  
suming this process to be guided by “*active swelling*”. Finally, Chiarello et al.

125 [7] study “*macroscopic limits of car-following traffic dynamics*” by means of tools of statistical mechanics.

Concerning the application of mathematical models to biological and medical sciences, this Special Issue addresses the problems of cell motion [8, 9, 10], stability of viral capsids [11], neurodegenerative diseases [12], mechanical behaviour of biological tissues [13, 14] and cancer growth and invasion [15, 16, 17].  
130 Specifically, the works of Stotsky and Othmer [8] and Chelly et al. [9] investigate the transmission of “*intra- and extracellular forces*” to the cell membrane and their influence in determining cell shape [8] and “*cell crawling on a compliant substrate*” [9], respectively. The mathematical findings in [9] are confirmed by  
135 experiments with “*PDMS stamps*” designed ad-hoc [9]. Likewise, the work by Braun et al. [10] is inspired by laboratory experiments, developed to investigate the “*cross-talk between immune and cancer cells in a confined environment given by a microfluidic chip, the so called Organ-on-Chip*”. The calibration algorithm for a system of coupled reaction-diffusion-transport equations with chemotactic  
140 terms, proposed therein [10], could be of great interest also in other experimental settings. Another technique to infer the model parameters from the medical/biological data is also proposed in Medaglia et al. [15], using “*suitable numerical methods for uncertainty quantification of the resulting kinetic equations*” applied to tumour growth and response to therapeutic protocols. Lorenzi  
145 and Painter [16] extended the standard Keller-Segel model for chemotaxis-driven cell invasions to “*account for the possibility of phenotypic heterogeneity*” in the chemotactic term and proliferation rate of different cells inside the same population. The mathematical problem of describing a cellular population composed by cell with different genotypes and phenotypes is also investigated in Chiari  
150 et al. [17], by proposing a multiscale hybrid modelling framework in which a discrete structuring variable differentiates cells and “*a specific mathematical representation (i.e., individual/pointwise vs. collective/density-based) is assigned to each individual on the basis of its phenotypic hallmarks*”. Another way “*to bridge the gap between microscopic and macroscopic approaches*” is proposed  
155 in Sampaoli et al. [12], where the theoretical framework of mixtures theory is

used to derive a macroscopic Cahn-Hilliard type equation to describe the evolution of misfolded proteins inside the healthy brain, during neurodegenerative diseases. Looking at the mechanical behaviour of tissues at the macroscopic scale, the paper by Di Stefano et al. [13] provides an analytical model of focal adhesions and evaluates how the elasticity of an adhesion affects the “*transition from a ductile to a fragile decohesion regime*”. Moreover, Marzocchi and Musesti [14] propose a mathematical model of the skin, described as a hyperelastic material, and prove “*the existence and uniqueness of the solution for a general measure-valued external load*”. Finally, in [11], mathematical models for viral capsids present in the literature are reviewed, to identify “*fundamental structural units*” (i.e., either single proteins or groups of them) in “*coarse-grained mechanical models*”.

All the research themes addressed above are very dear to Prof. Luigi Preziosi, who has been brightly dedicating his scientific career to many of them. Indeed, this special issue of IJNLM is the *Festschrift* celebrating the 60th birthday of Luigi Preziosi. Luigi’s recognized research is a typical example of how mechanics may be the fuel for interesting applied mathematics. Luigi’s PhD thesis was about the mechanics of immiscible Newtonian fluids [18] and viscoelastic media [19]. For many years, Luigi has been engaged principally in the framework of non-Newtonian fluid mechanics [20] and in the study of heat propagation [21, 22]. Then, he moved to work on kinetic theories [23, 24] and on some industrial problems involving porous materials [25, 26, 27]. In the second half of the nineties of the last century, Luigi’s skills in mechanics allowed him to focus his research activity on the study of the interactions between tumour evolution and immune system [28, 29], with some innovative and breakthrough ideas. This was the beginning of a prolific activity in biomechanics and mathematical biology, which today allows to recognise Luigi as one of the top scientists in the field of tumour and cell mechanics [30, 31, 32, 33, 34, 35, 36, 37, 38]. Besides all this, Luigi has been involved in the development of Mathematical Physics along the Italian tradition, which once again is strictly related to classical mechanics, mainly at the *Politecnico di Torino*, but also all along Italy, where he has played



a major role in the organisation of the community of applied and industrial mathematics. Recently, Luigi's scientific activity has been awarded with his membership of the *Accademia dei Lincei*.

190 Last but not least, it must be clear that the most important quality of Luigi is that he is a true gentleman in science and life: Luigi is a true friend, an extremely loyal and collaborative colleague, always willing to help, and proactive in resolving problems.

### References

- 195 [1] P. V. Coveney, E. R. Dougherty, R. R. Highfield, Big data need big theory too, *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences* 374 (2080) (2016) 1 – 11.
- [2] P. Villaggio, *Mathematical Models for Elastic Structures*, Cambridge University Press, 1997.
- 200 [3] L. Fusi, A. Farina, K. R. Rajagopal, L. Vergori, Channel flows of shear-thinning fluids that mimic the mechanical response of a bingham fluid, *International Journal of Non-Linear Mechanics* 138 (2022) 1–10 (103847).
- [4] A. Farina, L. Fusi, F. Rosso, G. Saccomandi, Creep, recovery and vibration of an incompressible viscoelastic material of the rate type: Simple  
205 tension case, *International Journal of Non-Linear Mechanics* 138 (2022) 1–9 (103851).
- [5] T. Arima, M. C. Carrisi, S. Pennisi, T. Ruggeri, Which moments are appropriate to describe gases with internal structure in rational extended thermodynamics?, *International Journal of Non-Linear Mechanics* 137 (2022)  
210 1–11 (103820).
- [6] I. Colorado Cervantes, M. Curatolo, P. Nardinocchi, L. Teresi, Morphing of soft structures driven by active swelling: a numerical study, *International Journal of Non-Linear Mechanics* 141 (2022) 1–8 (103951).

- 215 [7] F. A. Chiarello, B. Piccoli, A. Tosin, A statistical mechanics approach to macroscopic limits of car-following traffic dynamics, *International Journal of Non-Linear Mechanics* 137 (2022) 1–11 (103806).
- [8] J. Stotsky, H. G. Othmer, How surrogates for cortical forces determine cell shape, *International Journal of Non-Linear Mechanics* 140 (2022) 1–15 (103907).
- 220 [9] H. Chelly, A. Jahangiri, M. Mireux, J. Étienne, D. Dysthe, C. Verdier, P. Recho, Cell crawling on a compliant substrate: A biphasic relation with linear friction, *International Journal of Non-Linear Mechanics* 139 (2022) 1–17 (103897).
- 225 [10] E. C. Braun, G. Bretti, R. Natalini, Parameter estimation techniques for a chemotaxis model inspired by cancer-on-chip (coc) experiments, *International Journal of Non-Linear Mechanics* 140 (2022) 1–16 (103895).
- [11] G. Indelicato, P. Cermelli, Coarse-grained mechanical models for viral capsids, *International Journal of Non-Linear Mechanics* [to appear in this VSI].
- 230 [12] S. Sampaoli, A. Agosti, G. Pozzi, P. Ciarletta, A toy model of misfolded protein aggregation and neural damage propagation in neurodegenerative diseases, *International Journal of Non-Linear Mechanics* [to appear in this VSI].
- 235 [13] S. Di Stefano, G. Florio, G. Napoli, N. M. Pugno, G. Puglisi, On the role of elasticity in focal adhesion within the passive regime, *International Journal of Non-Linear Mechanics* [to appear in this VSI].
- [14] A. Marzocchi, A. Musesti, Measure-valued loads for a hyperelastic model of soft tissues, *International Journal of Non-Linear Mechanics* 137 (2021) 1–5 (103826).
- 240 [15] A. Medaglia, G. Colelli, L. Farina, A. Bacila, P. Bini, E. Marchioni, S. Figini, A. Pichiecchio, M. Zanella, Uncertainty quantification and control of

- kinetic models of tumour growth under clinical uncertainties, *International Journal of Non-Linear Mechanics* 141 (2022) 1–14 (103933).
- [16] T. Lorenzi, K. J. Painter, Trade-offs between chemotaxis and proliferation shape the phenotypic structuring of invading waves, *International Journal of Non-Linear Mechanics* 139 (2022) 1–14 (103885).
- [17] G. Chiari, D. Morselli, M. Scianna, M. Delitala, A hybrid modeling environment to describe aggregates of cells heterogeneous for genotype and behavior with possible phenotypic transitions, *International Journal of Non-Linear Mechanics* [to appear in this VSI].
- [18] D. D. Joseph, L. Preziosi, Stability of rigid motions and coating films in bicomponent flows of immiscible liquids, *Journal of Fluid Mechanics* 185 (5) (1987) 323–351.
- [19] L. Preziosi, D. D. Joseph, Stoke’s first problem for viscoelastic fluids, *Journal of Non-Newtonian Fluid Mechanics* 25 (3) (1987) 239–259.
- [20] L. Preziosi, On an invariance property of the solution to stoke’s first problem for viscoelastic fluids, *Journal of Non-Newtonian Fluid Mechanics* 33 (2) (1989) 225–228.
- [21] D. D. Joseph, L. Preziosi, Heat waves, *Review of Modern Physics* 61 (1) (1989) 41–73.
- [22] L. Preziosi, An inverse “source-sink” problem for the nonlinear heat equation, *Mathematical and Computer Modelling* 17 (7) (1993) 3–11.
- [23] N. Bellomo, L. Preziosi, G. Forni, On a kinetic (cellular) theory for competition between tumors and the host immune system, *Journal of Biological Systems* 4 (4) (1996) 479–502.
- [24] L. Preziosi, L. Rondoni, Conservative energy discretization of boltzmann collision operator, *Quarterly of Applied Mathematics* 57 (4) (1999) 699–721.

- [25] D. Ambrosi, L. Preziosi, Modeling injection molding processes with deformable porous preforms, *SIAM Journal on Applied Mathematics* 61 (1) (2000) 22–42.
- 270
- [26] L. Preziosi, D. Fransos, L. Bruno, A multiphase first order model for non-equilibrium sand erosion, transport and sedimentation, *Applied Mathematics Letters* 45 (2015) 69–75.
- [27] A. Lo Giudice, L. Preziosi, A fully eulerian multiphase model of windblown sand coupled with morphodynamic evolution: Erosion, transport, deposition, and avalanching, *Applied Mathematical Modelling* 79 (2020) 68–84.
- 275
- [28] N. Bellomo, L. Preziosi, Modelling and mathematical problems related to tumor evolution and its interaction with the immune system, *Mathematical and Computer Modelling* 32 (3) (2000) 413–452.
- [29] D. Ambrosi, N. Bellomo, L. Preziosi, Modelling tumor progression, heterogeneity, and immune competition, *Journal of Theoretical Medicine* 4 (2002) 351243.
- 280
- [30] D. Ambrosi, L. Preziosi, On the closure of mass balance models for tumor growth, *Mathematical Models and Methods in Applied Sciences* 12 (5) (2002) 737–754.
- 285
- [31] L. Preziosi, A. Farina, On darcy’s law for growing porous media, *International Journal of Non-Linear Mechanics* 37 (3) (2002) 485–491.
- [32] H. Byrne, L. Preziosi, Modelling solid tumour growth using the theory of mixtures, *Mathematical Medicine and Biology* 20 (4) (2003) 341–366.
- [33] M. A. J. Chaplain, L. Graziano, L. Preziosi, Mathematical modelling of the loss of tissue compression responsiveness and its role in solid tumour development, *Mathematical Medicine and Biology* 23 (3) (2006) 197 – 229.
- 290
- [34] A. Tosin, D. Ambrosi, L. Preziosi, Mechanics and chemotaxis in the morphogenesis of vascular networks, *Bulletin of Mathematical Biology* 68 (7) (2006) 1819 – 1836.
- 295

- [35] A. Chauviere, T. Hillen, L. Preziosi, Modeling cell movement in anisotropic and heterogeneous network tissues, *Networks and Heterogeneous Media* 2 (2) (2007) 333 – 357.
- [36] C. Giverso, L. Preziosi, Modelling the compression and reorganization of cell aggregates, *Mathematical Medicine and Biology* 29 (2) (2012) 181 – 204.
- [37] M. Scianna, L. Preziosi, Multiscale developments of the cellular potts model, *Multiscale Modeling and Simulation* 10 (2) (2012) 342 – 382.
- [38] C. Giverso, S. Di Stefano, A. Grillo, L. Preziosi, A three dimensional model of multicellular aggregate compression, *Soft Matter* 15 (48) (2019) 10005 – 10019.